ON SURFACE PHOTOMETRY OF THE MOON

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Abstract. The surface photometric observations of the Moon made by various authors are compared with the author's theoretical scattering law (Lumme, 1971), and a good agreement is obtained. The integrated brightness of the Moon has also been calculated from these different sets of data and then compared with the observed brightness. Some differences between the different observations were found.

1. Introduction

Of the non-atmospheric bodies in the solar system, it is possible to make reliable surface photometric observations only for the Moon. Therefore, it is interesting to investigate how the photometric function of the Moon varies as a function of the brightness longitude at a constant phase angle.

There are several papers by various authors about the photometric functions of different kinds of the lunar soil. Common to all these observations is that no systematic variations as a function of the brightness latitude have been found.This is exactly the case which can be predicted theoretically if no systematic variations exist between the polar and equatorial regions in the Moon's soil.

The aim of the present paper is to investigate the extent to which the theoretical scattering law, given by the present author (Lumme, 1971) for the non-atmospheric bodies in the solar system, can explain the existing surface photometric observations. In the said paper (hereafter called Paper 1) it was found that at least the observed integrated brightness of the Moon – when the opposition effect correction observed by Apollo 8 is taken into account – can well be explained by the theoretical law, and that the agreement is much better than in the case of Hapke's law (Hapke, 1963).

In the following sections only the brightness variations at a constant phase will be considered. The treatment of local phase-variations must still be left for the future, when these will have been observed with sufficiently small phase angles ($\alpha \leqslant 1\overset{\circ}{.}5$).

2. Theoretical Scattering Law for Lunar Surface Photometry

In Paper 1 a scattering law for a porous surface layer was given (Equation (14)). Introducing for the optical thickness of the layer $\tau_0 \to \infty$, it follows that

$$I(\alpha, \imath, \varepsilon) = b\pi F \Phi(\alpha) \frac{\cos \imath}{\cos \imath + \cos \varepsilon} \Psi(\eta), \tag{1}$$

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where

$$\begin{aligned} \eta &= \frac{\cos \iota + \cos \varepsilon}{x\sqrt{\cos^2 \iota + \cos^2 \varepsilon - 2\cos \iota \cos \varepsilon \cos \alpha}}, \\ \Psi(\eta) &= \sum_{k=0}^{\infty} \left(\frac{\eta}{2}\right)^k \frac{\Gamma(1+\eta)}{\Gamma(1+\eta+k)}, \end{aligned} \tag{2}$$

Where ι is the angle of incidence, ε the angle of reflection, α the phase angle, πF the incident solar flux, $\Phi(\alpha)$ the normalized phase function of an individual surface

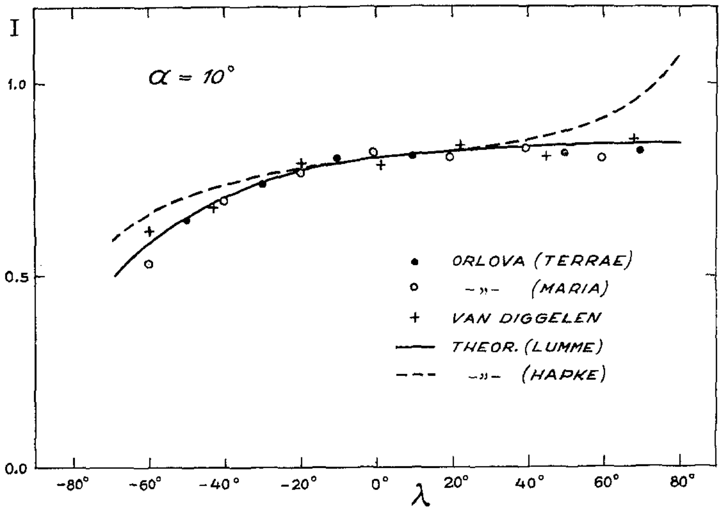

Fig. 1.

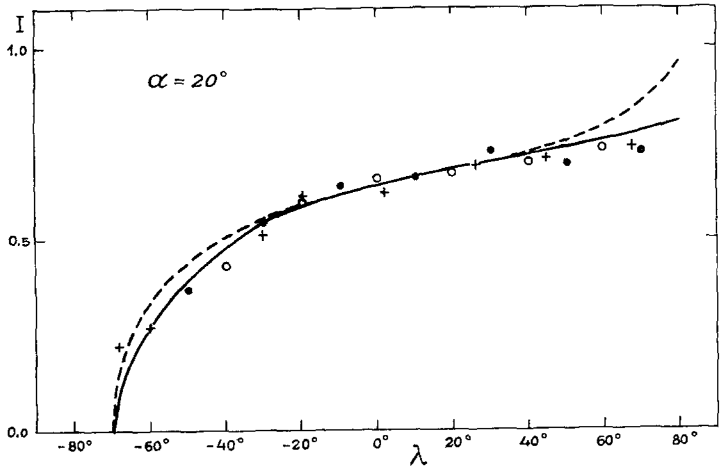

Fig. 2.

particle and Γ the ordinary gamma function. The parameter x describes how densely the surface matter is packed. In the case of a solid surface, $x=0$; if there is no matter at all, $x \to \infty$. The brightness longitude λ and latitude β are defined by

$$\begin{aligned} \cos \imath &= \cos\beta \cos(\lambda-\alpha), \\ \cos\varepsilon &= \cos\beta\cos\lambda . \end{aligned} \tag{3}$$

Inserting these into Equation (1) we obtain

$$I(\alpha,\lambda) = b\pi F\Phi(\alpha)\frac{\cos(\lambda-\alpha)}{\cos\lambda+\cos(\lambda-\alpha)}\Psi(\eta), \tag{4}$$

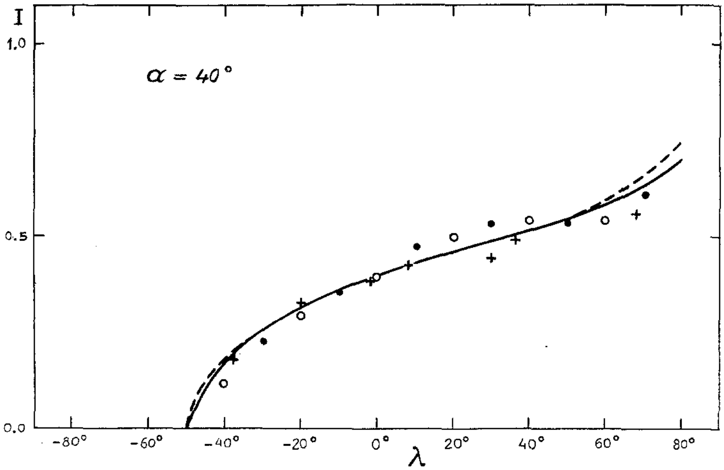

Fig. 3.

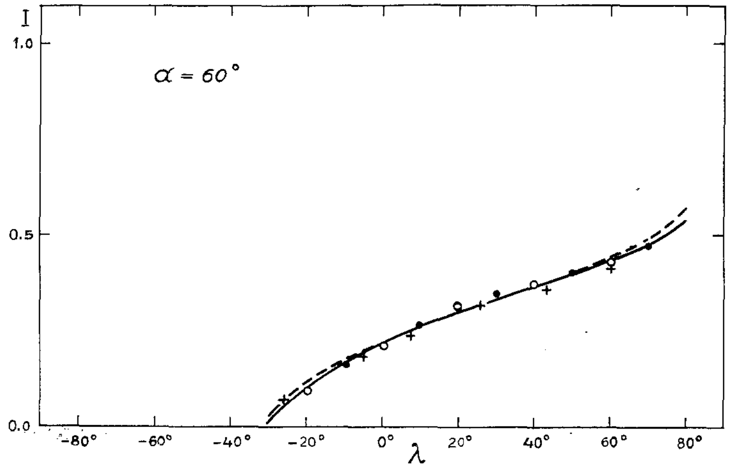

Fig. 4.

Figs. 1–4. Comparison of the theory presented by the present author with Orlova's and Van Diggelen's observations and with Hapke's theory.

where

$$\eta = \frac{\cos\lambda + \cos(\lambda - \alpha)}{x \sin\alpha}. \tag{5}$$

3. Comparison with Observations

Equation (4) will now be compared with the observations. As already mentioned, the comparison is carried out by keeping α constant. Then the phase function $\Phi(\alpha)$ is constant for every set of α values, and the only parameter is x. But, according to Paper 1, the integrated brightness of the Moon can be nicely fitted with the theory if $x=8$. Further, because Equation (4), when the extreme cases $x=0$ and $x \to \infty$ are excluded, is rather insensitive for x, this same value for x is now adopted. It is also interesting to compare the different sets of observations with each other. In doing so we calculate the integrated brightness $L(\alpha)$ for every set from

$$L(\alpha) = B \int\limits_{\alpha-\pi/2}^{\pi/2} I(\alpha, \lambda) \cos\lambda \, \mathrm{d}\lambda, \tag{6}$$

where B is a normalizing constant and $I(\alpha, \lambda)$ the observed surface brightness. To smooth the observations, we assume for I the parabolic form

$$I(\alpha, \lambda) = a_0(\alpha) + a_1(\alpha)\lambda + a_2(\alpha)\lambda^2, \tag{7}$$

where the coefficients $a_k(\alpha)$ can be calculated for every set separately by means of the least-squares procedure. The values of $a_k(\alpha)$ found are given in Table I. If λ is expres-

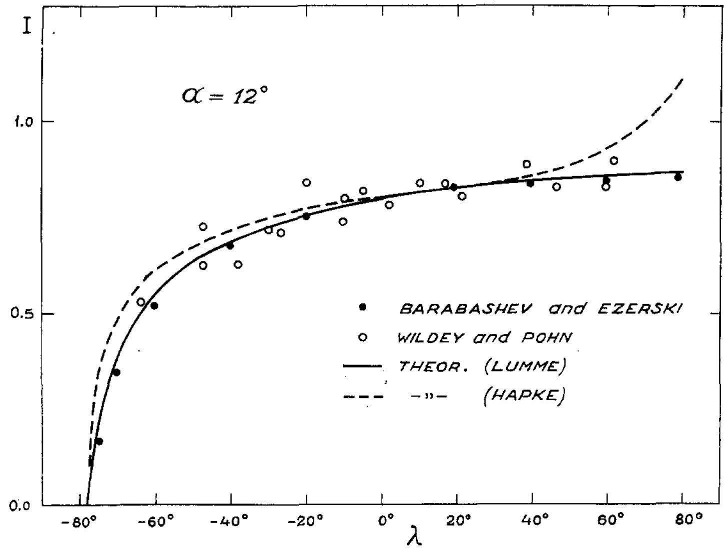

Fig. 5.

sed in degrees, an insertion of (7) into (6) gives

$$L(\alpha) = B\left\{a_0(\alpha)\,(1+\cos\alpha) + \frac{180}{\pi}\,a_1(\alpha)\left[\frac{\pi}{2} - \left(\frac{\pi}{2} - \alpha\right) - \sin\alpha\right] + \left(\frac{180}{\pi}\right)^2 a_2(\alpha)\left[\left(\frac{\pi}{2}\right)^2 - 2 + 2\left(\frac{\pi}{2} - \alpha\right)\sin\alpha + \left(\left(\frac{\pi}{2} - \alpha\right)^2 - 2\right)\cos\alpha\right]\right\}. \qquad (8)$$

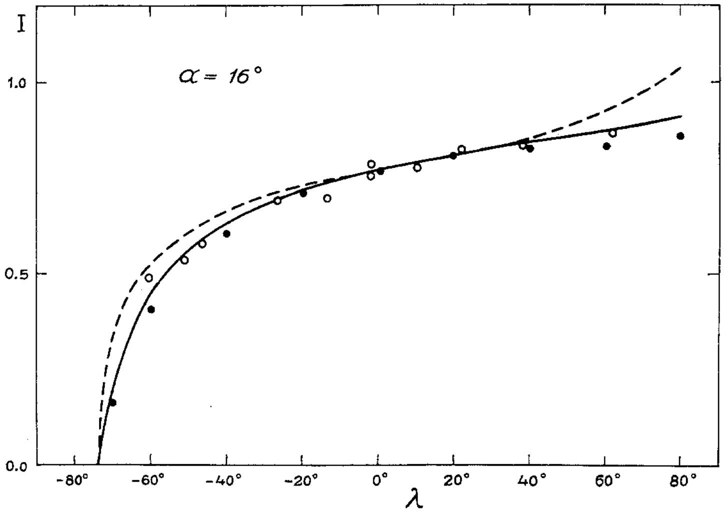

Fig. 6.

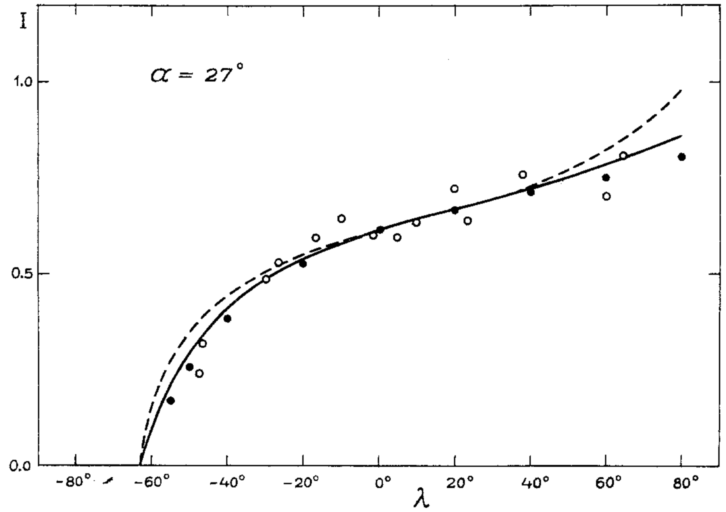

Fig. 7.

Figs. 5–7. Comparison of the theory presented by the present author with Barabashev's and and Ezerski's, and Wildey's and Pohn's observations, and with Hapke's theory.

TABLE I

The coefficients $a_k(\alpha)$ as obtained by a least-squares procedure, for different sets of observations

	Orlova Terrae			Orlova Maria			Van Diggelen			Wildey and Pohn		
α	a_0	$a_1 \times 10$	$a_2 \times 10^3$	a_0	$a_1 \times 10$	$a_2 \times 10^3$	a_0	$a_1 \times 10$	$a_2 \times 10^3$	a_0	$a_1 \times 10$	$a_2 \times 10^3$
10°	83.0	1.65	−2.18	79.0	1.90	−3.84	80.3	1.26	−1.27			
12°										80.4	2.22	−1.40
16°										77.7	2.82	−2.35
20°	65.9	3.38	−4.04	60.6	3.15	−5.07	63.6	3.35	−3.79			
27°										61.9	3.90	−1.92
30°	50.7	4.53	−4.68	45.3	4.00	−4.95	51.3	4.30	−4.11			
40°	37.6	4.87	−4.04	33.6	4.32	−3.96	40.0	4.69	−4.04			
50°	27.2	4.90	−3.31	24.5	4.57	−3.34	30.3	5.32	−4.26			
60°	18.6	4.53	−2.26	17.5	4.25	−1.86	22.0	5.02	−2.82			
70°	11.5	4.17	−1.37	12.0	4.20	−1.48	15.2	5.23	−3.22			

The observations which we shall use in the present analysis are: Orlova's tables (1956) for lunar terrae and maria. In these tables Orlova gives the surface brightness along the Moon's photometric equator as a function of ι and ε. The corresponding values for α and λ can be calculated from Equation (3). These observations show, however, large deviations form Minnaert's reciprocity theorem (Minnaert, 1961), and that is why we shall calculate the mean brightness $\bar{I}$ as

$$\bar{I}(\iota, \varepsilon) = \tfrac{1}{2}\left[I(\iota, \varepsilon)\frac{\cos\varepsilon}{\cos\iota} + I(\varepsilon, \iota)\right]. \tag{9}$$

Van Diggelen (1959) has constructed mean lunation curves for some selected craters. In the following we assume that no systematic differences exist between the left and right-hand side of the Moon and, therefore,

$$\bar{I}(\alpha, \lambda) = \tfrac{1}{2}[I(\alpha, \lambda) + I(-\alpha, -\lambda)], \tag{10}$$

where the minus sign refers to phases before the full Moon. Also Wildey and Pohn (1964) have made observations from some lunar formations – mainly craters. The defect in these observations is, however, that the phase angle interval is rather narrow, *i.e.*, $2° \leqslant \alpha \leqslant 27°$, and that the scattering of the values obtained in the individual points is remarkably large. For these observations we also use Equation (10). Barabashev and Ezerski (1965) constructed their mean lunation curves by reducing and homogenizing the extensive photometric work of Fedoretz (1952) and that is why no corrections are now needed. Lately, Jones (1969) has given data of 199 lunar surface features at five values of the phase angle. He has used a least-squares solution to fit a polynomial of the fourth degree to the observations. Comparison of the observations with theory is given in Figures 1–9. Inspection of these immediately reveals the large deviations of Hapke's model from the observations and from the present theory, particularly at large values of λ and small values of α. Figures 10 and 11 give the observed integrated brightness (see Paper 1) compared with the calculated ones (Equation (6)). It is seen that the lunation curves by Van Diggelen and Barabashev and Ezerski agree quite well, while in the other cases the agreement is rather poor. Jones's observations, although they fit quite well with the surface photometry, seem to agree so poorly with the integrated brightness that they have been completely omitted.

It is now possible to calculate from Equation (4) a table for the surface brightness of the Moon as a function of α and λ. To do this we must first evaluate the function $\Phi(\alpha)$ from

$$\frac{L(\alpha)}{L(0^{0})} = \tfrac{1}{2}\Phi(\alpha) \int\limits_{\alpha-\pi/2}^{\pi/2} \frac{\cos(\lambda-\alpha)\cos\lambda}{\cos\lambda + \cos(\lambda-\alpha)}\,\Psi(\eta)\,\mathrm{d}\lambda, \tag{11}$$

where the left-hand side stands for the observed integrated brightness. The results are given in Table II.

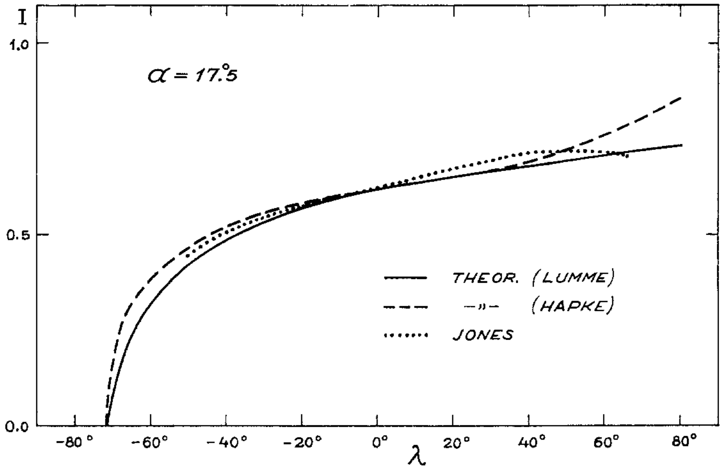

Fig. 8.

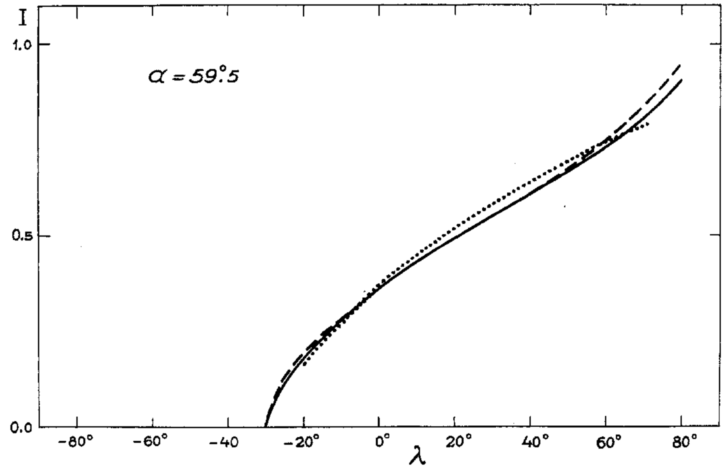

Fig. 9.

Figs. 8–9. Comparison of the theory presented by the present author with Jones' observations and with Hapke's theory.

4. Discussion

The theoretical scattering law for a porous surface layer developed in Paper 1 also seems to give a satisfactory explanation for the surface photometric observations of the Moon. However, it must be emphasized that all the different sets of observations were first corrected to take into account both the reciprocity theorem and the assumed similarity between the two halves of the Moon. Therefore nothing can be said about possible differences in photometric behaviour between the halves. It is also interesting

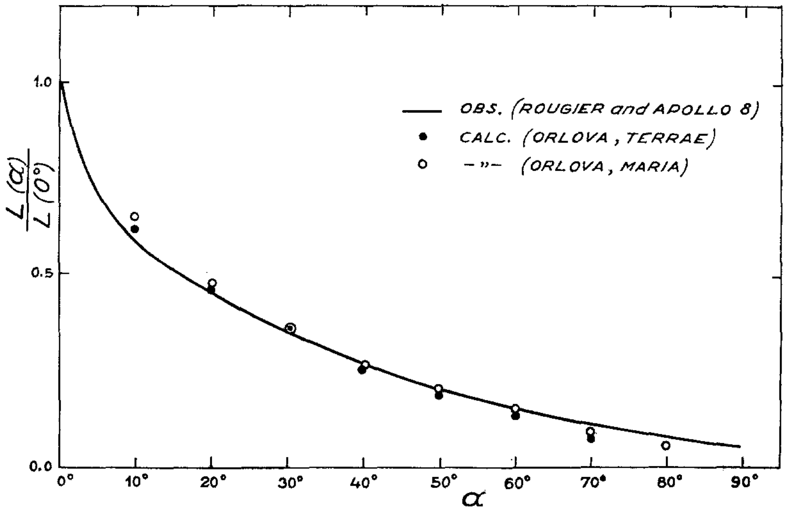

Fig.10.

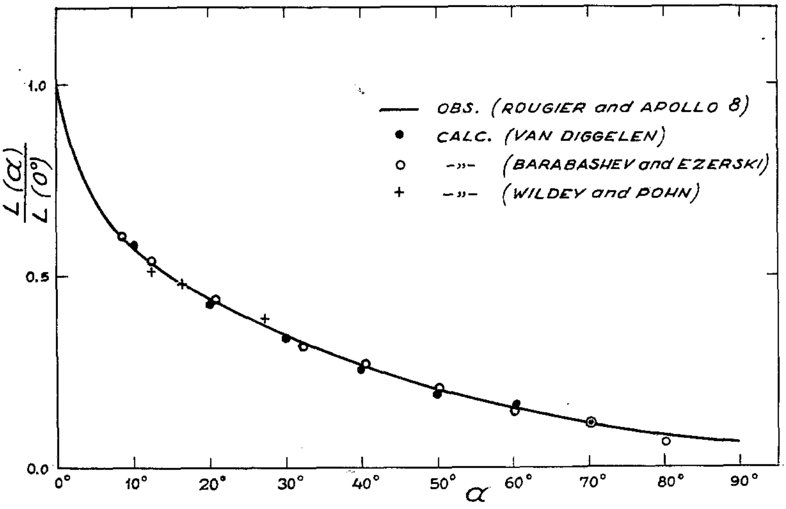

Fig. 11.

Figs. 10–11. Observed integrated brightness of the Moon as compared with the calculations when the surface photometric observations are used (Equation (6)).

to note that the integrated brightness calculated from the mean lunation curves of the craters (Van Diggelen) agrees well with the measured brightness.

The scattering law given might also be suitable for use in the photometry of Mercury and Mars. We already know the great similarity between the Moon and Mercury, but in the case of Mars the situation is far more complicated because of the atmosphere of this planet. The law given above could be used together with the atmospheric scattering to explain the observed opposition effect of Mars.

TABLE II

Normalized surface brightness of the Moon as a function of α and λ

$\lambda \backslash \alpha$	0°	10°	20°	30°	40°	50°	60°	70°
−70°	1.00	0.37	0.00					
−60°	1.00	0.44	0.23	0.00				
−50°	1.00	0.49	0.31	0.16				
−40°	1.00	0.53	0.37	0.24	0.12	0.00		
−30°	1.00	0.56	0.41	0.29	0.19	0.09	0.00	
−20°	1.00	0.58	0.44	0.33	0.23	0.15	0.08	0.00
−10°	1.00	0.60	0.46	0.36	0.27	0.19	0.13	0.08
0°	1.00	0.61	0.48	0.38	0.30	0.23	0.17	0.12
10°	1.00	0.62	0.50	0.40	0.33	0.25	0.20	0.15
20°	1.00	0.62	0.52	0.42	0.35	0.28	0.23	0.19
30°	1.00	0.63	0.53	0.44	0.37	0.30	0.26	0.22
40°	1.00	0.63	0.54	0.45	0.39	0.33	0.28	0.25
50°	1.00	0.64	0.55	0.47	0.41	0.35	0.31	0.28
60°	1.00	0.64	0.57	0.50	0.44	0.38	0.34	0.31
70°	1.00	0.65	0.59	0.53	0.47	0.41	0.37	0.34
80°	1.00	0.66	0.61	0.57	0.52	0.47	0.42	0.39

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