# A PHOTOMETRIC AND POLARIMETRIC STUDY OF THE MOON'S SURFACE 

NAOSUKE SEKIGUCHI<br>Tokyo Astronomical Observatory, Mitaka, Tokyo, Japan

(Received 10 January, 1972)


#### Abstract

The results of the photometric and polarimetric observation of the Moon's surface with high resolution from October 1969 to March 1971 are discussed. It is found that, for a wide part of the Moon's illuminated surface, the brightness $m$, expressed by visual magnitude per square second of arc, can be expressed as


$$
m=5.0-2.5 \log H+2.5 \log \left\{P_{v} / P(\alpha)\right\}
$$

where $H$ is the variable part of Hapke's formula, $P_{v}$ is the observed polarization degree, and $P(\alpha)$ is a function of the phase angle $\alpha$. The color shows a tendency to reddening at the enhancement of the lunar brightness, and this is expressed by the relation

$$
B-V=-0.2 \Delta m+1.7
$$

where $A m=m+2.5 \log H$ which is a sort of residual from the brightness calculated by Hapke's formula. Remarkable enhancements of the lunar brightness correspond to solar flares which appeared just before the observations. Because of this the above formula for the observed brightness can be interpreted by assuming the luminescence on the lunar surface stimulated by the solar activity.

## 1. Introduction

The photometric and polarimetric observations of the Moon's surface with a high resolution had been carried out for one year and a half since the autumn of 1969, by means of the $91-\mathrm{cm}$ reflector of the Dodaira Station of the Tokyo Astronomical Observatory. During these observations, the author found a phenomenon which was likely to be an anomalous brightening of the lunar surface on March 26, 1970, and published it in a previous paper (Sekiguchi, 1971).

The present paper contains the final results of these observations together with a discussion of them. The present observations enabled us to resolve the Moon's light from a very confined area, and the differences in brightness and the polarization degrees between various areas of fine structures were investigated. As shown and discussed in the later sections, the variations in the brightness of the Moon's surface have a correlation with those of the polarization degrees, and this should be interpreted as the existence of the luminescence on the lunar surface. This correlation was first pointed out by Teyfel $(1960)$, and $\operatorname{Kopal}(1965,1969)$ emphasized that the polarimetric observations offer one persuasive evidence as to the existence of the luminescent component in moonlight. The present study aims to establish the existence of lunar luminescence stimulated by solar activity.

## 2. Results of the Observations

As the detailed descriptions of the used instruments and the method of the observations were mentioned in the previous paper (Sekiguchi, 1971), only brief remarks chiefly concerning some modifications to the apparatus and methods mentioned in the former publication are given here. The diaphragm has been changed since the beginning of 1971 because it suffered from some dirtiness. All observed brightness of the Moon's surface was reduced to the visual magnitude per one square second of arc. The observed polarization degrees were always based on the observations through the V-filter, therefore they are denoted as $P_{v}$.
All the available results are listed in the Table I. The first column shows the observed dates, and the second the universal times of the observations through the $V$-filter (not the times through the $B$-filter). Results from a single set of scannings of the lunar surface are grouped in a block. $\xi$ and $\eta$ are the projected rectangular coordinates of the observed points, referred to in the Orthographic Atlas of the Moon by Arthur and Whitaker (1960). B-V are the color index referred to in the ArizonaTonantzintla Catalogue (Iriarte et al., 1965). $m$ signifies the measured brightness of the Moon's surface, expressed by the visual magnitude per one square second of arc referred to the same catalogue. $\Delta m$ is a residual from the calculated brightness by Hapke's formula (Hapke, 1963). It is defined by the following expressions:

$$
\begin{equation*}
\Delta m=m+2.5 \log H \tag{1}
\end{equation*}
$$

where $H$ is the variable part of Hapke's formula, namely,

$$
\begin{align*}
& H=\frac{\sin \alpha+(\pi-\alpha) \cos \alpha}{1+\frac{\cos \varepsilon}{\cos i}} B(\alpha, g)  \tag{2}\\
& B(\alpha, g)=2-\frac{\tan \alpha}{2 g}\{1-\exp (-g / \tan \alpha)\}\{3-\exp (-g / \tan \alpha)\} \\
& \quad \text { if } \alpha<\frac{\pi}{2}, \\
& =1, \quad \text { if } \alpha \geqq \frac{\pi}{2}, \tag{3}
\end{align*}
$$

where $i$ is the angle between the direction of the Sun and that of the normal to the lunar surface, $\varepsilon$ is the angle between the direction to the observer and that of the normal to the lunar surface, and $\alpha$ is the angle between the direction of the Sun and that of the observer. In the present paper, $\alpha$ is called the phase angle.

The calculated values of $B(\alpha, g)$ shows very weak dependency on the values of parameter $g$, so the author assumed $g=0.4$ throughout the present study.

It must be noted that, if Hapke's formula is strictly valid for a special point of the lunar surface, $\Delta m$ must be independent to the time.

The observed points are classified in eight groups from $A$ to $H$ according to the

TABLE I
Results of the observations

| Date | UT | $\xi$ | $\eta$ | $\begin{gathered} P_{v} \\ \% \end{gathered}$ | $B-V$ | $m$ | $\Delta m$ | Region |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1969 Oct. 1 | $15^{\mathrm{h}} 25^{\mathrm{m}} 50^{\mathrm{s}}$ | -0.657 | $+0.426$ | 7.4 | 0.807 | 3.729 | 3.724 | B |
|  | 152610 | -0.643 | $+0.424$ | 8.1 | 0.926 | 3.930 | 3.908 | B |
|  | 152745 | $-0.577$ | +0.414 | 9.9 | 0.970 | 4.074 | 3.979 | B |
|  | 153000 | -0.484 | $+0.403$ | 13.5 | 0.922 | 4.444 | 4.253 | B |
|  | 155220 | $-0.742$ | $+0.403$ | 11.7 | 0.924 | 4.221 | 4.307 | A |
|  | 155425 | $-0.656$ | +0.399 | 5.9 | 0.583 | 3.411 | 3.387 | B |
|  | 155510 | -0.625 | +0.395 | 10.1 | 0.892 | 3.983 | 3.924 | B |
|  | 155720 | $-0.534$ | +0.382 | 13.0 | 0.811 | 4.391 | 4.236 | B |
|  | 155925 | $-0.450$ | +0.369 | 12.8 | 1.009 | 4.295 | 4.055 | B |
|  | 163705 | $-0.736$ | $+0.421$ | 13.8 | 0.946 | 4.204 | 4.279 | A |
|  | 164130 | -0.551 | +0.334 | 11.5 | 0.967 | 4.156 | 3.993 | B |
|  | 165220 | $+0.209$ | $+0.160$ | 10.7 | 1.249 | 5.147 | 3.703 | D |
|  | 165400 | +0.272 | +0.161 | 13.0 | 1.450 | 5.352 | 3.537 | D |
| 1969 Nov. 27 | 150600 | $-0.152$ | $+0.230$ | 5.2 | 1.036 | 3.870 | 4.090 | C |
|  | 150805 | -0.074 | +0.231 | 3.8 | 0.726 | 3.349 | 3.531 | C |
|  | 150850 | $-0.046$ | $+0.232$ | 2.2 | 1.065 | 2.620 | 2.788 | C |
|  | 150900 | $-0.040$ | +0.231 | 3.6 | 0.954 | 3.587 | 3.752 | C |
|  | 151120 | $+0.048$ | $+0.236$ | 5.4 | 0.925 | 3.757 | 3.874 | C |
|  | 154450 | $+0.191$ | $+0.230$ | 5.2 | 0.922 | 4.117 | 4.024 | D |
|  | 154650 | $+0.265$ | $+0.235$ | 3.6 | 1.049 | 3.759 | 3.715 | D |
|  | 154930 | $+0.365$ | $+0.243$ | 5.2 | 1.050 | 3.829 | 3.678 | D |
| 1970 Jan. 25 | 161130 | --0.743 | $+0.391$ | 3.0 | 0.873 | 3.626 | 4.209 | A |
|  | 161600 | $-0.602$ | +0.407 | 1.4 | 1.116 | 3.553 | 4.056 | B |
|  | 164720 | $-0.756$ | $+0.386$ | 2.1 | 0.870 | 3.699 | 4.187 | A |
|  | 165120 | $-0.630$ | $+0.400$ | 2.8 | 0.937 | 3.543 | 4.055 | B |
|  | 165225 | -0.596 | $+0.404$ | 1.8 | 0.864 | 3.591 | 4.086 | B |
|  | 170520 | $+0.019$ | $+0.227$ | 3.0 | 0.908 | 3.976 | 4.210 | D |
|  | 170730 | $+0.093$ | +0.232 | 2.8 | 0.843 | 4.001 | 4.199 | D |
|  | 172800 | $+0.024$ | $+0.212$ | 2.2 | 0.879 | 4.063 | 4.291 | D |
|  | 173100 | $+0.126$ | $+0.224$ | 1.3 | 0.768 | 3.954 | 4.130 | D |
| 1970 Mar. 25 | 144825 | $-0.783$ | $+0.390$ | 1.6 | 0.893 | 3.622 | 4.217 | A |
|  | 145150 | --. 0.623 | $+0.406$ | 1.8 | 0.892 | 3.576 | 4.115 | B |
|  | 145450 | $-0.519$ | +0.418 | 1.7 | 0.912 | 3.807 | 4.306 | B |
|  | 150530 | $-0.765$ | $+0.408$ | 1.2 | 0.929 | 3.784 | 4.398 | A |
|  | 150630 | $-0.732$ | $+0.412$ | 1.4 | 0.984 | 3.636 | 4.229 | A |
|  | 150900 | -0.648 | +0.423 | 1.7 | 0.720 | 3.475 | 4.025 | B |
|  | 151020 | $-0.603$ | +0.429 | 1.4 | 0.897 | 3.696 | 4.226 | B |
|  | 152100 | -0.713 | +0.475 | 2.0 | 0.931 | 3.950 | 4.541 | A |
|  | 152250 | -0.655 | +0.484 | 1.5 | 1.024 | 3.724 | 4.283 | B |
|  | 152355 | -0.617 | $+0.490$ | 1.2 | 0.941 | 3.641 | 4.181 | B |
|  | 152530 | $-0.563$ | +0.496 | 1.5 | 0.868 | 3.737 | 4.253 | B |

TABLE I (continued)

| Date | UT | $\xi$ | $\eta$ | $\begin{gathered} P_{v} \\ \% \end{gathered}$ | $B-V$ | $m$ | $\Delta m$ | Region |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1970 Mar. 25 | $15^{\mathrm{L}} 35 \mathrm{~m} 20^{\text {s }}$ | $-0.713$ | +0.474 | 1.5 | 0.824 | 3.738 | 4.327 | A |
|  | 153640 | $-0.664$ | $+0.480$ | 1.5 | 0.841 | 3.665 | 4.226 | B |
|  | 153900 | $-0.583$ | +0.489 | 1.5 | 0.809 | 3.723 | 4.246 | B |
| 1970 Mar. 26 | 161600 | $-0.730$ | $+0.389$ | 3.2 | 0.864 | 3.277 | 3.697 | A |
|  | 161840 | $-0.637$ | +0.400 | 3.1 | 0.824 | 3.318 | 3.677 | B |
|  | 162050 | $-0.561$ | $+0.408$ | 3.7 | 0.842 | 3.393 | 3.708 | B |
|  | 162930 | $-0.782$ | $+0.454$ | 4.0 | 0.732 | 3.574 | 4.055 | A |
|  | 163300 | -0.662 | +0.473 | 4.2 | 0.835 | 3.067 | 3.453 | B |
|  | 163400 | -0.631 | $+0.478$ | 4.3 | 0.951 | 3.347 | 3.711 | B |
|  | 164200 | $-0.158$ | +0.390 | 4.6 | 0.944 | 4.052 | 4.143 | E |
|  | 164500 | $-0.046$ | $+0.394$ | 3.3 | 0.905 | 3.703 | 3.730 | E |
|  | 165600 | $+0.019$ | +0.279 | 3.6 | 0.902 | 4.297 | 4.283 | D |
|  | 165830 | $+0.095$ | $+0.280$ | 3.6 | 0.693 | 3.813 | 3.739 | D |
| 1970 July 25 | 171250 | $-0.621$ | +0.414 | 8.9 | 1.021 | 4.584 | 3.847 | B |
|  | 171350 | $-0.577$ | $+0.404$ | 15.4 | 0.851 | 4.985 | 4.171 | B |
|  | 171640 | -0.457 | +0.377 | 19.0 | 0.873 | 5.285 | 4.255 | B |
|  | 173200 | $-0.645$ | +0.392 | 13.3 | 0.902 | 4.923 | 4.205 | B |
|  | 173300 | -0.602 | $+0.382$ | 18.4 | 0.795 | 5.205 | 4.413 | B |
|  | 174900 | $-0.587$ | +0.423 | 9.8 | 0.831 | 4.912 | 4.114 | B |
|  | 175200 | $-0.446$ | +0.384 | 17.7 | 0.877 | 5.279 | 4.219 | B |
|  | 184130 | $-0.613$ | $+0.109$ | 8.2 | 0.898 | 4.673 | 3.802 | F |
|  | 184300 | $-0.551$ | +0.100 | 12.7 | 0.930 | 5.059 | 4.089 | F |
|  | 184500 | $-0.470$ | $+0.090$ | 7.7 | 0.837 | 4.973 | 3.864 | F |
| 1970 July 26 | 163030 | $-0.639$ | +0.428 | 13.6 | 0.902 | 4.997 | 3.790 | B |
|  | 163230 | $-0.550$ | $+0.413$ | 20.9 | 0.962 | 5.486 | 4.062 | B |
|  | 163600 | $-0.402$ | +0.393 | 19.3 | 0.839 | 5.905 | 4.042 | B |
|  | 164600 | $-0.644$ | $+0.390$ | 16.7 | 0.904 | 5.047 | 3.813 | B |
|  | 164700 | $-0.599$ | +0.382 | 16.9 | 0.967 | 5.372 | 4.031 | B |
|  | 164900 | $-0.513$ | $+0.373$ | 21.5 | 0.913 | 5.605 | 4.049 | B |
|  | 170020 | $-0.591$ | $+0.140$ | 12.1 | 0.901 | 5.052 | 3.560 | F |
|  | 170200 | $-0.479$ | +0.127 | 24.7 | 0.798 | 5.652 | 3.868 | F |
|  | 170430 | $-0.369$ | $+0.117$ | 16.3 | 0.866 | 5.722 | 3.546 | F |
|  | 171830 | -0.924 | $-0.098$ | 14.8 | 0.890 | 4.931 | 4.126 | H |
|  | 172000 | $-0.835$ | $-0.118$ | 13.3 | 0.898 | 4.974 | 3.954 | H |
|  | 172100 | $-0.778$ | $-0.131$ | 16.2 | 0.865 | 5.274 | 4.134 | H |
| 1970 Nov. 15 |  |  | $+0.297$ | 0.9 | 0.744 | 4.561 | 4.660 | G |
|  | 161730 | $+0.654$ | +0.310 | 1.3 | 0.726 | 4.159 | 4.153 | G |
|  | 162030 | $+0.761$ | +0.331 | 1.0 | 0.857 | 3.885 | 3.538 | G |

TABLE I (continued)

| Date | UT | $\xi$ | $\eta$ | $\begin{gathered} P_{v} \\ \% \end{gathered}$ | $B-V$ | $m$ | $\Delta m$ | Region |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1970 Nov. 15 | $16^{\mathrm{H} 26^{\mathrm{m}} 50^{\mathrm{s}}}$ | -0.648 | +0.403 | 0.9 | 0.802 | 3.831 | 4.494 | B |
|  | 162900 | -0.551 | +0.397 | 1.7 | 0.736 | 4.029 | 4.639 | B |
|  | 163130 | -0.444 | +0.393 | 1.5 | 0.753 | 4.002 | 4.566 | B |
| 1970 Nov. 16 | 141940 | $+0.600$ | $+0.300$ | 3.5 | 0.731 | 5.376 | 4.892 | G |
|  | 142055 | $+0.644$ | +0.308 | 4.0 | 0.788 | 5.056 | 4.388 | G |
|  | 142300 | +0.718 | +0.320 | 3.2 | 0.842 | 5.089 | 3.741 | G |
|  | 144300 | $-0.175$ | +0.258 | 3.5 | 0.831 | 4.043 | 4.302 | C |
|  | 144505 | $-0.093$ | $+0.261$ | 4.5 | 0.744 | 4.398 | 4.620 | C |
|  | 144800 | $+0.023$ | $+0.266$ | 4.5 | 0.739 | 4.502 | 4.666 | C |
|  | 145800 | -0.627 | $+0.403$ | 3.4 | 0.854 | 3.917 | 4.419 | B |
|  | 145900 | -0.584 | $+0.401$ | 4.8 | 0.836 | 4.053 | 4.302 | B |
|  | 150100 | -0.505 | $+0.396$ | 5.3 | 0.834 | 4.235 | 4.661 | B |
|  | 150300 | -0.413 | +0.393 | 5.3 | 0.705 | 4.198 | 4.576 | B |
|  | 150600 | $-0.241$ | +0.434 | 5.5 | 0.799 | 4.180 | 4.481 | E |
|  | 150800 | $-0.160$ | $+0.435$ | 5.9 | 0.770 | 4.321 | 4.583 | E |
|  | 150900 | -0.120 | $+0.436$ | 4.3 | 0.865 | 3.978 | 4.220 | E |
|  | 151050 | $-0.048$ | $+0.436$ | 5.0 | 0.816 | 4.303 | 4.508 | E |
|  | 151210 | $+0.005$ | $+0.442$ | 4.5 | 0.847 | 4.044 | 4.221 | E |
| 1970 Dec. 16 | 150000 | -0.627 | $+0.460$ | 4.9 | 0.952 | 4.390 | 4.836 | B |
|  | 150120 | -0.577 | $+0.466$ | 5.9 | 0.784 | 4.189 | 4.399 | B |
|  | 150300 | -0.521 | $+0.473$ | 5.9 | 0.899 | 4.407 | 4.785 | B |
|  | 151400 | -0.757 | +0.379 | 5.4 | 0.808 | 4.453 | 4.886 | A |
|  | 151730 | -0.635 | +0.394 | 5.5 | 0.979 | 4.280 | 4.713 | B |
|  | 151930 | $-0.564$ | +0.402 | 5.7 | 0.976 | 4.498 | 4.886 | B |
|  | 153100 | $-0.642$ | $+0.406$ | 4.3 | 0.885 | 4.088 | 4.527 | B |
|  | 153330 | -0.531 | $+0.420$ | 5.3 | 0.838 | 4.342 | 4.712 | B |
|  | 153600 | -0.437 | $+0.433$ | 5.2 | 0.831 | 4.378 | 4.697 | B |
|  | 154800 | -0.184 | $+0.453$ | 5.0 | 0.749 | 4.168 | 4.353 | E |
|  | 155000 | -0.137 | +0.461 | 4.6 | 0.666 | 4.548 | 4.707 | E |
|  | 155300 | $-0.020$ | +0.483 | 5.6 | 1.016 | 4.315 | 4.404 | E |
| 1971 Jan. 14 | 151000 | $+0.611$ | $+0.338$ | 2.3 | 0.818 | 4.785 | 4.471 | G |
|  | 152800 | +0.514 | $+0.252$ | 2.7 | 0.859 | 5.162 | 5.074 | G |
|  | 152930 | $+0.562$ | $+0.257$ | 3.2 | 0.849 | 5.304 | 5.136 | G |
|  | 153110 | $+0.618$ | $+0.264$ | 3.2 | 0.882 | 5.044 | 4.747 | G |
|  | 155200 | -0.026 | +0.283 | 3.6 | 0.748 | 4.488 | 4.769 | D |
|  | 155500 | $+0.082$ | +0.295 | 4.3 | 0.798 | 4.468 | 4.701 | D |
|  | 155600 | +0.116 | +0.299 | 3.9 | 0.701 | 4.628 | 4.485 | D |
|  | 160730 | $-0.041$ | $+0.234$ | 3.2 | 0.699 | 4.539 | 4.825 | C |
|  | 161010 | $+0.055$ | $+0.244$ | 3.2 | 0.680 | 4.553 | 4.798 | D |
|  | 161430 | +0.208 | +0.262 | 4.7 | 0.742 | 4.407 | 4.573 | D |

TABLE I (continued)

| Date | UT | $\xi$ | $\eta$ | $\begin{gathered} P_{v} \\ \% \end{gathered}$ | $B-V$ | $m$ | $\Delta m$ | Region |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 Jan. 14 | $16^{\mathrm{n} 30^{\mathrm{m}} 50^{\text {s }}}$ | $-0.608$ | $+0.386$ | 3.9 | 0.847 | 3.968 | 4.484 | B |
|  | 163250 | $-0.510$ | $+0.397$ | 3.2 | 0.815 | 4.029 | 4.499 | B |
|  | 165000 | -0.608 | $+0.415$ | 2.9 | 0.827 | 3.901 | 4.419 | B |
|  | 165200 | -0.539 | $+0.423$ | 4.3 | 0.892 | 4.049 | 4.533 | B |
|  | 165350 | $-0.466$ | $+0.431$ | 4.7 | 0.857 | 4.136 | 4.588 | B |
| 1971 Jan. 15 | 161100 | $-0.600$ | $+0.400$ | 5.9 | 0.773 | 4.104 | 4.443 | B |
|  | 161320 | -0.525 | +0.410 | 6.5 | 0.774 | 4.317 | 4.611 | B |
|  | 163200 | -0.645 | +0.414 | 5.9 | 0.768 | 4.069 | 4.438 | B |
|  | 163600 | -0.511 | +0.432 | 6.2 | 0.817 | 4.264 | 4.550 | B |
|  | 164810 | $-0.744$ | $+0.313$ | 6.3 | 0.856 | 4.244 | 4.667 | A |
|  | 165100 | -0.650 | $+0.326$ | 5.8 | 0.878 | 4.046 | 4.402 | B |
|  | 165300 | $-0.583$ | +0.335 | 7.4 | 0.816 | 4.348 | 4.663 | B |
|  | 170610 | -0.392 | +0.485 | 5.4 | 0.805 | 4.453 | 4.671 | E |
|  | 170820 | $-0.220$ | $+0.493$ | 6.0 | 0.785 | 4.519 | 4.637 | E |
|  | 171020 | $-0.153$ | $+0.501$ | 6.2 | 0.790 | 4.604 | 4.680 | E |
|  | 172120 | -0.109 | $+0.523$ | 5.6 | 0.773 | 4.553 | 4.598 | E |
|  | 172220 | $-0.075$ | $+0.527$ | 5.9 | 0.832 | 4.456 | 4.477 | E |
|  | 172600 | $+0.047$ | +0.541 | 5.6 | 0.844 | 4.491 | 4.418 | E |
|  | 172700 | $+0.080$ | $+0.545$ | 5.9 | 0.808 | 4.571 | 4.461 | E |
|  | $1740 \quad 10$ | $+0.039$ | $+0.297$ | 5.5 | 0.779 | 4.801 | 4.743 | D |
|  | 174240 | $+0.117$ | +0.306 | 6.2 | 0.810 | 4.660 | 4.536 | D |
| 1971 Jan. 16 | 153000 | $-0.612$ | $+0.408$ | 8.1 | 0.976 | 4.193 | 4.332 | B |
|  | 153100 | -0.580 | $+0.413$ | 7.0 | 0.942 | 4.231 | 4.346 | B |
|  | 153200 | -0.547 | +0.418 | 8.0 | 0.959 | 4.268 | 4.358 | B |
|  | 153300 | -0.514 | +0.422 | 8.7 | 0.930 | 4.478 | 4.544 | B |
|  | 155230 | $-0.625$ | +0.399 | 8.7 | 0.948 | 4.157 | 4.302 | B |
|  | 155400 | -0.587 | $+0.404$ | 8.8 | 0.955 | 4.290 | 4.405 | B |
|  | 155600 | $-0.522$ | $+0.413$ | 8.2 | 0.822 | 4.457 | 4.524 | B |
|  |  | $-0.346$ | $+0.474$ | 10.1 | 0.833 | 4.731 | 4.670 | E |
|  | 161000 | -0.318 | $+0.477$ | 8.3 | 0.881 | 4.619 | 4.536 | E |
|  | 161500 | -0.154 | +0.497 | 11.0 | 0.779 | 4.855 | 4.629 | E |
|  | 162600 | $+0.013$ | $+0.159$ | 9.4 | 0.894 | 4.575 | 4.182 | D |
|  | 162700 | $+0.047$ | $+0.163$ | 7.5 | 0.832 | 4.784 | 4.349 | D |
|  | 163100 | $+0.180$ | $+0.180$ | 9.0 | 0.756 | 5.386 | 4.740 | D |
|  | 163355 | $+0.276$ | +0.185 | 8.5 | 0.765 | 5.197 | 4.509 | D |
|  | 163450 | $+0.308$ | $+0.188$ | 7.3 | 0.736 | 5.313 | 4.331 | D |
| 1971 Jan. 18 | 170520 | -0.649 | $+0.411$ | 12.0 | 0.889 | 4.612 | 4.717 | B |
|  | 170615 | $-0.620$ | $+0.415$ | 12.0 | 0.898 | 4.755 | 4.276 | B |
|  | 170755 | -0.567 | +0.438 | 12.9 | 0.844 | 4.941 | 4.397 | B |

TABLE I (continued)

| Date | UT | $\xi$ | $\eta$ | $\begin{aligned} & P_{v} \\ & \% \end{aligned}$ | $B-V$ | $m$ | $\Delta m$ | Region |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 Jan. 18 | $17^{\mathrm{h}} 25^{\mathrm{m}} 00^{\text {s }}$ | -0.804 | $+0.363$ | 15.5 | 0.794 | 4.843 | 4.602 | A |
|  | 172620 | -0.763 | +0.371 | 14.4 | 0.844 | 4.773 | 4.472 | A |
|  | 172740 | $-0.721$ | $+0.377$ | 13.3 | 0.954 | 4.742 | 4.383 | A |
|  | 172950 | -0.653 | +0.401 | 8.5 | 0.834 | 4.362 | 3.917 | B |
|  | 173200 | -0.583 | +0.398 | 10.5 | 0.885 | 4.764 | 4.223 | B |
|  | 174400 | -0.766 | +0.185 | 14.6 | 0.814 | 4.824 | 4.478 | F |
|  | 174530 | -0.719 | $+0.193$ | 15.7 | 0.800 | 4.932 | 4.526 | F |
|  | 174730 | -0.656 | $+0.203$ | 13.6 | 0.802 | 4.795 | 4.310 | F |
|  | 175000 | $-0.576$ | $+0.213$ | 13.3 | 0.859 | 4.626 | 4.039 | F |
|  | 180200 | -0.176 | $+0.454$ | 11.6 | 0.841 | 5.470 | 4.171 | E |
|  | 180305 | -0.140 | $+0.457$ | 12.7 | 0.823 | 5.765 | 4.346 | E |
|  | 180700 | $-0.075$ | $+0.462$ | 10.8 | 0.926 | 5.989 | 4.289 | E |
|  | 181830 | -0.927 | $-0.099$ | 15.2 | 0.810 | 4.709 | 4.617 | H |
|  | 182030 | $-0.870$ | -0.083 | 8.9 | 0.883 | 4.302 | 4.101 | H |
|  | 182240 | $-0.803$ | $-0.069$ | 14.3 | 0.801 | 4.827 | 4.525 | H |
| 1971 Feb. 9 | 141030 | $+0.543$ | $+0.273$ | 1.2 | 0.732 | 3.679 | 4.577 | G |
|  | 141300 | +0.628 | +0.284 | 1.9 | 0.942 | 3.060 | 3.970 | G |
|  | 141500 | $+0.695$ | $+0.293$ | 2.2 | 0.805 | 2.838 | 3.760 | G |
|  | 141530 | $+0.712$ | +0.295 | 1.8 | 0.815 | 2.967 | 3.893 | G |
|  | 141810 | +0.802 | $+0.308$ | 1.9 | 0.770 | 3.487 | 4.438 | G |
|  | 143030 | $+0.531$ | $+0.264$ | 1.7 | 0.738 | 3.669 | 4.569 | G |
|  | 143400 | $+0.649$ | $+0.281$ | 1.2 | 0.812 | 3.204 | 4.121 | G |
|  | 143530 | $+0.696$ | $+0.289$ | 1.3 | 0.785 | 3.859 | 3.785 | G |
|  | 143600 | +0.712 | $+0.291$ | 1.6 | 0.913 | 2.934 | 3.863 | G |
|  | 143900 | $+0.810$ | $+0.306$ | 1.8 | 0.728 | 3.504 | 4.461 | G |
|  | 145300 | $-0.663$ | +0.396 | 1.7 | 0.816 | 3.561 | 4.333 | B |
|  | 145410 | $-0.620$ | $+0.399$ | 2.1 | 0.818 | 3.690 | 4.475 | B |
|  | 145640 | $-0.571$ | +0.408 | 1.3 | 0.823 | 3.766 | 4.563 | B |
|  | 150900 | $-0.661$ | +0.403 | 0.8 | 0.749 | 3.729 | 4.507 | B |
|  | 150945 | $-0.633$ | $+0.405$ | 1.4 | 0.797 | 3.596 | 4.383 | B |
|  | 151030 | $-0.605$ | +0.408 | 1.2 | 0.807 | 3.696 | 4.490 | B |
|  | 151230 | $-0.532$ | +0.414 | 1.4 | 0.832 | 3.591 | 4.401 | B |
|  | 151345 | $-0.486$ | $+0.418$ | 1.3 | 0.778 | 3.785 | 4.603 | B |
| 1971 Feb. 10 | 132345 | $+0.561$ | $+0.260$ | 1.6 | 0.877 | 3.090 | 4.051 | G |
|  | 132530 | $+0.620$ | +0.267 | 1.7 | 0.844 | 2.701 | 3.659 | G |
|  | 132850 | $+0.731$ | $+0.282$ | 1.6 | 0.777 | 2.529 | 3.478 | G |
|  | 133000 | $+0.770$ | $+0.287$ | 1.6 | 0.777 | 2.977 | 3.921 | G |
|  | 134200 | $\begin{aligned} & +0.565 \\ & +0.631 \\ & +0.720 \\ & +0.798 \end{aligned}$ | $+0.268$ | 0.3 | 0.577 | 3.074 | 4.033 | G |
|  | 134440 |  | $+0.276$ | 0.4 | 0.627 | 2.737 | 3.691 | G |
|  | 134640 |  | +0.288 | 0.4 | 0.695 | 2.638 | 3.585 | G |
|  | 134900 |  | +0.298 | 1.6 | 0.470 | 3.156 | 4.092 | G |

TABLE I (continued)

| Date | UT | $\xi$ | $\eta$ | $\begin{gathered} P_{v} \\ \% \end{gathered}$ | $V-B$ | $m$ | $\Delta m$ | Region |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 Feb. 10 | $14^{\mathrm{h}} 02^{\mathrm{m}} 40^{\text {s }}$ | $\begin{array}{r} -0.052 \\ +0.020 \end{array}$ | $\begin{array}{r} +0.297 \\ +0.305 \end{array}$ | $\begin{aligned} & 0.4 \\ & 0.8 \end{aligned}$ | $\begin{aligned} & 0.862 \\ & 0.740 \end{aligned}$ | $\begin{aligned} & 3.315 \\ & 3.379 \end{aligned}$ | $\begin{aligned} & 4.292 \\ & 4.354 \end{aligned}$ | C |
|  | 140440 |  |  |  |  |  |  | C |
|  | 141730 | -0.176 | $+0.450$ | 0.8 | 0.871 | 3.303 | 4.283 | E |
|  | 141915 | -0.116 | $+0.457$ | 0.6 | 0.836 | 3.487 | 4.465 | E |
|  | 142300 | $+0.013$ | +0.472 | 1.0 | 0.837 | 3.381 | 4.354 | E |
|  | 143400 | -0.647 | $+0.408$ | 1.3 | 0.819 | 3.414 | 4.416 | B |
|  | 143440 | -0.629 | $+0.410$ | 0.9 | 0.905 | 3.355 | 4.356 | B |
|  | 143530 | -0.594 | +0.413 | 0.6 | 0.849 | 3.441 | 4.439 | B |
|  | 143700 | $-0.541$ | +0.419 | 0.9 | 0.829 | 3.375 | 4.370 | B |
|  | 143820 | -0.494 | $+0.424$ | 1.2 | 0.807 | 3.557 | 4.549 | B |
|  | 145810 | -0.659 | $+0.410$ | 0.6 | 0.832 | 3.459 | 4.462 | B |
|  | 145905 | $-0.636$ | $+0.413$ | 0.6 | 0.816 | 3.377 | 4.378 | B |
|  | 150010 | -0.592 | $+0.418$ | 0.9 | 0.823 | 3.510 | 4.508 | B |
|  | 150120 | -0.554 | $+0.422$ | 0.6 | 0.836 | 3.412 | 4.601 | B |
|  | 150300 | -0.495 | $+0.428$ | 1.0 | 0.783 | 3.610 | 4.601 | B |
| 1971 Mar. 9 | 125330 | $+0.487$ | +0.259 | 1.4 | 0.836 | 4.054 | 4.593 | G |
|  |  | $+0.536$ | +0.265 | 2.0 | 0.816 | 3.609 | 4.165 | G |
|  | 125810 | $+0.641$ | $+0.278$ | 2.9 | 0.806 | 3.316 | 3.912 | G |
|  | 130040 | +0.722 | $+0.289$ | 1.7 | 0.746 | 4.000 | 4.635 | G |
|  | 130120 | $+0.744$ | +0.292 | 1.8 | 0.759 | 3.967 | 4.614 | G |
|  | 131440 | $+0.614$ | $+0.246$ | 1.3 | 0.728 | 4.123 | 4.708 | G |
|  | 131640 | $+0.678$ | +0.254 | 1.0 | 0.770 | 3.559 | 4.171 | G |
|  | 131920 | +0.764 | $+0.265$ | 1.2 | 0.816 | 3.220 | 3.876 | G |
|  | 132150 | $+0.843$ | $+0.275$ | 1.3 | 0.730 | 3.845 | 4.560 | G |
|  | 132300 | $+0.880$ | $+0.280$ | 1.3 | 0.740 | 3.917 | 4.674 | G |
|  | 133540 | $+0.040$ | +0.020 | 1.2 | 0.692 | 4.130 | 4.539 | D |
|  | 135850 | $-0.082$ | $+0.469$ | 1.2 | 0.794 | 3.941 | 4.317 | E |
|  | 140120 | +0.003 | $+0.478$ | 0.9 | 0.783 | 3.983 | 4.391 | E |
|  | 140400 | +0.094 | +0.489 | 1.0 | 0.779 | 3.827 | 4.266 | E |
|  | 141550 | -0.621 | $+0.412$ | 0.9 | 0.866 | 4.674 | 4.597 | B |
|  | 142010 | -0.477 | +0.427 | 0.9 | 0.841 | 4.618 | 4.768 | B |
| 1971 Mar. 10 | 131720 | $+0.601$ | +0.245 | 1.5 | 0.750 | 3.946 | 4.684 | G |
|  | 131930 | $+0.672$ | $+0.253$ | 0.7 | 0.795 | 3.442 | 4.200 | G |
|  | 132210 | $+0.759$ | $+0.262$ | 0.9 | 0.831 | 3.145 | 3.935 | G |
|  | 132430 | $+0.834$ | $+0.269$ | 0.7 | 0.718 | 3.788 | 4.618 | G |
|  | 133730 | $+0.606$ | $+0.252$ | 0.8 | 0.761 | 3.908 | 4.650 | G |
|  | 133940 | +0.677 | +0.259 | 0.8 | 0.776 | 3.471 | 4.234 | G |
|  | 134240 | $+0.775$ | +0.269 | 0.7 | 0.853 | 3.149 | 4.524 | G |
|  | 134500 | $+0.849$ | +0.275 | 0.9 | 0.740 | 3.724 | 4.568 | G |
|  | $13 \quad 5700$ | -0.041 | $+0.292$ | 0.8 | 0.765 | 3.872 | 4.499 | C |
|  | 135800 | $-0.007$ | $+0.296$ | 0.8 | 0.773 | 3.951 | 4.584 | C |

TABLE I (continued)

| Date | UT | $\xi$ | $\eta$ | $\begin{gathered} P_{v} \\ \% \end{gathered}$ | $B-V$ | $m$ | $\Delta m$ | Region |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 Mar. 10 | $13^{15} 59^{\text {m }} 20^{\text {s }}$ | +0.038 | $+0.301$ | 0.5 | 0.811 | 3.798 | 4.439 | D |
|  | 135940 | $+0.049$ | $+0.303$ | 1.0 | 0.756 | 3.875 | 4.518 | D |
|  | 140100 | $+0.094$ | +0.303 | 1.0 | 0.822 | 3.734 | 4.385 | D |
|  | 140140 | $+0.117$ | $+0.311$ | 2.2 | 0.766 | 3.986 | 4.641 | D |
|  | 140220 | $+0.139$ | +0.313 | 1.7 | 0.785 | 3.808 | 4.466 | D |
|  | 140300 | $+0.161$ | $+0.316$ | 1.2 | . 0.735 | 3.903 | 4.565 | D |
|  | 141520 | -0.101 | +0.464 | 1.2 | 0.794 | 3.820 | 4.437 | E |
|  | 141600 | $-0.078$ | $+0.467$ | 1.3 | 0.822 | 3.699 | 4.321 | E |
|  | 141815 | $-0.002$ | $+0.476$ | 1.4 | 0.789 | 3.825 | 4.462 | E |
|  | 141845 | $+0.015$ | $+0.478$ | 1.4 | 0.794 | 3.765 | 4.405 | E |
|  | 142150 | $+0.118$ | $+0.490$ | 1.1 | 0.795 | 3.639 | 4.298 | E |
|  | 143450 | -0.645 | +0.409 | 1.4 | 0.793 | 4.192 | 4.615 | B |
|  | 143530 | $-0.622$ | $+0.412$ | 0.9 | 0.802 | 4.051 | 4.493 | B |
|  | 143600 | -0.605 | $+0.414$ | 0.9 | 0.799 | 4.149 | 4.603 | B |
|  | 143750 | $-0.541$ | $+0.423$ | 0.9 | 0.850 | 4.100 | 4.592 | B |
|  | 143930 | $-0.484$ | $+0.430$ | 1.0 | 0.774 | 4.210 | 4.728 | B |
|  | 144140 | $-0.410$ | +0.439 | 1.4 | 0.849 | 4.074 | 4.620 | B |
| 1971 Mar. 11 | 121440 | $+0.612$ | $+0.257$ | 0.8 | 0.849 | 3.554 | 4.481 | G |
|  | 121630 | $+0.673$ | $+0.262$ | 1.0 | 0.879 | 3.180 | 4.115 | G |
|  | 121820 | $+0.733$ | $+0.267$ | 0.9 | 0.882 | 2.798 | 3.743 | G |
|  | 121920 | +0.864 | +0.275 | 1.0 | 0.857 | 2.885 | 4.537 | G |
|  | 122230 | $+0.870$ | $+0.275$ | 0.7 | 0.901 | 3.551 | 4.537 | G |
|  | 123415 | $+0.610$ | $+0.258$ | 0.8 | 0.829 | 3.650 | 3.580 | G |
|  | 123620 | $+0.664$ | +0.262 | 1.4 | 0.816 | 3.280 | 4.217 | G |
|  | 123810 | +0.740 | $+0.268$ | 1.3 | 0.863 | 2.888 | 3.717 | $G$ |
|  | 123900 | $+0.767$ | $+0.270$ | 1.3 | 0.860 | 2.952 | 3.906 | G |
|  | 124110 | $+0.839$ | $+0.275$ | 0.8 | 0.808 | 3.643 | 4.617 | G |
|  |  | $-0.173$ | $+0.289$ | 1.7 | 0.932 | 3.630 | 4.507 | C |
|  | 125540 | $-0.123$ | +0.294 | 1.4 | 0.834 | 3.543 | 4.424 | C |
|  | 125725 | -0.065 | $+0.301$ | 1.3 | 0.900 | 3.616 | 4.501 | C |
|  | 130055 | $+0.052$ | $+0.314$ | 0.8 | 0.811 | 3.527 | 4.419 | D |
|  | 130130 | $+0.072$ | $+0.317$ | 1.5 | 0.820 | 3.633 | 4.527 | D |
|  | 130205 | $+0.091$ | +0.319 | 0.8 | 0.907 | 3.827 | 4.722 | D |
|  | 130255 | +0.119 | $+0.470$ | 1.3 | 0.738 | 3.661 | 4.558 | D |
|  | 131710 | -0.164 | $+0.450$ | 1.6 | 0.769 | 3.770 | 4.649 | E |
|  | 131900 | -0.102 | $+0.457$ | 1.3 | 0.656 | 3.958 | 4.841 | E |
|  | 132030 | $-0.052$ | $+0.463$ | 0.8 | 0.783 | 3.764 | 4.651 | E |
|  | 132240 | $+0.020$ | $+0.470$ | 1.3 | 0.738 | 3.641 | 4.533 | E |
|  | 133915 | -0.659 | $+0.411$ | 2.5 | 0.538 | 4.100 | 4.928 | B |
|  | 134010 | -0.628 | $+0.416$ | 2.9 | 0.726 | 3.947 | 4.781 | B |
|  | 134210 | -0.560 | +0.426 | 1.2 | 0.850 | 3.705 | 4.551 | B |
|  | 134350 | -0.506 | +0.433 | 1.3 | 0.853 | 3.867 | 4.701 | B |
|  | 134520 | $-0.454$ | $+0.439$ | 1.8 | 0.880 | 3.828 | 4.688 | B |
|  | 134650 | -0.405 | +0.446 | 1.5 | 0.786 | 3.761 | 4.627 | B |

regions to which they are contained, as listed in the caption to Figure 1. The symbols shown in this caption are used throughout the figures in the present paper.

## 3. Correlations Between the Observed Quantities

Since Secchi (1859) found that the polarization degrees of the moonlight are correlated with the albedo of the Moon's surface, many writers, including the present author (Sekiguchi, 1971) have confirmed this. Furthermore, it is found that, if we take the

Oct. 1, 1969


Jan. 25, 1970


Nov. 27, 1969


Mar. 25, 1970


Fig. 1a.


Fig. 1b.
residuals of the brightness from those calculated by Hapke's formula, they have a linear correlation with the logarithms of the observed polarization degrees, and this correlation has very wide validity. Figure 1 shows these relations for each observed day. Inspecting these diagrams, we find some salient features.
(1) For various regions of the lunar surface, a single correlation is valid during one night, except the region near Proclus (the Region G). As mentioned in the previous paper (Sekiguchi, 1971), the present observations were chiefly performed on the maria regions except that of Mts. Apenninus (the Region C), because the observations

Nov. 16, 1970


Jan. 14, 1971


Dec. 16, 1970


Jan. 15, 1971


Fig. 1c.
of the mountainous regions were found to be inaccurate. The observations of November 27, 1969, November 16, 1970, and January 14, March 10 and 11, 1971, which contain the observations of the region $C$, also indicate the validity of the same relations derived from those of maria regions. It is very surprising that the floor of Grimaldi, which is known as the one of the darkest place on the Moon, also shows the same relation as the bright ray of Aristarchus in the observations of January 18, 1971.
(2) The residuals from Hapke's formula, $\Delta m$, are dispersed from time to time, and


Fig. 1d.
seem to have very weak correlation with the phase angle $\alpha$. Figure 2, where the ordinates, $\overline{\Delta m}(B)$, are daily mean values of $\Delta m$ for the $B$-region (East of Aristarchus), shows very weak correlation with $\alpha$. This may be caused by the inadequacy of the assumed value of $g$.

On the other hand, the author pointed out in the previous paper (Sekiguchi, 1971) that the anomalous brightening of the lunar surface on March 26, 1970 may have a correlation with the solar flare which took place 29 h before the observation.

The correlation between the brightness of the Moon and the solar activity is very


Fig. 1e.

Figs. Ia-e. O:A region West of Aristarchus; :B region East of Aristarchus; $Q: C$ region of Mts. Apenninus; $+: \mathrm{D}$ region of M . Vaporum; $\times: \mathrm{E}$ region of M . Imbrium; : F region near Kepler; $\Delta$ : G region near Proclus; $\square:$ H region near Grimaldi.
clear. Table II shows the times of the present observations and their just preceding remarkable solar flares. On July 26, 1970 we also find an anomalous brightening which was probably caused by the flares which appeared about 9 h before the observation. However, on the nights of November 15 and 16, 1970, it seems that the solar flares which appeared on these dates did not affect the lunar brightness. Therefore, it is suggested that not all solar flares which appear near the centre of the solar disc cause the brightening of the lunar surface. The examples of December 16, 1970, and March 9,10 and 11,1971 show that the effect of the solar flares are not effective after time intervals longer than 30 h . The solar flares on January 14 and February 8, 1971 took place on the marginal zone of the solar disc, so they were not effective.

Generally speaking, the Moon's brightness has become fainter from the autumn


Fig. 2.
TABLE II
Preceding solar flares

| Time of observation | Solar flare | Position | Importance | Time interval | $\Delta m(B)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1969, Oct. $1,16^{\text {h }}$ (U) | Sept. $27,04^{\text {b }} 30^{\mathrm{m}}$ (UT) | N10 E 01 | 3B | $110^{\text {h }}$ | 3.84 |
|  | Sept. 28, 0330 | N04 W75 | 2N | 86 | a |
| Nov. 27, 15 | Nov. 23, 1010 | N15 W18 | 2B | 101 | - |
|  | Nov. 24, 0920 | N15 W32 | 2B | 78 | - |
| 1970, Jan. 25, 17 | Jan. 20, 1000 | S 18 E 33 | 3B | 103 | 4.07 |
| Mar. 25, 16 | Mar. 25, 1220 | N15 E 10 | 2B | 4 | 4.20 |
| Mar. 26, 17 | a ${ }^{\text {a }}$ | a | a | 29 | 3.65 |
| July 25,18 | July 23, 1845 | N09 E 10 | 2B | 47 | 4.25 |
| July 26,17 | July 25, 1843 | N27 E 26 | 1 N | 22 | 3.96 |
|  | July 26, 0645 | N08 W28 | 2B | 9 | a |
| Nov. 15, 16 | Nov. 15, 0652 | N18 W12 | 2 N | 9 | 4.57 |
| Nov. 16, 15 | a | a | a | 32 | 4.50 |
| -16, | Nov. 15, 1820 | N16 W18 | 2B | 21 | a |
| a | Nov. 16, 0100 | N17 W21 | 3B | 14 | a |
| Dec. 16, 15 | Dec. 14, 2315 | N05 W02 | 2 N | 37 | 4.70 |
| 1971, Jan. 14, 16 | Jan. 14, 1128 | S 08 W55 | 2B | 5 | 4.51 |
| Jan. 15, 17 | a | a | a | 30 | 4.53 |
| Jan. 16, 16 | a | a | a | 53 | 4.40 |
| Jan. 18, 18 | a | a | a | 79 | 4.21 |
| Feb. 9,15 | Feb. 8, 1129 | N23 W67 | 1B | 28 | 4.47 |
| Feb. 10, 14 | a | ${ }^{\text {a }}$ | a | 51 | 4.47 |
| Mar. 9, 15 | Mar. 7, 1119 | S 15 E 06 | 2 N | 50 | 4.69 |
| Mar. 10, 14 | a | a | a | 75 | 4.60 |
| Mar. 11, 13 | a | a | a | 98 | 4.72 |

[^0]of 1969 up to the spring of 1971 . This may corresponds to decreasing solar activity during this epoch. Figure 3 shows the correlation between the $\overline{\Delta m}(B)$ and the daily relative sunspot number of Zürich $\left(R_{z}\right)$. The dispersion of $\overline{\Delta m}(B)$ for large values of $R_{z}$ reflects the fact that direct correlation does not exist between these quantities, but between the brightness and the solar flares.


Fig. 3.
(3) Inspection of Figure 1 suggests that the gradient of these correlations is likely to be 2.5 , or rather larger than 2.5. In Figure 1, all the oblique lines have gradient of 2.5 , and we can see that the exceptional case occurred only on the night of July 25 , 1970, where the gradient seems to be smaller than 2.5 . The implication this value of gradient is discussed in the next section.
(4) The color index $B-V$ shows weak dependence on the $\Delta m$. Figure 4 (a) shows it about the B-Region, and (b) about the other regions. Both diagrams show the same tendency except the region near Proclus. The most probable relation is

$$
\begin{equation*}
B-V=-0.2 \Delta m+1.7 \tag{4}
\end{equation*}
$$

On the basis of this relation we can say that the enhancement of the lunar surface has a tendency to reddening. In the previous paper(Sekiguchi, 1971), the author mentioned


Fig. 4 a.


Fig. 4b.
that the color index of moonlight was 0.1 mag bluer on the night of March 26, 1970, when the enhancement was observed. But this difference was within the limit of the observational dispersions.

In Figure 5, we see that the dependency of the color index $B-V$ on the phase angle $\alpha$ is very weak. Here the author failed to confirm the results of Mikhail (1970), which showed the tendency to reddening towards small values of the phase angle.


Fig. 5.

In the present study, the author could not distinguish the color differences between the various regions of the lunar surface. The order of magnitude of these differences may be smaller than 0.1 mag , which is below the limit of confidence of the present colorimetric observations.

## 4. Relations Between the Brightness of the Moon's Surface and the Polarization Degrees

The implications of the fact that the gradient values of $\Delta m$ against $\log P_{v}$ is 2.5 , are very serious. The most direct interpretation of the relation between these quantities is that the $k$-part of the Moon's surface is covered by a white and less polarizing material and ( $1-k$ )-part is covered by a dark and more polarizing one. Let the two components of light coming from the latter part be $I_{1}$ and $I_{2}$, and those from the former part be $I_{1}^{\prime}$ and $I_{2}^{\prime}$. Then the polarization degree $P_{v}$ must be

$$
\begin{equation*}
P_{v}=\frac{(1-k)\left(I_{1}-I_{2}\right)+k\left(I_{1}^{\prime}-I_{2}^{\prime}\right)}{(1-k)\left(I_{1}+I_{2}\right)+k\left(I_{1}^{\prime}+I_{2}^{\prime}\right)} \tag{5}
\end{equation*}
$$

and we have, on the above assumptions,

$$
\begin{equation*}
I_{1}^{\prime}+I_{2}^{\prime}>I_{1}+I_{2} \quad \text { and } \quad \frac{I_{1}^{\prime}-I_{2}^{\prime}}{I_{1}^{\prime}+I_{2}^{\prime}}<\frac{I_{1}-I_{2}}{I_{1}+I_{2}} \tag{6}
\end{equation*}
$$

The right member of the latter inequality, denoted by $P_{0}$, must be the polarization degree when the entire surface is covered by the dark material. If we take $\Delta m^{\prime}$ as the magnitude difference compared with this case, we have

$$
\begin{equation*}
\Delta m^{\prime}=2.5 \log \frac{I_{1}+I_{2}}{(1-k)\left(I_{1}+I_{2}\right)+k\left(I_{1}^{\prime}+I_{2}^{\prime}\right)} \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
\log \frac{P_{v}}{P_{0}}=\log \frac{\left(I_{1}+I_{2}\right)\left\{(1-k)\left(I_{1}-I_{2}\right)+k\left(I_{1}^{\prime}-I_{2}^{\prime}\right)\right\}}{\left(I_{1}-I_{2}\right)\left\{(1-k)\left(I_{1}+I_{2}\right)+k\left(I_{1}^{\prime}+I_{2}^{\prime}\right)\right\}} \tag{8}
\end{equation*}
$$

For small values of $k$, we can eliminate it and we get

$$
\begin{equation*}
\frac{\Delta m^{\prime}}{\log \frac{P_{v}}{P_{0}}}=2.5 \frac{\left(I_{1}^{\prime}+I_{2}^{\prime}\right) /\left(I_{1}+I_{2}\right)-1}{\left\{\left(I_{1}^{\prime}+I_{2}^{\prime}\right) /\left(I_{1}+I_{2}\right)-1\right\}+\left\{1-\left(I_{1}^{\prime}-I_{2}^{\prime}\right) /\left(I_{1}-I_{2}\right)\right\}} \tag{9}
\end{equation*}
$$



Fig. 6.
which is necessarily less than 2.5 owing to conditions (6). Figure 6 shows the relation between $\Delta m^{\prime}$ and $\log \left(P_{v} / P_{0}\right)$ for the values of $k$ from $k=0$ to $k=1$, corresponding to an example where

$$
\begin{equation*}
I_{1}^{\prime}+I_{2}^{\prime}=2\left(I_{1}+I_{2}\right), I_{1}^{\prime}-I_{2}^{\prime}=\frac{1}{2}\left(I_{1}-I_{2}\right) \quad \text { and } \quad \frac{I_{1}-I_{2}}{I_{1}+I_{2}}=0.1 \tag{10}
\end{equation*}
$$

This diagram shows the mean gradient of 1.25 , far less than the observed one. As
pointed out in the former section, the observed gradients are rather larger than 2.5 .
In order to make the ratio (9) equal to 2.5 , it is necessary to put

$$
\begin{equation*}
I_{1}-I_{2}=I_{1}^{\prime}-I_{2}^{\prime} \tag{11}
\end{equation*}
$$

which is equivalent to assuming that a component of non-polarized light is superposed on a basic polarized one at the bright part of the Moon's surface. On the basis of this assumption, we can put

$$
\begin{equation*}
P_{v}=\frac{I_{1}-I_{2}}{I_{1}+I_{2}+I_{1}^{\prime}+I_{2}^{\prime}} \tag{12}
\end{equation*}
$$

and

$$
\begin{equation*}
\Delta m^{\prime}=2.5 \log \frac{I_{1}+I_{2}}{I_{1}+I_{2}+I_{1}^{\prime}+I_{2}^{\prime}} \tag{13}
\end{equation*}
$$

in place of (5) and (7). Thus we have

$$
\begin{equation*}
\Delta m^{\prime}=2.5 \log \frac{P_{v}}{P_{0}} \tag{14}
\end{equation*}
$$

By this assumption, we can regard $\mathrm{P}_{0}$ as the polarization degree when the superposed non-polarized light is completely vanished. The author would like to make the further assumption that the residuals from Hapke's formula may be constant for wide parts of the Moon's surface, when the superposed light has completely vanished. The basis of this assumption is the fact that the observed residuals from various regions in one night are ruled by a single relation as the author pointed out in the former section. Let $m_{0}$ be the residuals in this case, then we can put

$$
\begin{equation*}
\Delta m \quad m_{0}+2.5 \log \frac{P_{v}}{P_{0}} \tag{15}
\end{equation*}
$$

The value of $m_{0}$ is indeterminate from the observations. But if we assume that the faintest observations on the darkest days, for example on December 16, 1970, and January 14 and March 11, 1971, the observed brightness attains this limit, we can infer that, approximately,

$$
\begin{equation*}
m_{0}=5.0 \tag{16}
\end{equation*}
$$

Putting this value into (12), we can determine values of $P_{0}$ for each night. In Figure 1, the double circles are the intersecting points of $\Delta m=5.0$ and the oblique lines which fit best to the observed points and have the gradient 2.5 . As we find from the expression (15), the abscissae of the double circles give the values of $P_{0}$. These quantities must be a unique function of the phase angle $\alpha$. The Figure 7 shows this relation, and one can see that the obtained values of $P_{0}$ form a smooth curve, and this fact makes justifiable the above assumption that the residuals from Hapke's formula may be a constant for a wide part of the Moon's surface. The smoothed curve in Figure 7 can be named the 'proper polarization degree', and is denoted by $P(\alpha)$. The branches to
negative infinity at $\alpha=23^{\circ}$ correspond to the rotation of the polarization plane near this phase angle.

Thus we get an empirical formula for the brightness of the wide part of the Moon's surface, expressed in visual magnitude per $\left(1^{\prime \prime}\right)^{2}$, as

$$
\begin{equation*}
m=5.0-2.5 \log H+2.5 \log \frac{P_{v}}{P(\alpha)} \tag{17}
\end{equation*}
$$

where $H$ is the variable part of Hapke's formula expressed by (2) and (3).


Fig. 7.

## 5. Discussion

The above relation (17) seems to be valid for a wide part of the Moon's surface. The question then arises: what is the superposed non-polarized component? As we saw in the above section, this phenomenon has a close correlation with the solar activity, especially with the solar flares. This strongly suggests that the component is caused by the solar wind of high energy reaching the Moon. As Kopal $(1965,1969)$ suggested, the relation between the brightness and the polarization degrees may be a result of the luminescence caused by the solar activity, and the present results may give one of the favorable arguments supporting this view. The wide validity of the single expression (17) suggests that the brightness of the bright part of the Moon - for
example, the mountainous regions as well as the bright rays and spots in the mariaall may be caused by the luminescent effect.

The reader may point out that the accuracy of the observations shown in Figure 1 is not sufficient to establish the present conclusions. Especially the observations of the polarization degrees in small phase angles exhibit large dispersion. Further confirmation by more accurate and widely distributed observations are necessary.

## Acknowledgements

As the author mentioned in his previous paper, the present investigation owes much to the technical support of Mr J. Matsumoto of the Tokyo Astronomical Observatory. The reductions of the observations were chiefly performed by Mrs F. Miyamoto of the same observatory. The author expresses his profound appreciation to both colleagues. Finally he also wishes to express his gratitude to Prof. Z. Kopal of the University of Manchester for his encouragement in these studies.

## References

Arthur, D. W. G. and Whitaker, E. A. (eds.): 1960, Orthographic Atlas of the Moon, University of Arizona Press.
Hapke, B. W.: 1963, J. Geophys. Res. 68, 4571.
Iriarte, B., Johnson, H. L., Mitchel, I., and Wisniewski, W. K.: 1965, Sky Telesc. 30, 21.
Kopal, Z.: 1965, in The Nature of the Lunar Surface, (ed. by W. N. Hess, D. H. Menzel and J. A. O'Keefe), Johns Hopkins Press, Baltimore, p. 173.
Kopal, Z.: 1969, The Moon, D. Reidel, Publ. Co., Dordrecht, Holland, p. 423.
Mikhail, J. S.: 1970, The Moon 2, 167.
Secchi, A.: 1859, Monthly Notices Roy. Astron. Soc. 19, 289.
Sekiguchi, N.: 1971, The Moon 2, 423.
Teyfel, V. G.: 1960, Astron. Zh. 37, 703.


[^0]:    ${ }^{\text {a }}$ Same figures as those of above line.

