STABILITY OF THE SOLAR SYSTEM: EVIDENCE FROM THE ASTEROIDS*

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Abstract. An analysis of the distribution of the orbital periods of the asteroids has shown that there is a preference for these periods to be near-commensurate with that of Mars. We suggest that this preference is associated with a formation process and implies that the orbital period of Mars has not changed greatly since the time of asteroid formation. We deduce from this that the solar system is highly stable and long-period gravitational perturbations have probably had little influence on the gross evolution of the solar system.

Anders (1965) considers that the present distribution of asteroids, despite the effects of collisions and perturbations by Jupiter, is not too far removed from the original distribution. Thus a study of the asteroids and their distribution may give an insight into the process of planetary formation. Dermott (1968a, b and 1971) has suggested that the marked preference for near-commensurability found among pairs of mean motions in the solar system reflects a condition of formation, secondary bodies forming preferentially in near-commensurate orbits. In this paper we present a method of testing this hypothesis. If the terrestrial planets and the asteroids condensed out of the primeval solar nebula before the jovian planets then we should expect to find some evidence that the asteroids were formed preferentially in orbits near-commensurate with that of Mars.

A histogram of the orbital periods of the 1644 asteroids with period $3<T<6$ yr tabulated in the *Ephemeridy Malych Planet* (1971) is shown in Figure 1. The well-known gaps in the distribution produced by resonance with Jupiter correspond to $T/T_{♃}=\frac{1}{2}, \frac{1}{3}, \frac{2}{5}$, and $\frac{3}{7}$ The presence of these large gaps makes it difficult to determine the original distribution of the asteroids. We present a method, based on that used by Dermott (1968a), which enables us to detect any preference for the asteroid periods to be near-commensurate with any other period and, for periods not commensurate with Jupiter, reduces the effect of the Jupiter gaps.

For each asteroid form the ratio R, where

$$R = T/T_p \quad \text{if} \quad T < T_p,$$
$$R = T_p/T \quad \text{if} \quad T > T_p,$$

and T_p is some orbital period. If $A_2/A_1\,(A_2<A_1)$ and $A_2'/A_1'\,(A_2'<A_1')$ (where A_1,

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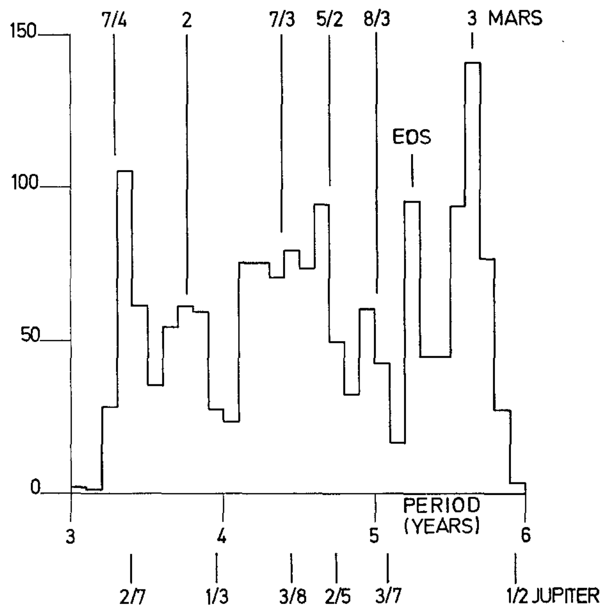

Fig. 1. Histogram of asteroid orbital periods. Periods commensurate with the orbital period of Mars are shown at the top of the diagram, those commensurate with the orbital period of Jupiter at the bottom. All multiples of these periods which can be formed with two integers $\leqslant 8$ and which lie in the range 3 to 6 yr are shown.

A_2, A'_1, A'_2 are small integers $\leqslant$ some arbitrary limit I) are the two fractions which bound R most closely from above and below, then we define

$$a = \frac{R - A'_2/A'_1}{A_2/A_1 - A'_2/A'_1}.$$

The number a varies from 0 to 1, the preference for commensurability increasing both as $a \to 0$ and as $a \to 1$. For ease of presentation we define the deviation of R from a whole number ratio to be

$$\delta = a - b$$

where

$$b = 0 \quad \text{if} \quad a \leqslant \tfrac{1}{2}$$
$$b = 1 \quad \text{if} \quad a > \tfrac{1}{2}.$$

Thus $-\frac{1}{2} \leqslant \delta \leqslant +\frac{1}{2}$ and the preference for commensurability increases as $\delta \to 0$.

It can be shown from classical perturbation theory that the dynamical consequences of near-commensurability decrease as $A_1 - A_2$ increases. The above procedure is followed using $I = 3$, 4, 5 and 7. The distribution of δ for T_p = Jupiter's period and T_p = Mars' period is shown in histogram form in Figure 2. The results, particularly for $I = 4$, indicate that the orbital periods of the asteroids have a preference to be near-commensurate with that of Mars.

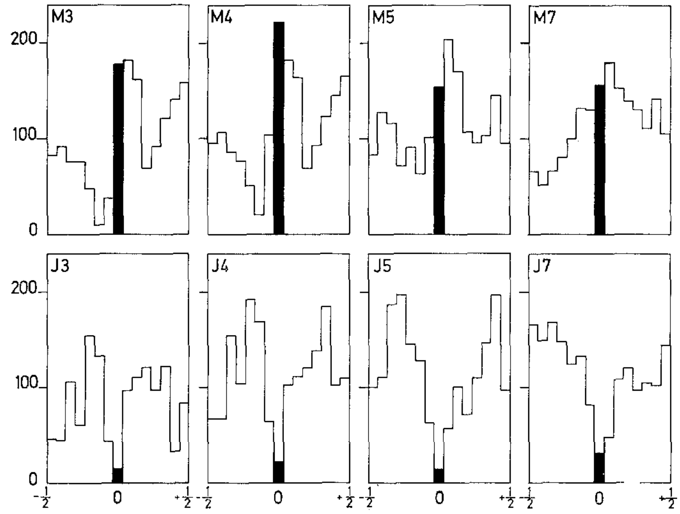

Fig. 2. Histograms of δ distributions. The four histograms at the top refer to Mars, the four at the bottom to Jupiter. The number in the top left corner of each histogram is I. Each histogram is divided into 15 boxes. The central box is shaded; the content of this box gives a measure of the preference for near-commensurability.

The content of the central box in any one of the histograms in Figure 2 gives a measure of the preference for commensurability of the asteroid orbital periods with respect to a period T_p. We define P, the preference for commensurability, by

$$P = \frac{BN_C}{N_B},$$

where B, which is always chosen to be odd, is the number of boxes in the histogram, N_C the content of the central box, and N_B the total content of the B boxes.

Let f be the total number of fractions (F) formed with mutually prime integers $\leqslant I$. If $I = 4$ the fractions are

$$\tfrac{1}{4}, \tfrac{1}{3}, \tfrac{1}{2}, \tfrac{2}{3}, \tfrac{3}{4}$$

and $f = 5$. If $I = 7$ then $f = 17$. If peaks in the period histogram (Figure 1) occur at periods $K_n (n = 1, 2, \ldots, m)$ and gaps at periods $G_n (n = 1, 2, \ldots, l)$ then a plot of P against T_p will have maxima and minima at periods X_n and N_n respectively, where

$$\begin{aligned} X_n &= K_n F \quad \text{if} \quad X_n < K_n, \\ &= K_n / F \quad \text{if} \quad X_n > K_n, \\ N_n &= G_n F \quad \text{if} \quad N_n < G_n, \\ &= G_n / F \quad \text{if} \quad N_n > G_n. \end{aligned}$$

The total number of maxima and minima will be $2mf$ and $2lf$ respectively.

A plot of P against T_p is a particularly sensitive method of detecting any preference for the asteroid periods to be near-commensurate with some unknown period. If the peaks in the period histogram (Figure 1) occur at periods K_n which are near-commensurate with some period T_c then at T_c in the plot of P against T_p, providing I is suitably chosen, the peaks are superposed and a large P results: effectively, the superposition of the peaks, which only occurs if the K_n are near-commensurate with T_c, gives an enhanced 'signal to noise' ratio. The gaps in the period histogram (Figure 1) will produce deficiencies of numbers in certain parts of the δ-histogram (Figure 2), but if the G_n are not near-commensurate with T_c then these deficiencies will be randomly distributed throughout the δ distribution and thus their effect minimised.

Plots of P against T_p for $I = 4$ and periods near that of Mars are shown in Figures 3 and 4. With $I = 4$ the asteroid distributions around $T = 2T_♂$ and $T = 3T_♂$ are superposed. P is a maximum at $T_p = 1.871$y: the period of Mars is 1.881y. The peak is much larger

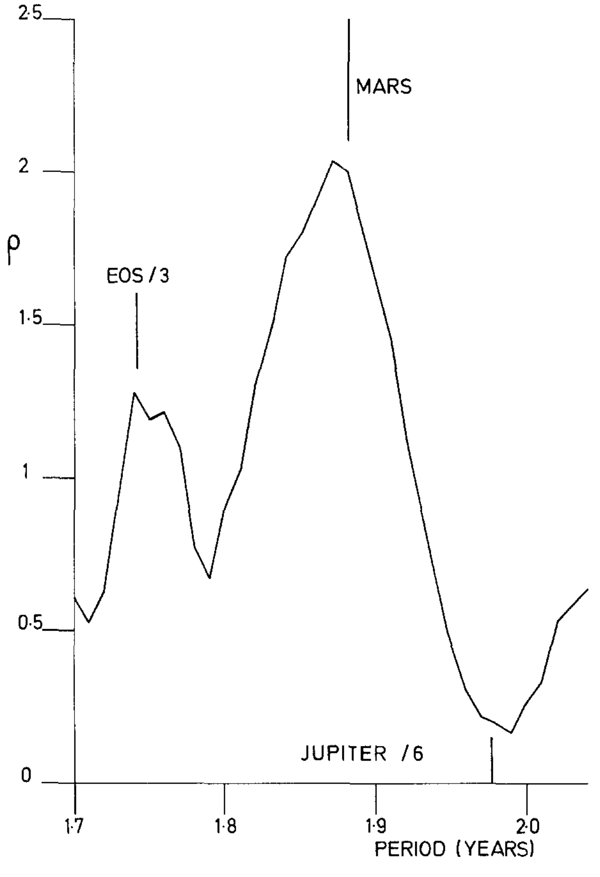

Fig. 3. Variation of the preference for near-commensurability, P, with period, T_p. $I = 4$ and P is calculated at intervals of 0.01 yr.

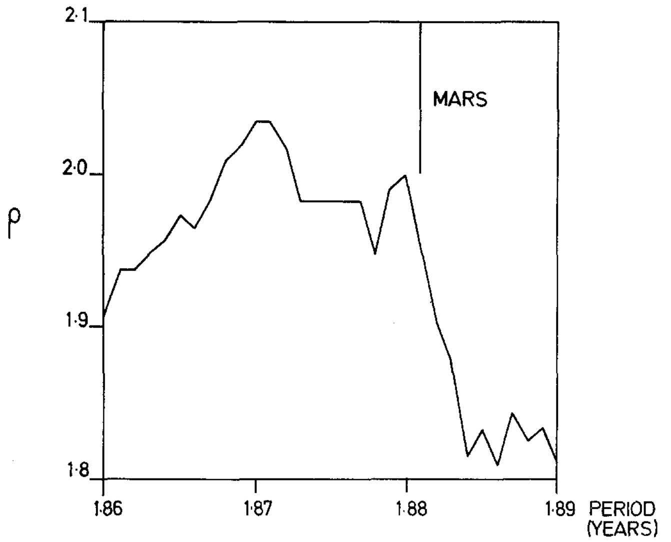

Fig. 4. $I = 4$ and P is calculated at intervals of 0.001 yr. The top of the peak shown in Figure 3 is shown here in more detail.

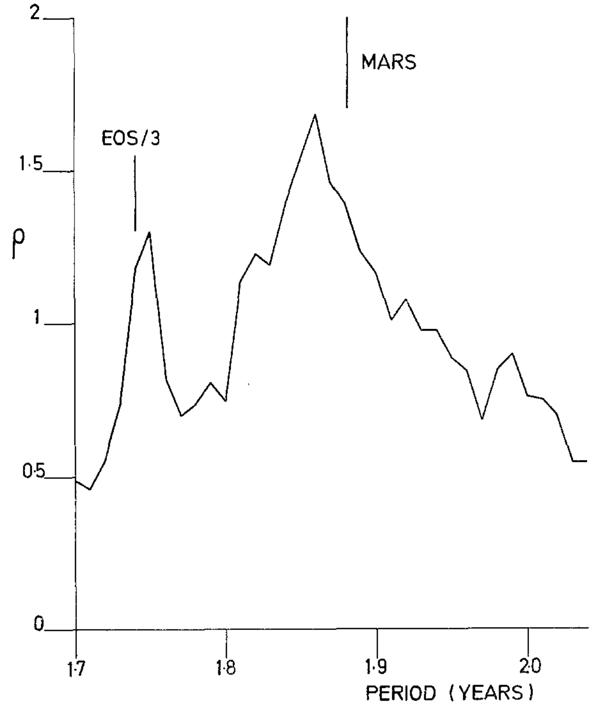

Fig. 5. $I = 7$ and P is calculated at intervals of 0.01 yr.

than that at $T_p = 1.74$y which can be ascribed to the Eos family of asteroids. At $T_p = 1.99$y a minimum occurs; this can, of course, be ascribed to Jupiter.

At $T_p = T_♂$ $P = 1.94$; there are almost twice as many asteroids at the near-commensurate points ($2T_♂$, $3T_♂$) than would be expected if the asteroids were randomly distributed. As there are 1743 asteroids involved in the analysis and $15 (= B)$ boxes in the histogram then the excess number of asteroids is ~ 10 times the standard deviation. Note, however, the distribution of δ cannot strictly be compared with a random distribution. The number of pronounced gaps in the period histogram (Figure 1) is not large compared with the number (B) of boxes in the δ histogram (Figure 2) and therefore the effects of Jupiter on the distribution cannot be completely ignored.

A plot of P against T_p for $I = 7$ and periods near that of Mars is shown in Figure 5. At $T_p = T_♂$, with $I = 7$, the asteroid distributions around $T = 7T_♂/4$, $T = 2T_♂$, $T = 7T_♂/3$, $T = 5T_♂/2$ and $T = 3T_♂$ are superposed: P is a maximum at $T_p = 1.86$y.

Plots of P against T_p for $I = 7$ and periods near that of Jupiter are shown in Figures 6, 7 and 8. Figure 6 is particularly interesting as it shows a pronounced peak at $T_p \simeq 7T_♂$. As $T_♃/T_♂ = 6.30 \simeq 25/4$, $5T_♂/2 \simeq 2T_♃/5$ and thus we cannot hope to detect a peak in Figure 1 at exactly $5T_♂/2$ because of the effect of Jupiter. At $T_p = 7T_♂$, with $I = 7$, the asteroid distributions around $T = 7T_♂/4$, $T = 2T_♂$, $T = 7T_♂/3$ and $T = 3T_♂$, but not around $T = 5T_♂/2$, are superposed: P is a maximum at $T_p = 7 \times 1.873$y. Also shown in Figure 6 are peaks at $35T_♂/6$ and $6T_♂$. At $T_p = 35T_♂/6$, with $I = 7$, the asteroid distribu-

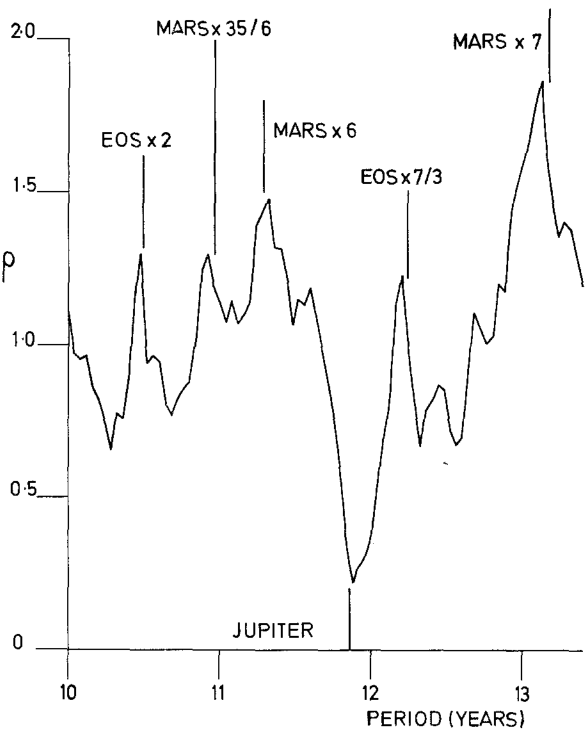

Fig. 6. $I = 7$ and P is calculated at intervals of 0.04 yr.

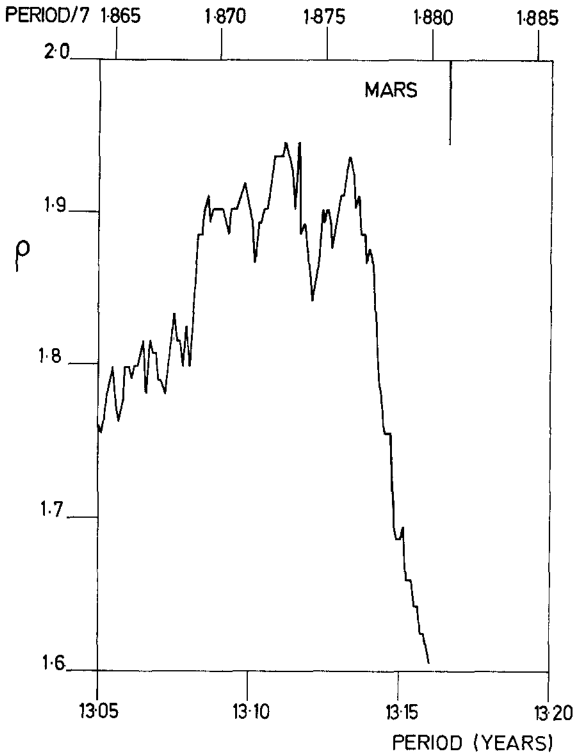

Fig. 7. $I = 7$ and P is calculated at intervals of 0.001 yr. The top of the peak shown in Figure 6 is shown here in more detail. From the scale at the top of the diagram, it can be seen that the peak occurs at approximately $7T_♂$.

tions around $T = 7T_♂/4$, $7T_♂/3$ and $5T_♂/2$ are superposed: P is a maximum at $T_p = 35/6 \times 1.874$y. At $T_p = 6T_♂$, with $I = 7$, the asteroid distributions around $T = 2T_♂$ and $T = 3T_♂$ are superposed: P is a maximum at $T_p = 6 \times 1.884$y.

It is important to note that the peak in Figure 6 at $T_p \simeq 7T_♂$ is larger than the other peaks and that there are other, smaller, peaks which are not related to the same peaks in the period histogram (Figure 1). An extended plot of P against T_p can be interpreted in the same way as that in Figure 6: the peaks can be ascribed to Mars and the Eos family and the dips to Jupiter.

We conclude that our results are consistent with the hypothesis that the asteroids were formed preferentially in orbits near-commensurate with that of Mars. We have not, however, proved that this is the only reasonable explanation of the observed asteroid distribution. Although the effects of Jupiter can be minimized they cannot be ignored completely. But the close agreement between those periods (for various I) which make P a maximum and the period of Mars and the fact that the maximum value of P is high (~ 2) suggest that the results are not due to chance alone. This requires further investigation.

The implications of this work are twofold:

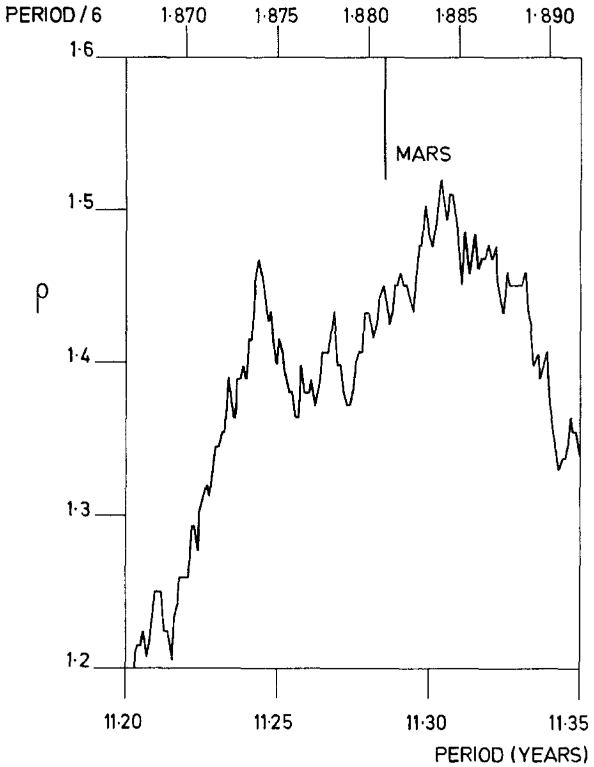

Fig. 8. $I = 7$ and P is calculated at intervals of 0.001 yr.

(i) The asteroids were formed under the dynamic influence of Mars in the absence of Jupiter.

(ii) The solar system is dynamically stable over long periods of time. If the preference for the asteroid periods to be near-commensurate with that of Mars is associated with a formation process then as the preference can be observed today the orbital period of Mars cannot have changed greatly since the time of asteroid formation. Long period gravitational perturbations, therefore, have probably had little influence on the gross evolution of the solar system.

Other features of the asteroid distribution which suggest that the solar system is dynamically stable are:

(i) Gaps in the orbital period distribution produced by resonance with Saturn at $T_♄/6$, $T_♄/7$, $T_♄/8$ and $T_♄/9$.

(ii) Groups of large asteroids with almost the same orbital periods. These features will be discussed in a following paper.

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