LUNAR PHOTOELECTRON LAYER DYNAMICS*

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Abstract. Possible waves and oscillations in the lunar photoelectron layer (PEL) are investigated. The steady state PEL is reviewed as a basis for discussing PEL motions. Magnetic fields are neglected, so that there are four possible wave modes to consider. The propagation through the PEL of the two electromagnetic modes is discussed. Positive-ion waves, the third mode, are dismissed and plasma waves are considered at length. It is concluded that there are no propagating waves in the PEL other than electromagnetic. However, there is a type of oscillation which appears to be new and which may not be strongly damped. With these oscillations, termed flight-time oscillations, the height of the PEL fluctuates as does the electric field. These oscillations appear to be analogous to the height oscillations of the vertical jet of water in a city park water fountain. If flight-time oscillations are not much damped then it would be simplest to interpret them as plasma oscillations continually driven by the upwelling photoelectron stream. A possible laboratory investigation of these oscillations is discussed. For the surfaces of the Moon and the planet Mercury, the flight-time oscillation frequency, ω_F, is found to be respectively $\sim 4 \times 10^6$ and $\sim 10^7$ rad s^{-1}. The PEL's of those surfaces may be in a state of continual vertical 'quivering' due to flight-time oscillations, or may be quiescent.

1. Introduction

In an earlier paper (Walbridge, 1973), hereafter referred to as Paper 1, we examined the structure and consequences of the steady state lunar photoelectron layer (PEL). In this paper we look into the possibility of dynamic phenomena, that is, waves and oscillations, in the PEL. Dynamic phenomena have hitherto been little investigated. However, Manka and Anderson (1968) found an upper limit to the electric field components of MHD waves should such waves exist in the PEL.

As concluded in Paper 1, the PEL is a sea of hot ($\sim$15000K) photoelectrons extending to a height of about 2 m above the lunar surface. It is natural to think that if this electron sea is disturbed at some point a series of ripples will spread through the PEL, such as ripples in a pond. Indeed, these ripples might expand over the entire sunlit hemisphere of the Moon, providing the damping length were sufficiently great.

2. Possible Wave Modes

In order to discuss PEL waves, it is first necessary to fix our picture of the steady state PEL. We will consider the waves to be perturbations in the steady state 'most likely' range 2 PEL discussed in Paper 1 (see Figures, 1, 7, 8, 9 and 10 and Table 2 of Paper 1).

The scale height of the PEL is one Debye length (λ_D), and since this scale height is not large compared to λ_D the PEL is not a plasma by Langmuir's definition. The PEL's

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non-electrical neutrality is fully consistent with the λ_D scale height. Indeed, it is precisely this non-neutrality that limits the scale height to λ_D.

We take as negligible any intrinsic magnetic field of the Moon (Ness *et al.*, 1967), and the photoelectron gyroradius in the 5γ interplanetary field is ~ 1 km which is large compared to the 2 m height of the PEL. Hence, magnetic field effects can be neglected altogether. The four wave modes (Spitzer, 1962) then divide into two types: electromagnetic and electrostatic. For the two electromagnetic modes the phase velocity V_{PH} is given by

$$V_{PH}^2 = \frac{c^2}{1 - \omega_p^2/\omega^2},$$

where c is the velocity of light and here ω is 2π times the frequency of the wave. $\omega_p = (4\pi n_e e^2/m)^{1/2}$ is the plasma frequency, n_e the electron density, and e and m the electron charge magnitude and mass respectively. For wavelengths small compared to the dimension of the PEL the wave will be little effected by the PEL. For longer wavelengths there will be more of an effect. Since ω_p increases toward $x=0$ (x is the coordinate perpendicular to the lunar surface, and increases upward, with $x=0$ at that surface), the PEL will tend to direct the wave vector away from the Moon's surface.

The other two wave modes are 'positive-ion waves', which can not exist in the PEL since there are no ions in the PEL, and plasma waves. By plasma waves we mean the electrostatic electron waves first investigated by Tonks and Langmuir (1929) and later by Landau (1946).

For plasma waves the electric field and perturbation electron density at a point in space both vary sinusoidally in time with the frequency

$$\omega = \pm \sqrt{\omega_p^2 + \left(\frac{2\pi}{\lambda}\right)^2 \frac{3kT}{m}},$$

where λ is the wavelength; k, Boltzmann's constant; and T, the photoelectron temperature. This expression for ω is valid for

$$\left[\left(\frac{2\pi}{\lambda}\right)^2 \frac{3kT}{m} \bigg/ \omega_p^2\right] = 3\left(\frac{2\pi}{\lambda}\right)^2 \lambda_D^2 \ll 1 .$$

In the long wavelength limit approximately $\omega = \pm\omega_p$.

For the PEL, $\lambda_D = 13$ cm (Paper 1). Plasma waves are strongly damped unless $(2\pi/\lambda)\,\lambda_D \ll 1$. Since most of the electrons are confined to below 30 cm, we take $\lambda = 30$ cm as the maximum wavelength for vertical oscillations. Clearly any vertical plasma oscillations will be strongly damped. However, horizontally propagating waves could have $(2\pi/\lambda)\,\lambda_D \ll 1$ and so be only weakly Landau-damped.

Now plasma waves are usually considered in a uniform infinite plasma. For the finite height PEL, one expects horizontally propagating plasma waves to be modified to the form shown in Figure 1. There is an important qualitative difference between the electrons in the uniform, infinite medium associated with conventional plasma waves and the PEL electrons, viz. the PEL electrons are continually created and absorbed and this occurs at the lunar surface. A typical PEL electron is emitted, rises up, and then is pulled back again to $x=0$. In plasma waves energy is alternately stored as electrostatic potential energy and as kinetic energy of the electrons. The PEL

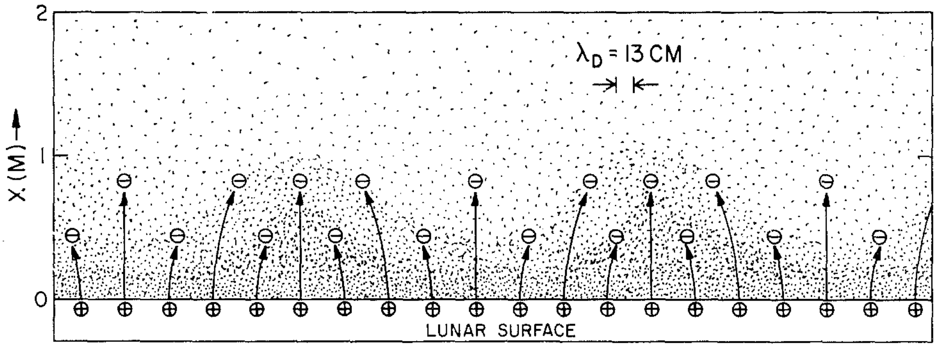

Fig. 1. The electric field configuration (solid lines) and electron density variation for horizontally propagating plasma waves in the PEL. The density of dots is proportional to the electron density. These waves are strongly damped.

electrons which are absorbed at the lunar surface will carry some of the stored kinetic energy with them, and hence damping will occur. This damping will be small if τ_p/τ_T is small compared to one, where $\tau_p = 2\pi/\omega_p$ and τ_T is the average transit time for a PEL electron. The transit time (flight time) is the time that elapses between the instant an electron leaves the lunar surface and the instant it returns to that surface. Now $\tau_p = (\pi m/n_e e^2)^{1/2}$ and we use the simple expression for τ_T obtained from the mono-energetic, vertical photoemission model of case 1, Paper 1, viz. $\tau_T = (\sqrt{2}/\pi)(\pi m/n_e e^2)^{1/2}$. This expression gives the order of magnitude of τ_T in the actual PEL. We have $\tau_p/\tau_T = 2.1$ and so plasma waves are strongly damped. This ratio tells us that before even one plasma oscillation can be completed the electrons carrying the stored kinetic energy will return to the Moon's surface. We conclude then that there are no propagating waves in the PEL other than electromagnetic.

3. Flight Time Oscillations

However there is a type of oscillation which appears to be new, is not fully understood, and which may not be strongly damped, viz. flight-time oscillations. In these oscillations the height of the PEL fluctuates as does the electric field, although the electric field is always directed vertically upward.

It is not hard to understand the physical basis for flight time oscillations. If initially the number of PEL electrons is greater than its steady state value, then immediately after these surplus electrons have returned to the lunar surface the electric field E will have returned to its steady state value. However, those electrons emitted when the electron density and hence E were greater than in the steady state, will return to the surface before the time at which they would have returned in the steady state. These early returnings will decrease E below its steady state value. That is, E will decrease below its steady state value due to the lag between the time the electrons 'see' the stronger E and the time they return to the surface. When E has fluctuated below its steady state value electrons will begin to have somewhat longer than average flight times and these longer flight times will, after an appropriate lag, increase E again, etc.

These considerations can be made more quantitative. If the electric field at $x=0$ and time t is $E(0, t)$ and the surface charge density is $\sigma(t)$, then

$$E(0, t) = 4\pi\sigma(t). \tag{1}$$

Now we also have that

$$\sigma(t) = eF_0\tau_T(t), \tag{2}$$

where F_0 is the number of photoelectrons emitted per cm^2 s and $\tau_T(t)$ is the transit time for electrons which have just returned to $x=0$ at time t. The validity of Equation (2) is not hard to establish for the case of the monoenergetic, vertical flux model of Paper 1. We consider that if one electron is emitted before another then the second electron never catches up with or overtakes the first. If at time $t-\tau(t)$ we began painting each electron red as it was emitted then at time t the PEL would consist of only red electrons and there would be $F_0\tau_T(t)$ of these per cm^2 column. Equation (2) follows from this and is valid for a general PEL, not just a vertical monoenergetic model. Combining Equations (1) and (2) we obtain

$$E(0, t) = 4\pi eF_0\tau_T(t). \tag{3}$$

For the case of monoenergetic, vertical photoemission we obtain (Appendix) the following simple expression for $\tau_T(t)$

$$\tau_T(t) = 2um/[eE_{av}(t)], \tag{4}$$

where u is the electron speed at emission and $E_{av}(t)$ is the time averaged electric field seen by an electron which has at t, just arrived at $x=0$.

Though Equation (4) was derived for monoenergetic, vertical emission it will be valid, to within a factor of order unity, for the real PEL.

Taking into account the time lag we assume that

$$E_{av}(t) = \beta E(0, t-\tau_1),$$

where $0<\tau_1\leqslant\tau_T$ and β is a dimensionless constant lying in the range $0<\beta\leqslant 1$. $\tau_T = (\sqrt{2}/\pi)(\pi m/n(x=0)\,e^2)^{1/2}$ is the time averaged value of $\tau_T(t)$ (Equation (4)) for small amplitude time fluctuations in $E(x=0)$. For these small amplitude fluctuations $E_1(t)$,

$$E(0, t) = E_0 + E_1(t),$$

where E_0 is the steady-state electric field at $x=0$. Combining this expression with Equations (3) and (4) we find that

$$E(0,t) = E_0 + E_1(t) = [(8\pi F_0 um)/(\beta E_0)]\,[1 - E_1(t-\tau_1)/E_0],$$

where terms of order $(E_1/E_0)^2$ and higher have been dropped. When $E_1 = 0$,

$$E_0 = [(8\pi F_0 um)/(\beta E_0)]\,;$$

so that

$$E_1(t) = -\,E_1(t-\tau_1).$$

Letting

$$E_1(t) = A\,\exp(i\omega t)$$

we find that

$$\exp(i\omega t) = -\exp(iw[t-\tau_1]),$$

which is solved for

$$\omega\tau_1 = \pi.$$

These oscillations occur at a frequency $\omega = \pi/\tau_1$ which we designate as ω_F, the oscillation frequency for flight time oscillations, i.e. $\omega_F = \pi/\tau_1$. These consideration neglect damping so that, actually, ω_F is only the real part of the oscillation frequency.

It is clear from the definition of $E_{\mathrm{Av}}(t)$ (Appendix) that the electron is primarily affected by the electric field early in its trajectory. Hence we assume τ_1 is restricted to

$$\tau_T/2 \leqslant \tau_1 \leqslant \tau_T.$$

The corresponding range of ω_F is

$$2\pi/\tau_T \geqslant \omega_F \geqslant \pi/\tau_T.$$

The period $\tau_F = 2\pi/\omega_F$ is equal, to within a factor of two, to τ_T. This is the reason these oscillations are designated flight time oscillations. For 5×10^3 els cm^{-3} at $x=0$, $\omega_F \approx 4\times10^6$ rad s^{-1}. Since $\tau_T = \tau_p/2.1$ the above range for ω_F corresponds to

$$(2.1)\,\omega_p \geqslant \omega_F \geqslant \left(\frac{2.1}{2}\right)\omega_p,$$

i.e., ω_F is essentially the plasma frequency ω_p. If flight time oscillations are strongly damped then it is simplest to say that they are just plasma oscillations suffering, as expected, from strong Landau damping. If flight time oscillations are not damped then again it seems simplest to consider them to be plasma oscillations, but plasma oscillations which are continually driven by the flux of upwelling, newly ejected photoelectrons. The height oscillations of the vertical jet of a city park water fountain provide an attractive analogy to this latter case. For such a jet the oscillations are driven by the inflowing water. It would be of considerable interest to have a solution of the time dependent Vlasov equation–Poisson equation system for the PEL, so as to determine, theoretically, the extent of damping. Also it should be feasible to investigate flight time oscillations in a terrestrial laboratory by shining a strong UV source on an efficiently photoemitting surface (such as Cs_3Sb) and using an electron beam to look for electric field oscillations.

It might be possible to design an electron beam device that would detect the $\sim 10^6$ cycle s^{-1} flight time oscillations of the lunar surface PEL (Manka, 1970). We obtain $\omega_F \approx 10^7$ rad s^{-1} for the surface of Mercury at the subsolar point, assuming the surface of that planet to have the photoemissive properties assumed in Paper 1 for the Moon.

4. Conclusions

Horizontally propagating plasma waves in the PEL are strongly damped. Hence there are no horizontally propagating waves confined to the PEL. Flight time oscillations, which we identify as vertical plasma oscillations in the PEL, may be strongly Landau-damped as one would first expect, or may be driven by the flux of upwelling electrons. The lunar PEL may be quiescent or may be in a state of continual motion up and down, a 'quivering' motion requiring about 10^{-6} s for each oscillation. That is, a steady state solution such as that found in Paper 1 may not be stable and may decay to a state of continual flight time oscillations with parameters oscillating about their steady state values. Similarly the photoelectron layer on the sunlit side of a spacecraft may be in a state of continuous oscillation rather than in the steady state usually assumed (as, for example, by Whipple (1965) or Guernsey and Fu (1970)). Theoretical and experimental investigations of flight time oscillations would be of interest.

Appendix

Consider the monoenergetic, vertical photoemission PEL model discussed in Paper I but now for the case of vertical oscillations, i.e. for an electric field and electron density depending on time as well as x. The electrons move only in the x direction.

Let $x(t, t_0)$ denote the height at time t of an electron which was emitted at time t_0. We take this electron to move through an electric field $E(x, t)\,\hat{x}$ where $\hat{x}$ is the unit vector in the $+x$ direction. Upon integrating the equation

$$m\frac{\mathrm{d}x^2}{\mathrm{d}t^2} = -eE(x, t)$$

twice and transforming the double integral to a single integral (in the manner of Margenau and Murphy, 1956; p. 532) we obtain

$$x(t, t_0) = u[t - t_0] - \frac{e}{m}\int_{t_0}^{t} (t - t')\, E(x(t', t_0), t')\, \mathrm{d}t'. \tag{A1}$$

For $t=t_0+\tau_T(t)$,

$$x(t_0+\tau_T(t), t_0) = 0 = u\tau_T(t) - \frac{e}{m}\int_{t_0}^{t_0+\tau_T(t)} (t_0+\tau_T(t)-t')\, E(x(t', t_0), t')\, \mathrm{d}t'. \tag{A2}$$

If we define $E_{\mathrm{av}}(t)$ by

$$\tau_T(t)\, E_{\mathrm{av}}(t) = 2\int_{t_0}^{t_0+\tau_T(t)} \frac{(t_0+\tau_T(t)-t')}{\tau_T(t)}\, E(x(t', t_0), t')\, \mathrm{d}t', \tag{A3}$$

it follows that

$$\tau_T(t) = \frac{2u}{\frac{e}{m}E_{\mathrm{av}}(t)}.$$

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