MEASUREMENT OF PHYSICAL LIBRATIONS USING LASER RETROREFLECTORS

J. DERRAL MULHOLLAND
Dept. of Astronomy, University of Texas at Austin

and

ERIC C. SILVERBERG
McDonald Observatory, Ft. Davis, Texas, U.S.A.

1. Introduction

The Apollo Lunar Laser Ranging Experiment (LURE) is a multifaceted program rather than a monolith with a single justification or a single goal. Under the direction of a diverse Investigator Team*, it promises to provide information in the areas of lunar motion, geophysics, lunar physics and dynamical theory (Alley and Bender, 1968; Alley *et al.*, 1969; Bender *et al.*, 1970). Most of these topics are uninteresting in the context of this meeting, but the nature of the experiment is such that, in the actual analysis of the data, the topics cannot be so easily isolated into comfortable pigeonholes and treated separately. We do not propose to describe here the techniques employed in making the measurements or applying them, as they have been well treated elsewhere (Bender *et al.*, 1970; Silverberg and Currie, 1971; Faller, 1971), but rather to discuss only the applicability of these measures to lunar physics and to indicate where we now stand in terms of the adequacy of the data to supply the desired information.

2. Principles of Measurement

Information about the lunar interior can be obtained from measurements of the librations. The apparent librational motion of the Moon, i.e. the deviation of the origin of the selenographic coordinate system from the Earth–Moon centerline, is the combined result of two effects. The *optical libration* is the parallax effect due to the Moon's orbital eccentricity, assuming that the lunar rotation is absolutely constant and equal to the rate of orbital mean longitude. This effect is an angular displacement equal to the perturbed equation of the center, the difference between the osculating true anomaly and the Keplerian mean anomaly. Its maximum amplitude is about 7.9°. Superimposed on this is the *physical libration*, the small departures of the Moon from absolutely uniform rotation. This action is primarily a forced oscillation driven by the gravitational attraction of Earth and Sun acting differentially on the uneven mass

---

* The LURE Team is composed of the following Investigators, in addition to the authors: Carroll O. Alley and Douglas G. Currie, University of Maryland; Peter L. Bender, Joint Institute for Laboratory Astrophysics; Robert H. Dicke and David T. Wilkinson, Princeton University; James E. Faller, Wesleyan University; William M. Kaula, University of California at Los Angeles; Gordon J. F. MacDonald, Council on Environmental Quality; Henry H. Plotkin, Goddard Space Flight Center.

Communication presented at the Conference on Lunar Geophysics, held between October 18–21, 1971, at the Lunar Science Institute in Houston, Texas, U.S.A.

distribution of the Moon; its maximum amplitude is perhaps 0.04°, and it affects both longitude and latitude, shifting the direction of the selenographic pole. There is probably also a small free oscillation.

The optical libration, being purely geometric, is of no particular interest, but a study of the forced libration can reveal some of the global characteristics of the lunar interior. The problem is complicated by the fact that there is no way to measure the physical librations directly, short of installing a transit circle in a lunar-based observatory. Once the total libration is observed, however, the optical libration can be computed to reasonably high precision and removed. Another difficulty arises from the fact that the physical librations are forced by the same attractions that govern the orbital motion. All frequencies that appear in the librations are also dominant in the ephemeris, so that solutions for libration parameters from observations of a single reflector could be highly correlated with orbit parameters or reflector coordinates. In fact, the main parts of the librations due to second degree terms in the lunar gravity field are partly correlated with other parameters, and the addition of third degree terms may make the situation worse. In principle, this problem is overcome by making differential measurements, using one reflector as a reference benchmark. Thus, one requires three reflectors to separate the rotations about the x_2 and x_3 axes completely. Instead of measuring Mösting A with respect to the limb, we measure the difference in range between two arrays; Apollo 14 is about 40° distant from Apollo 11, primarily in longitude, while Apollo 15 completes what is very nearly an equilateral triangle. This should give quite satisfactory bi-rotational separation.

Ideally, the libration measures would consist of differential ranges between the Apollo 11 array and the other two arrays. This requires simultaneous measures to two arrays, which is clearly impossible in practice. Therefore, what one strives for is sequential observations as closely spaced as is operationally feasible, which can be used in a regression analysis to yield the desired solution parameters, in essence mapping the observations to achieve a quasi-simultaneity.

3. Lunar Properties

The Fourier representation of the physical librations will have frequencies established by the orbits of Earth and Moon. The amplitudes of the harmonic terms are functions of the differential moments of inertia (Koziel, 1961)

$$\alpha = \frac{C - B}{A}, \quad \beta = \frac{C - A}{B}, \quad \gamma = \frac{B - A}{C}, \quad \alpha - \beta + \gamma = \alpha\beta\gamma,$$

representing two degrees of freedom. Bender (1971) has estimated that 15 cm range accuracies will permit the present uncertainties in β and γ to be reduced by a factor of about 50. Recent developments in laser technology and in the treatment of systematic error sources promise to make 3 cm accuracies achievable within the coming 2 yr. This would give four figure accuracy in the mechanical ellipticity $f = \alpha/\beta$ and a corresponding improvement in the determination of the dynamical form factor J_2

and the harmonics C_{22} and S_{22}, providing an independent check on the analyses of Orbiter data. Additionally, very high precision should be realized in the differential coordinates Δx_i of the reflectors.

This improvement in f will provide an additional datum in the efforts to clear up the mystery of the anomalous discrepancies in the observed-minus-theoretical rates of lunar perigee and node discussed by Eckert (1965).

In a very interesting development, Eckhardt and Dieter (1971) have recently shown that dissipation effects in the lunar interior will be reflected in a phase shift in the physical librations. The phase shift is a function of γ and the dissipation parameter Q, and it will be particularly evident in the longitude libration, where the amplitude of one term is magnified by a small divisor arising from a nearby singularity. The phase lag σ in this term, whose basic argument is twice the argument of perigee ω, was shown to be

$$\sigma = 12/Q\,.$$

With 3 cm range accuracy, the physical libration should be determinable to about 0.″01 in selenocentric longitude. Since the sin 2ω term has an amplitude of roughly 15″, this corresponds to approximately 10^{-3} radians for σ. Thus, it appears that this term in the longitude libration will be capable of supplying the value of Q if it is anywhere with the expected range. Using the above numbers, the upper limit of determinability by this method is about 10000.

Adequate separation of the phase lag requires data taken over a sizable fraction of a cycle of the argument, so one to two years' accumulation of high-quality data is needed to make the determination.

The characteristic frequencies due to the libration parameters are significantly different from those for the absolute and relative coordinates of the reflectors. In principle, these parameters can be separated reasonably well with one or two years' data, as mentioned earlier. At present, however, such an analysis is limited in practice by the completeness with which the libration theory has been calculated. The best available work (Eckhardt, 1970) has an accuracy of perhaps 0.″3 per term, with a term-by-term truncation at 1″ prior to publication. Use of the published results would lead to range errors of about 5 m, even if exactly the correct values of the libration parameters were used. Hopefully this limitation will be removed by one or more of the several new libration studies now underway. For full utility, it will be necessary that third degree harmonics of the lunar gravity field be included in these computations.

This entire discussion will be compromised if it is found that the free libration has a sensible magnitude, but there is as yet no evidence that this is the case.

4. Status of Data Collection

Single-array ranging to the Apollo 11 retroreflector from the McDonald Observatory began in 1969 August. After early difficulties, the data rate rose to a satisfactory level in 1970 April, went down for various reasons until 1970 October, and has remained satisfactory since, except for occasional periods of bad weather. Differential measurements were begun during the Apollo 14 mission last February. Figure 1 shows the 1971 distribution of days on which ranges were obtained on multiple reflectors, compared with total number of days when any successful ranging occurred. The large jump for the September lunation resulted from a combination of good observing conditions and the availability of the third array carried on the Apollo 15 mission.

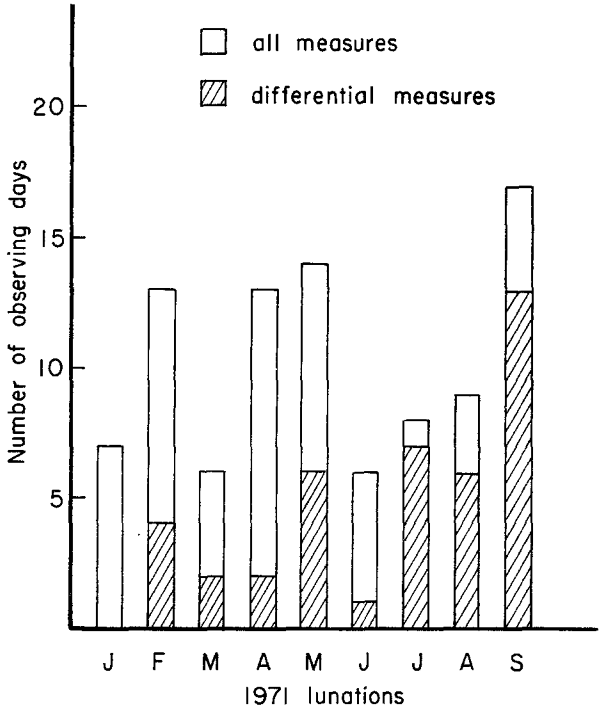

Fig. 1.

To date, these measurements have all been made at ±30 cm resolution, but we expect that ±15 cm will be achieved by the end of the present year. More exciting is the prospect that the year 1973 will see a second ranging station operating at 2 or 3 cm accuracy and modifications to the McDonald station achieving comparable precision. At present, we are substantially limited by the accuracy of the libration model available, but this problem is receiving attention by various investigators and will presumably be improved in the months ahead.

Obviously, our present status represents but a small start towards the determination of β, γ, the retroreflector coordinates and particularly Q, but the promise is there. We look forward with great anticipation towards a resolution of these questions.

Acknowledgements

This work is supported by the National Aeronautics and Space Administration under Grants NGR 44-012-169 and NGR 44-012-219.

References

Alley, C. O. and Bender, P. L.: 1968, in Wm. Markowitz and B. Guinot (eds.), 'Continental Drift, Secular Motion of the Pole, and Rotation of the Earth', *IAU Symp.* **32**, 86.

Alley, C. O., Bender, P. L. *et al.*: 1969, in Runcorn (ed.), *The Application of Modern Physics to the Earth and Planetary Interiors*, Wiley, New York.

Bender, P. L.: 1971, private communication.

Bender, P. L., Dicke, R. H., *et al.*: 1970, 'Laser Ranging to the Moon', Conf. on Experimental Tests of Gravitation Theories, Calif. Inst. of Technology, Pasadena.

Eckert, W. J.: 1965, *Astron. J.* **70**, 787.

Eckhardt, D. H.: 1970, *The Moon* **1**, 264.

Eckhardt, D. H. and Dieter, K.: 1971, *The Moon* **2**, 309.

Faller, J. E.: 1971, 'The Apollo Retroreflector Arrays and a New Multi-Lensed Receiver Telescope', *Space Research* **XII**, in press. Akademie-Verlag, Berlin.

Koziel, K.: 1961, in Z. Kopal (ed.), *Physics and Astronomy of the Moon*, Academic Press, New York.

Silverberg, E. C. and Currie, D. G.: 1971, 'A Description of the Lunar Ranging Station at McDonald Observatory', *Space Research* **XII**, in press, Akademie-Verlag, Berlin.