# A PROPOSED LUNAR ORBITING GRAVITY GRADIOMETER EXPERIMENT


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**Abstract.** Analysis of the gravity gradiometer developed by R. L. Forward and C. C. Bell at the Hughes Research Laboratories suggest than an accuracy, in the range 0.1 to 0.5 EU can be expected in a lunar orbiter application. This accuracy will allow gradient anomalies associated with mascons to be mapped with 1% accuracy and should reveal a great deal of new information about the lunar gravity field.

The proposed experiment calls for putting such a gradiometer into a closely circular polar orbit at an average height of about 30 km above the lunar surface. This orbit allows the entire lunar surface to be covered in fourteen days, the gradiometer to be checked twice per revolution and results in successive passes above the lunar surface being spaced at about the resolution limit of about 30 km set both by the satellite altitude and instrumental integration time. Doppler tracking will be employed and the spacecraft will carry an electromagnetic altimeter. Gradient and altitude data from the far side of the Moon can be stored for replay when communication is re-established.


## 1. The Gradiometer

A dumbbell pivoted about an axis through its center of gravity and perpendicular to its length experiences a torque due to gravity gradients. This torque (Figure 1) is given by

$$T = \tfrac{1}{2} I \sin 2\theta \left( \frac{\partial g_y}{\partial y} - \frac{\partial g_x}{\partial x} \right) + I \cos 2\theta \, \frac{\partial g_y}{\partial x}, \tag{1}$$

where $I = 2mr^2$ is the dumbbell's moment of inertia. This torque is measured statically in the well-known Eötvös torsion balance. In the rotating gravity gradiometer conceived by R. L. Forward and described by Bell (1970) the dumbbell is instead rotated about its axis and, as may be seen by substituting $\theta = \omega t$ in (1), experiences an oscillating torque at twice the rotational frequency. Two such dumbbells 90° apart spinning about a common axis thus experience a differential torque and, if they are coupled by a flexural pivot, will oscillate with a scissorslike motion. In operation the gradiometer is spun at one half the resonant frequency of the scissors oscillation, and the oscillations are detected by means of a piezo-electric transducer attached to the flexural pivot working into an electronic amplifier. The construction of a typical sensor is shown in Figure 2.

This gradiometer has many important advantages. Translational accelerations and


Communication presented at the Conference on Lunar Geophysics, held between October 18–21, 1971, at the Lunar Science Institute in Houston, Texas, U.S.A.

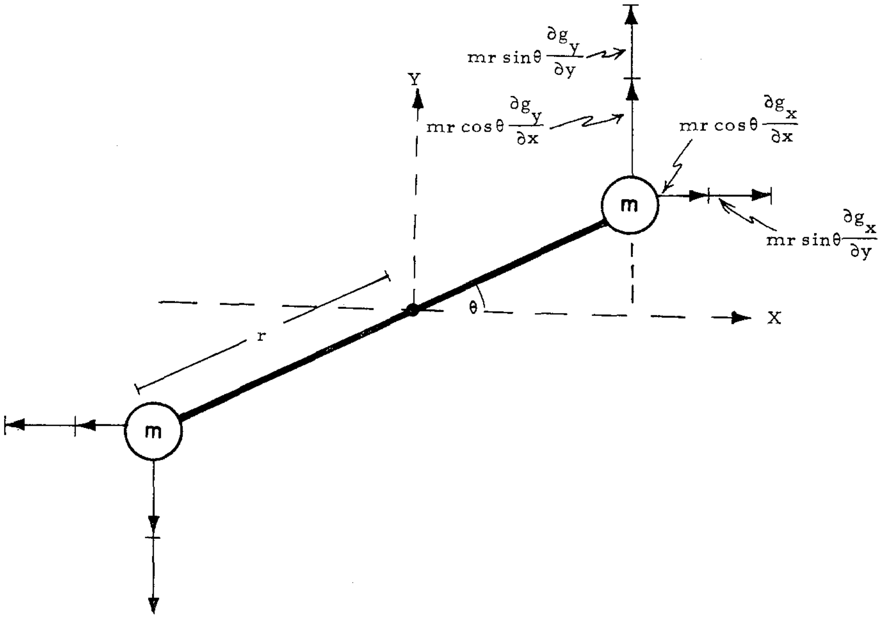


Fig. 1. Differential gravity forces, relative to force at dumbbell center center, acting on mass dumbbell in the $XY$ plane.

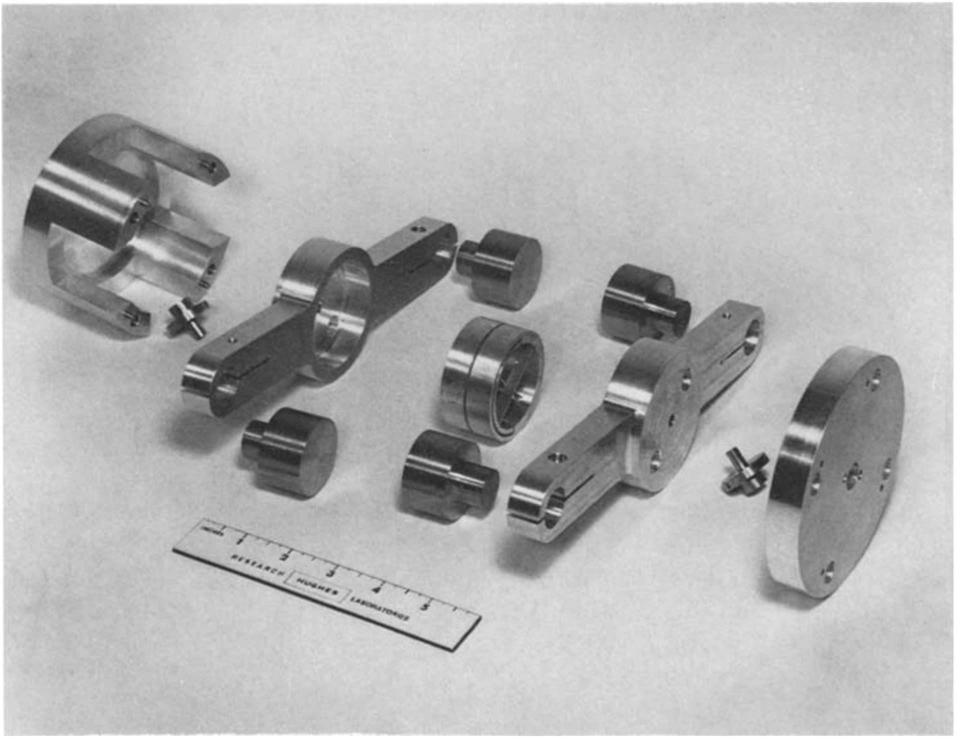

Fig. 2. Exploded view of Hughes gravity gradiometer.

centripetal accelerations due to rotation about the spin axis do not produce torques on the arms; hence it is well suited for use in a spin stabilized spacecraft in a free fall trajectory. At a spin rate of a few revolutions per second both the electronic and mechanical components are extremely stable thus compensating for the very low amplitude of the induced signals by allowing high gain amplifiers to be used. Operation at a mechanical resonance results in an immediate discrimination in favor of gradient induced signal against other large inertial forces produced by satellite nutations. This discrimination can be further increased by subsequent filtering, but the initial discrimination is important in reducing the requirements for linearity in the transducer and electronics. One does not have to worry about gradients due to the mass of the spacecraft itself, as these rotate with the sensor and do not excite oscillations. Finally an instrument of this type has been operating for a number of years and estimates of its performance can thus be based on hard engineering experience in the laboratory. However the instrument has the somewhat unusual property of being better suited to an orbital than to a terrestrial environment and its performance in the laboratory is limited by the terrestrial environment.

Experience with instruments of this type used statically (gravitational antennas) has shown that with carefully designed electronics it is possible to approach the noise limit set by Brownian motion of the sensor. Writing the angular positions of the arms 1 and 2 as $\theta_1$ and $\theta_2$ measured relative to their equilibrium positions 90° apart, in axes rotating with the sensor, the equations of motion may be written ideally as

$$I\ddot{\theta}_1 + \beta\dot{\theta}_1 + k_s\theta_1 + k_c(\theta_1 - \theta_2) = I\left\{\frac{1}{2}\left(\frac{\partial g_y}{\partial y} - \frac{\partial g_x}{\partial x}\right)\sin 2\omega t + \frac{\partial g_y}{\partial x}\cos 2\omega t\right\},$$

$$I\ddot{\theta}_2 + \beta\dot{\theta}_2 + k_s\theta_2 + k_c(\theta_2 - \theta_1) = -\,I\left\{\frac{1}{2}\left(\frac{\partial g_y}{\partial y} - \frac{\partial g_x}{\partial x}\right)\sin 2\omega t + \frac{\partial g_y}{\partial x}\cos 2\omega t\right\}.$$

Subtraction gives, for the differential movement $(\theta_1 - \theta_2)$,

$$I(\ddot{\theta}_1 - \ddot{\theta}_2) + \beta(\dot{\theta}_1 - \dot{\theta}_2) + (k_s + 2k_c)(\theta_1 - \theta_2) = \\ = I\left\{\left(\frac{\partial g_y}{\partial y} - \frac{\partial g_x}{\partial x}\right)\sin 2\omega t + 2\frac{\partial g_y}{\partial x}\cos 2\omega t\right\}; \qquad (2)$$

where $k_c$ is the constant of the spring coupling the gradiometer arms, $k_s$ that of the two support springs, assumed identical. $\beta$ is a damping term which will be written as

$$\beta = (\beta_s + \beta_e),$$

where $\beta_s$ refers to damping due to a process subject to statistical fluctuation and $\beta_e$ refers to electronic feedback damping. The statistical fluctuations in $\beta_s$ may be accounted for by adding a term $N(f)$ to the right-hand side of (2) where $N(f)$ has a flat power spectral density equal to $4\beta_s kT$ (Kittel, 1958, pp. 147–153). If $Z(f)$ is the response function of Equation (2), the response of the gradiometer to the gravity gradient is a line at the resonant frequency $f_r\,(=2\omega/2\pi)$ of squared amplitude

$$I^2\left\{\left(\frac{\partial g_y}{\partial y}-\frac{\partial g_x}{\partial x}\right)^2+4\left(\frac{\partial g_u}{\partial x}\right)^2\right\}|Z(f_r)|^2,$$

and the noise term a continuous spectrum of power spectral density

$$4\beta_s kT\,|Z(f_r)|^2.$$

The output of the gradiometer is now put into a phase sensitive rectifier which has two functions: firstly, by dividing the output into 'in phase' and 'quadrature' components it allows the signal to be divided between the $(\partial g_y/\delta y)-(\partial g_x/\delta x)$ and $2(\partial g_y/\partial x)$ components of gradient; and, secondly, by time averaging the output over an interval $t_{av}$ it allows one to use a narrow band of width $1/t_{av}$ centered on the resonant frequency $f_r$. In this band there will be a signal of amplitude squared

$$I^2\Gamma^2|Z(f_r)|^2,\quad \text{where}\quad \Gamma \quad \text{is either}\quad \left(\frac{\partial g_y}{\partial y}-\frac{\partial g_x}{\partial x}\right)\quad \text{or}\quad 2\frac{\partial g_y}{\partial x};$$

and noise of power of

$$2\beta_s kT\,|Z(f_r)|^2\;1/t_{av},$$

provided $1/t_{av}$ is small compared with the width of the resonance curve $Z(f)$. This noise will be indistinguishable from a randomly varying gradient of root mean square amplitude

$$\Gamma_{\text{rms}}=\frac{1}{I}\sqrt{\frac{2\beta_s kT}{t_{av}}}=2\sqrt{\frac{\pi kT}{t_{av}t_sQ_sI}},$$

with $Q_s=(2\pi I/\beta_s t)$, the quality factor of the gradiometer in the absence of electronic feedback damping and $t_s$, the resonant period of the gradiometer.

For a gradiometer with $t=0.5$ s, $t_{av}=5$ s, $Q_s=350$, $I=10^5$ gm cm$^2$ and $T=300\,°$K, this noise limit is 0.08 EU.

The largest noise source during laboratory operation is due to vibrations transmitted to the instrument through the spin bearings. This is not a problem when the gradiometer is mounted in a spacecraft and the entire craft is spinning, but instead one is faced with nutation and variations in rotational speed of the spacecraft. In addition it is unrealistic to assume that the sensor is ideal; mismatches in arm, moments, moments of inertia and the springs coupling the arms to the spacecraft will introduce unwanted torques.

Accordingly an analysis has been made for a typical gradiometer ($m=500$ g, $r=10$ cm, $f_r=2$ Hz) making rather conservative assumptions about the tolerances in the gradiometer and behavior of the spacecraft. These assumptions are summarized in Table I. The analysis is complex because the various rotational frequencies couple together in a non-linear manner. A summary of the more important differential torques grouped according to source and frequency is listed in Table II. The torques are expressed as ratios to that produced by a 1 EU. vertical gradient. Ratios less than 0.1 have been neglected.

Some of the torques at D.C. and at the nutational frequency are very large. D.C. outputs from the sensor can be blocked in the electronics and there is a 20:1 ratio between the sensor resonant and nutational frequencies, so these large torques do not prevent detection of small gradient signals if efficient frequency discrimination techniques are employed. Although the error torques at the signal frequency $2\omega_s$ are mostly small, there is one significant torque at this frequency which cannot be separated by frequency discrimination. This arises from the modulation of spin rate, due to the gradient torques acting on the unbalanced spacecraft, coupling into the sensor through unbalanced support springs. The 5% difference in transverse moments of inertia can probably be improved upon by careful balancing of the spacecraft

TABLE I

Assumptions in error analysis

| Parameter | |
|---|---|
| Spacecraft spin | $\omega_s = 2\pi$ rad/s |
| Frequency of nutation | $p = 0.1\ \omega_s$ |
| Amplitude of nutation | $A = 10^{-3}$ rad |
| Difference in principal moments of inertia of spacecraft orthogonal to spin axis | $\frac{I_1 - I_2}{I_3} = 0.05$ |
| Distance of gradiometer from c.m. of spacecraft | |
| along spin axis | 1 m |
| off spin axis | $10^{-3}$ m |
| Misalignment of spin axes of sensor and spacecraft | 0.2 rad |
| Mismatch of springs coupling sensor to spacecraft | 0.01 |
| Fractional difference of arm mass unbalance $\frac{\Delta mr}{mr}$ | $2 \times 10^{-6}$ |
| Fractional difference of gradiometer arm inertias | $10^{-4}$ |

TABLE II

Error as ratio of differential torque to that due to 1 EU gradient

| Source | Frequency | | | | |
|---|---|---|---|---|---|
| | $2\omega_s$ | $\omega_s$ | $2p$ | $p$ | 0 |
| Translation due to spin modulation coupled through arm moment unbalance | | | | 7.5 | |
| Translation due to nutation coupled through centripetal accelerations | | | | 75 | $9 \times 10^4$ |
| Spin modulation coupled through sensor arm unbalance | | | | 15 | |
| Misalignment of sensor and spacecraft spin axes | | 45 | 2500 | $3 \times 10^6$ | $2.5 \times 10^9$ |
| Spin rate modulation coupled through support coupling mismatch | 2.2 | 0.25 | 0.75 | $5 \times 10^3$ | |

before launch. A factor of five improvement here would reduce the term to 0.4 EU. As this error is proportional to the gravity gradient it appears as an error in calibration factor. Hence it is essentially removed by in orbit calibration, using the known magnitude of the main field gradient, and does not interfere with the sensor's ability to detect small gradient variations.

The largest terms arise from misalignment of the spin axis of the sensor and the principal axis of the spacecraft. Even though this misalignment does not generate a significant torque at the signal frequency, these terms certainly should be reduced for comfort. A 12° misalignment has been assumed – the difficulty here is in predicting the position of the principal axis of the satellite when the principal moment about the spin axis cannot be allowed to differ greatly from the other moments if the nutational frequency is to be kept low. A solution to this dilemma is to use the sensitivity to this misalignment at the nutational frequency to carry out the final alignment in orbit, either by an internal servo or by external command. In fact, final alignments of sensor position relative to spacecraft center of gravity and balancing of transverse moments of inertia can be accomplished in this manner.

A frequency discrimination equal to the $Q$ of the sensor is obtained immediately by the mechanical resonance. A further discrimination of 12 db per octave can be obtained from the phase lock amplifier by use of a two stage R-C filter on its output, even without any tuned input. A total rejection of terms at the nutational frequency of 10000:1 should be easily attainable and this, together with sensor axis alignment in orbit, should enable the gradiometer to work in the tenths of Eötvös Unit range, while sensor sensitivities better than 0.1 EU accuracy do not appear to be ruled out. The calibration factor must be stable to 1 part in $10^4$ for 0.27 EU accuracy; however the proposed polar orbit has the advantage that the gradiometer repeats measurements twice per orbital revolution and need therefore only maintain this stability over relatively short time intervals. The gradients due to the larger Mascons are expected to be about 40 EU and these can therefore be mapped with an accuracy of 1% or better. These predictions involve analytical extrapolation of ground-based laboratory tests. High confidence in better than say 1 EU performance must await an orbital test for verification.

## 2. The Experiment

The proposed polar orbit has the advantage of covering the entire lunar surface during 14 days. One is therefore free to change the orientation of the gradiometer spin axis, if all has gone well, and to determine a different set of gradient combinations during the next fourteen days. The lower the altitude of the orbit above the lunar surface the larger are the gradients and the greater the resolution. Probably a nominal height of 30 km is as low as is practicable in view of the danger of crashing the spacecraft; however, once the primary mission is accomplished one might wish to execute some low altitude passes over areas of special interest. A measurement spacing of about 10 km is needed to take advantage of the resolution attainable at 30 km height (Figures 3 and 4). An averaging time of 5 s would result in an observation spacing of 8.25 km along the orbit, and successive orbits would be spaced 30 km at the equator.

The spacecraft would be tracked by Doppler techniques while visible; on the far side of the moon the orbit will have to be interpolated from the observed segments. Gradient data from the far side would be stored for playback while the spacecraft is visible from Earth. This far-side information could then be used to improve the orbit determination on the far side. The near side Doppler tracking information should give improved information about its gravity field, owing to the low orbit. Recent estimates of the precision attainable with Doppler tracking techniques indicate that spacecraft positions can be obtained to about 30 m. The vertical gradient of gradient is $-4.5$ EU per km, so the corresponding error in gradient determination is 0.14 EU.

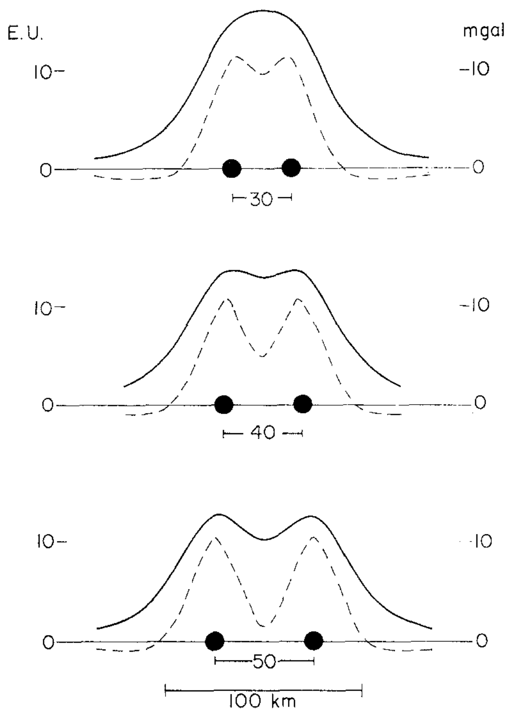


Fig. 3. Computed curves of vertical attraction. (Solid curve) and $(\partial g_y/\partial y)-(\partial g_x/\partial x)$ gradient combination at 30 km height, of two spheres on the lunar surface separated by 30, 40 and 50 km.

The sensor's spin axis would be nominally perpendicular to the orbital plane. Its output is relatively insensitive to errors in spin axis position as long as it remains horizontal (that is perpendicular to a radius), but an uncertainty in tilt of 0.6° will produce an error of 0.27 EU. Spin axis orientation will thus have to be determined to about 0.5° which appears quite practicable. However separation of the $(\partial g_y/\partial_y)-(\partial g_x/\partial x)$ and $\partial g_y/\partial x$ gradient combinations with an accuracy of 0.1 EU requires knowing the $x$ and $y$ directions with respect to the spacecraft to about 1 min of arc. This will probably be impossible and it will be necessary to settle for measuring only the gradient magnitude. In practice this means measuring $(\partial g_y/\partial y)-(\partial g_x/\partial x)$ with $x$ measured along the orbit and $y$ vertically, owing to the dominance of the lunar gravity field by the central term. This is analogous to interpreting terrestrial gravity anomalies as anomalies in the vertical component of gravity, neglecting deflections of the vertical, and should not be an important limitation.

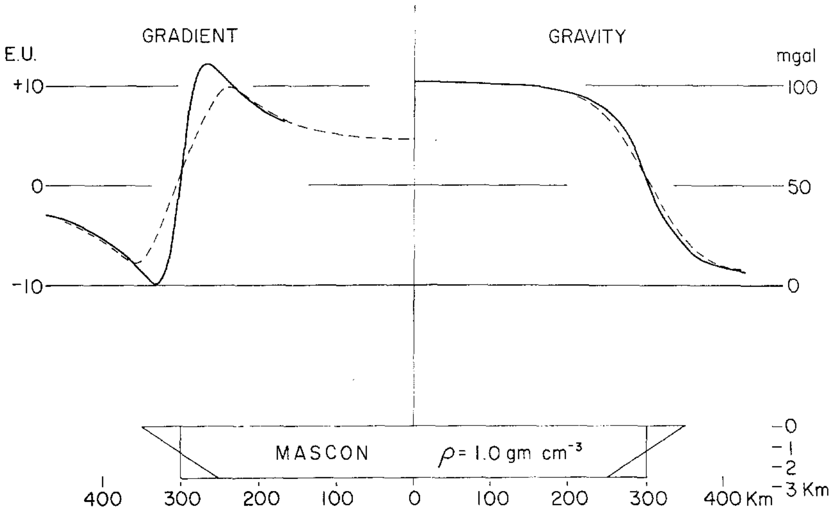


Fig. 4. Gradient and vertical attraction profiles for two mascon models, showing sensitivity of gradient profile to small changes in the mass distribution.

The selected spin rate is a compromise between the spacecraft mechanical design, which is unable to tolerate too rapid a spin, and the gradiometer electronics which are easier to design at the higher frequencies. Spin speeds between 1 and 5 revolutions per sec are probably satisfactory from both counts. A relatively low sensor $Q$ of about 20 would be employed to avoid undue sensitivity to the exact spacecraft spin rate. However the necessary damping can be obtained by electronic feed back, which has the advantage of not increasing the thermal noise over that introduced by the mechanical damping.

## 3. Discussion

This experiment will yield a complete map of $(\Gamma_{rr}-\Gamma_{\phi\phi})$ at a height of about 30 km. Gravity gradient techniques have not been used since the 1930's, (and then in a very different way) so careful thought needs to be given to making the best use of the information that this experiment is capable of yielding.

The gradient is a variety of gravity data and such a survey shares with conventional gravity techniques the advantage of being an excellent reconnaissance tool. The use of gradients, rather than attractions, will result in high resolution and sensitivity to detailed structure; in this respect it should be a fine complement to the Doppler methods which measure the change in velocity or the integral of attraction. The resolving power can be demonstrated by comparing the vertical attraction and $(\Gamma_{yy} - \Gamma_{xx})$ gradient of a pair of point masses on the lunar surface, at a height of 30 km. As may be seen from Figure 3, one starts to resolve the masses at 30 km separation with the gradient data and at 50 km the resolution is almost complete. In the attraction profiles the masses just begin to separate at 50 km. It also becomes clear that to take advantage of this inherent resolving power, one cannot afford to average over more than 10 km laterally, or 6 sec in time. The sensitivity of the technique is such that a mass of $1.35 \times 10^{17}$ g on the surface produces a gradient of 1 EU – this is the mass of a disk 10 km in radius and 150 m thick. Hence the lunar topography will have to be known to at least this resolution to make full use of the accuracy attainable with the gradiometer, and this implies that the gradient measurement must be combined with a topographic sounding experiment.

Two questions to which the gradient measurements will contribute directly are the degree of isostatic compensation of the lunar surface features and the detailing of the mascons. The gravity field of a circular mascon of 300 km radius should be reasonably well approximated by that of a two dimensional body at 30 km altitude. In Figure 4 we have used a two dimensional calculation to investigate the sensitivity of the gradiometer to small changes in the Mascon models – in one case the mascon is modelled as a slab with vertical sides, in the other it thins out gradually over a linear distance of 100 km. The vertical gravity profiles differ very slightly and certainly could not be distinguished by their effects in changing the velocity of a satellite; the gradient profiles do, however, differ significantly – by up to 5 EU or half the anomaly amplitude – but again spatial resolution of 10 km will be necessary to detect these differences.

Probably the most useful aspect of the results will, however, be in identifying new gravity features, too small to be detected by Doppler techniques but invaluable as a guide to planning further lunar explorations. The Moon is evidently strong enough to have supported the enormous mass excesses of the Mascons throughout most of its history, and it should therefore have retained a record of other processes involving movements of mass. A major consideration is that this information would be obtained for both the near and far sides.

It appears that the Hughes gradiometer is instrumentally capable of a few tenths of an EU or better in an orbital application. The most fundamental limitations are in knowing the position of the satellite and the lunar topography. Errors in position are clearly not random but arise from orbital perturbations due to the attractions whose gradients are being sensed. There appears to be a real possibility of combining the gradiometer and Doppler data to make a significant improvement in the orbit determination over that possible with the Doppler tracking alone. This would, apart from the information obtained about the distribution of mass near the lunar surface, lead to improved navigation near the Moon and hence to improved determination of topography. This is a question that needs careful examination and could be very significant in improving positions on the far side.

The instrument has two unusual features – it functions most efficiently in an orbital environment and it makes measurements of quantities which have not generally been used for 35 yr and then under conditions in which the principal uncertainties arose because of small masses very close to the observer. In view of the obvious promise of the technique we need to develop experience in two directions; firstly, to use the gradiometer in an orbital application in order to obtain direct information about its performance under these conditions and, secondly, to develop insights into the interpretation of this type of data. This latter insight can be developed to a great extent by computer modelling, both by developing a library of mass distributions with associated gradients and by attempting to recover a mass distribution from simulated orbital data. The Trace-66 computer program already developed (Wong *et al.*, 1971) provides a good starting point for studies of this latter type.

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