THE LUNAR SEISMOGRAM

ANTHONY F. GANGI

Department of Geophysics, Texas A & M University, College Station, Texas, U.S.A.

Abstract. The unexpected and unusual characteristics of the lunar seismogram have given rise to various speculations regarding their origin: secondary ejecta, diffusive wave propagation and wave propagation effects in a self-compacted powder layer with a linearly increasing velocity with depth. Many of the characteristics can be explained, qualitatively, by the simple theory of a self-compacting, dry powder layer for which the velocity varies as the sixth root of the depth. This gives a very low seismic velocity at the lunar surface which, in turn, allows the signal to have a long duration, a lack of correlation between horizontal and vertical displacements, a signal envelope that changes with source to receiver separation and a varying spectrum over the duration of the signal. To explain the long duration of the seismic signal quantitatively, it is necessary to include scattering of the normally incident rays at the surface by shallow surface undulations. The sixth root velocity-depth dependence is consistent with the measured variation, with pressure, of the compressibility and velocity of lunar samples.

1. Introduction

The unusual and unexpected characteristics of the lunar seismogram (see Latham *et al.*, 1970a, b; Latham, 1971) have given rise to many speculations regarding their origin. The unusual characteristics of the lunar seismic signal that have been noted are: (1) the long duration of the signal (Latham *et al.*, 1970a, b; Latham, 1971); (2) the variable character of the signals (variable durations, variable onset and shape of the envelope – see Latham *et al.*, 1970a); (3) the lack of correlation between the horizontal and vertical components of displacement (Latham, 1971); (4) the variation of the spectrum of a single signal over its duration (Latham *et al.*, 1970b) and (5) the variation of near surface *p*-wave velocity measurements with values from 45 m/s (Sutton and Duennebier, 1970) to 104 m/s (Watkins and Kovach, 1971). Any theory used for the lunar seismic signal must explain all the above characteristics satisfactorily.

Chang *et al.* (1970) have proposed that the long duration may be explained by a "spray of secondary ejecta around the seismometer rather than the result of seismic waves propagated through the Moon". Mukhamedzhanov (1970) proposed that the long duration of the Apollo 12 seismogram was due to a multiple-cascade fall of material ejected by the impact of the Apollo 12 lunar module. Latham *et al.* (1970a) have proposed a diffusive wave propagation mechanism which requires a high Q ($Q \simeq 3000$) and intense wave scattering. Gold and Soter (1970) have proposed a self-compacting deep layer of powder which has a linear velocity increase with depth (from 150 m/s at the surface to 6 km/s at depths of the order of 4.3 km). The long duration of the signal is explained by having waves multiply-reflected and scattered from an undulating surface (surface slopes between $\pm 10°$).

The diffusive wave propagation theory of Latham *et al.* (1970a) explains the long duration of the signal and, qualitatively, many of the other characteristics but it

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appears to require an inordinately high Q ($Q \approx 3000$). The thick powder layer theory of Gold and Soter (1970) with its linear velocity-depth variation and surface scattering also explains many of the characteristics of the lunar seismic signal. However, they have used an assumed velocity variation with depth for the self-compacting powder layer. Because of the success of the Gold and Soter powder layer theory in explaining the lunar seismic signal characteristics, it was decided to look more closely at elastic wave propagation in a self-compacting powder.

2. Velocity-Depth Variation of Self-Compacting Powders

The characteristics of self-compacting spheres have been investigated by soil mechanicists and exploration seismologists as a first approximation to the characteristics of self-compacting soils and sediments near the surface of the earth. These calculations are based on Hertz's theory (see Love, 1944) of the deformation of two spheres (or ellipsoids) in contact. Gassmann (1951) obtains (for dry, *hexagonal* close-packed, self-compacting, uniform spheres) a p-wave velocity given by

$$V_{hp} = [4E/\pi\varrho(1-\sigma^2)]^{1/3}(gz)^{1/6} F(\theta), \tag{1}$$

where

$$\begin{aligned} F(\theta) &= \{25 + 15\cos 2\theta + [(15 + 9\cos 2\theta)^2 + 256\sin^2 2\theta]^{1/2}\}^{1/2}/8 = \\ &= 1.0;\ \theta = 0 \\ &= 0.5;\ \theta = \pi/2 \end{aligned}$$

E = Young's Modulus = 0.98 Mb; ϱ = density = 3.1 g/cm^3; σ = Poisson's Ratio = 2.7; g = accel. of gravity = 1.62 m/s^2; θ = angle between the vertical and the direction of propagation of the p-wave.

The function, $F(\theta)$, expresses the velocity anisotropy introduced into the powder layer by the vertical force of gravity.

The above values of the Young's modulus, density and Poisson's Ratio of lunar material have been obtained from Anderson *et al.* (1970). The distribution of the measured properties are given in Figure 1 (note; $E = 3K(1-2\sigma)$ where K is the incompressibility). A powder layer of spheres in hexagonal packing would have a maximum porosity of 26% (when there is no deformation of the spheres).

Using the above values of the lunar material properties, Equation (1) gives: (for $z_0 = 1$ km)

$$V_{hp} \approx 1200\,(z/z_0)^{1/6} F(\theta)\ \text{m/s}. \tag{2}$$

The vertical p-wave velocity for dry, *cubic* packed, self-compacting, uniform spheres has been determined by White and Sengbush (1953) to be:

$$\begin{aligned} V_{cp} &= [4E/\pi\varrho(1-\sigma^2)]^{1/3}(27gz/32)^{1/6} \\ &= (27/32)^{1/6} V_{hp} = 0.972\,V_{hp}. \end{aligned} \tag{3}$$

For this packing, the maximum (no-deformation) porosity is 47.6%; however, we notice that there is less than 3% variation in the velocity predicted for this packing as compared to that predicted by the hexagonal packing with 26% porosity. Since neither velocity given by Equation (1) or (3) depends upon the sphere size and since the predicted velocities are the same to within 3% (even though the porosities are quite different), it is assumed that Equation (1) or (2) represents the velocity distribution in a powder layer at the surface of the Moon. This sixth root depth dependence of the

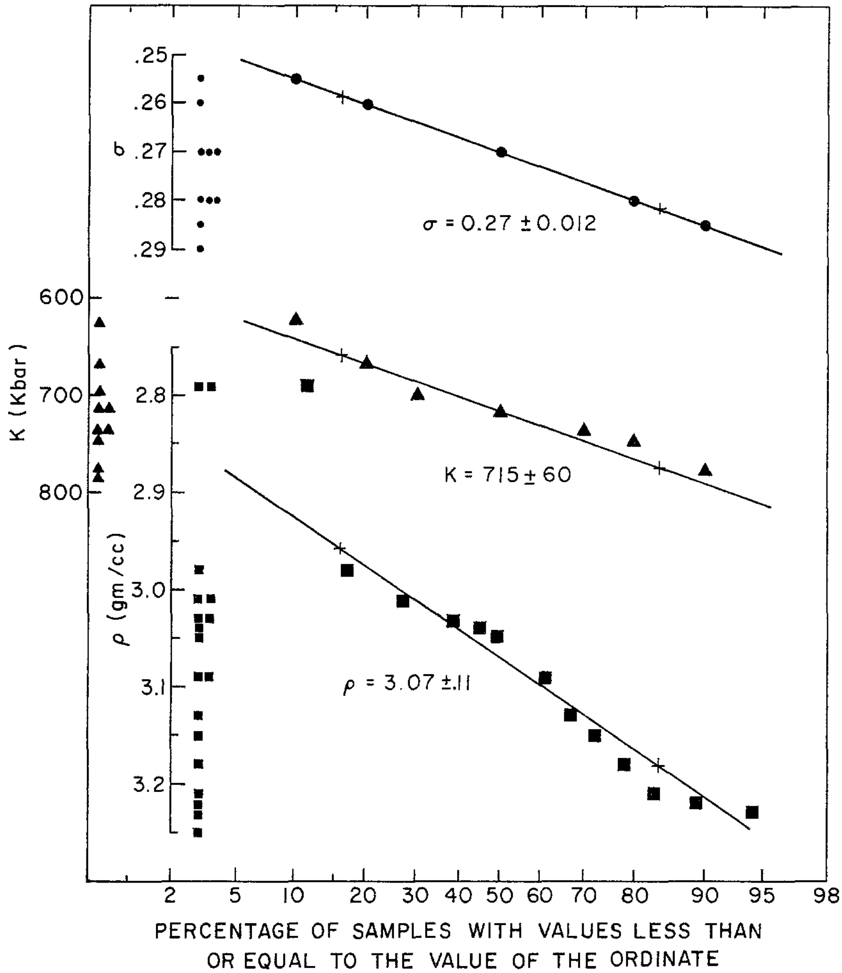

Fig. 1. Properties of lunar glass spheres (LG-101 through LG-118). Ref.: Anderson *et al.* (1970).

vertical p-wave velocity gives rise to a very rapidly varying velocity with depth (see Figure 2).

This velocity dependence appears to be consistent with the measured dependence of the compressibility with pressure shown in Figure 3 (for a constant density of 3.0 gm/cm^3, the lunar static-pressure-gradient is about $\frac{1}{20}$ Kb/km). The data given in Figure 3 were obtained by Anderson *et al.* (1970) and the straight line is the best

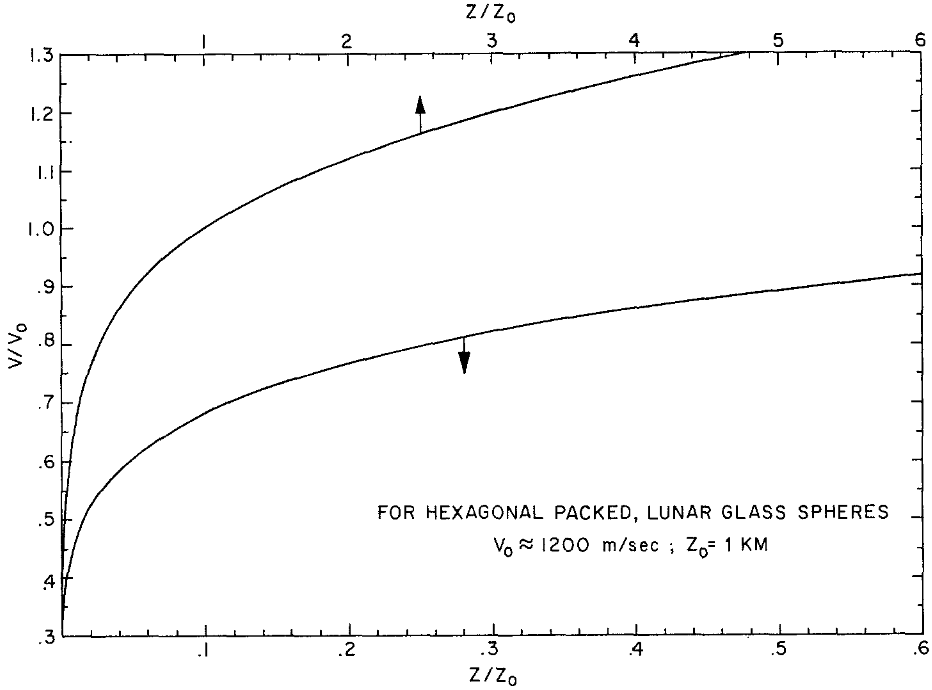

Fig. 2. Velocity-depth variation for self-compacting dry spheres.

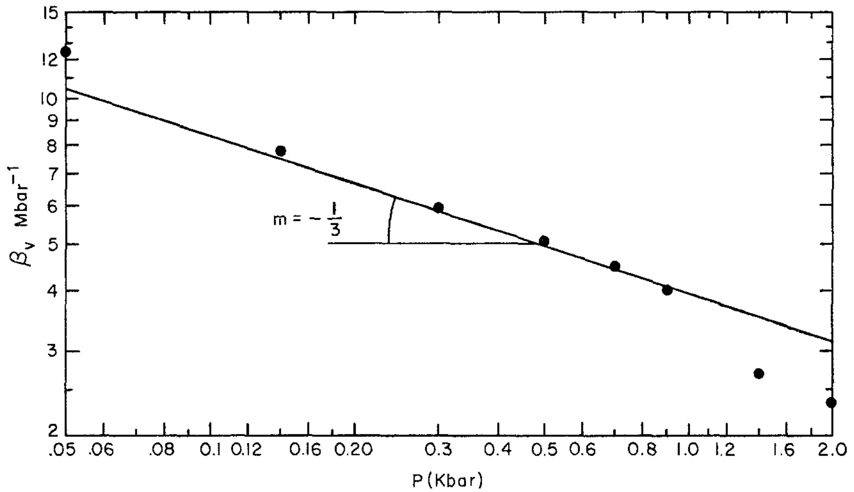

Fig. 3. Volume compressibility versus pressure (Lunar sample 10017). Ref.: Anderson *et al.* (1970).

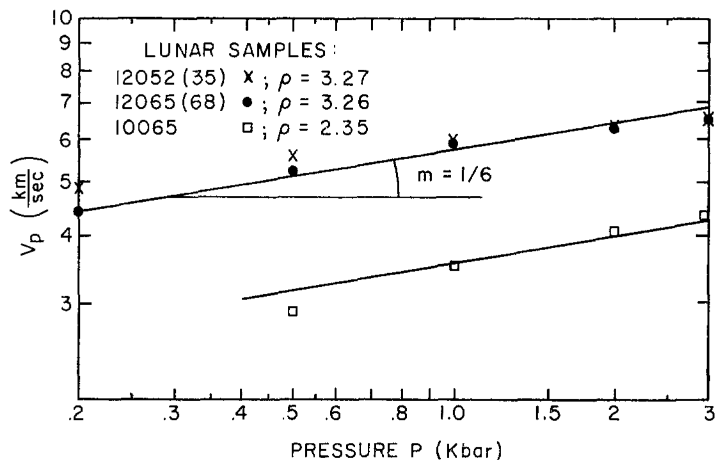

Fig. 4. *P*-wave velocity versus pressure (Lunar samples 12052(35), 12065(68) and 10065). Refs.: Kanamori *et al.* (1971) and Kanamori *et al.* (1970).

fitting straight line with a slope $m = -\frac{1}{3}$; it is *not* the best fitting straight line through the data. Figure 4 shows the measured variation of the p-wave velocity with pressure of lunar samples. The measurements on sample 10065 were made by Kanamori *et al.* (1970) while those on samples 12052(35) and 12065(68) were made by Kanamori *et al.* (1971). The lines are the best fitting (by eye) straight lines with slope $m = \frac{1}{6}$ that could be passed through the data.

3. Elastic Wave Propagation Characteristics

The above results are consistent with the sixth root velocity-depth relationship expected for self-compacting spheres. Consequently we assume Equation (1) represents

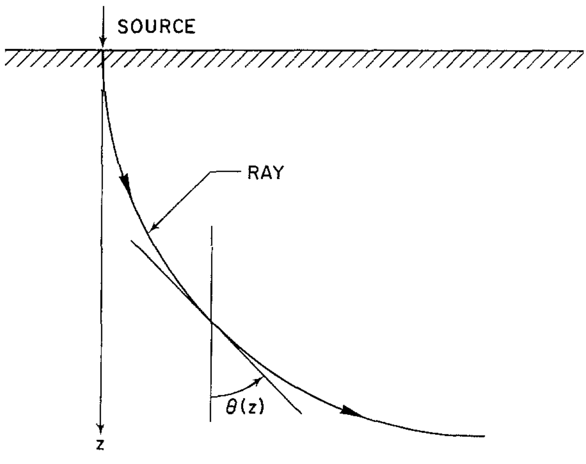

Fig. 5. A ray in a vertically inhomogeneous elastic medium.

the p-wave velocity-depth relationship and determine the resulting characteristics for elastic waves in such an inhomogeneous medium. First we recall that the *ray parameter p* (see Figure 5), given by

$$p = [\sin\theta(z)]/v(z), \tag{4}$$

is a constant along any ray in a horizontally homogeneous, but vertically inhomogeneous elastic medium. This is just an alternate statement of Snells' Law.

Since the velocity at the surface ($z=0$) is zero, Equation (4) shows that the rays are normally incident to the surface. This would explain why there is no correlation between the horizontal and vertical components of displacement measured at the surface. Incident p-waves would, therefore, have displacement at the surface in the vertical direction only. Also, horizontally (SH) and vertically (SV) polarized shear waves would have displacements only in the horizontal plane at the surface. In addition, rays emanating from a surface source would all be directed vertically into the lunar interior at the surface. This could explain why surface waves, as such, are not recognized in the lunar seismogram. Finally, the zero (or, at best, very small) angle of the rays at the surface with respect to the vertical means that very small undulations of the lunar surface will be very effective in scattering the elastic waves.

The travel-times for rays from a surface source to a surface receiver in a medium with a velocity variation

$$v(z) = v_0 (z/z_0)^{1/6} \tag{5}$$

may be obtained from the results given by Kaufman (1953) and are given by

$$v_0 t/z_0 = 1.2 (15\pi/8)^{1/6} (x/z_0)^{5/6}.$$

For lunar materials we obtain (for $\theta=\pi/2$, $z_0=1$ km)

$$t_0 = 2.69 (x/z_0)^{5/6} \text{ s}. \tag{6}$$

The depth of penetration of the direct wave is

$$h_0 = 8x/15\pi = x/5.89. \tag{7}$$

The travel-time for a ray reflected $m-1$ times from the surface is then

$$t_m(x) = m t_0 (x/m) = m^{1/6} t_0 (x), \tag{8}$$

and its depth of penetration is

$$h_m(x) = h_0 (x/m) = h_0 (x)/m. \tag{9}$$

Travel-times for short distances ($x \leqslant 160$ m) have been calculated using Equation (6). These are plotted in Figure 6 (curve labelled $T_{1/6}$) along with the travel-times computed using the preliminary near-surface lunar model of Watkins and Kovach (1971). The curve labelled T_{p1} is the travel-time for the direct p-wave in the upper layer while T_{121} is the travel-time for the refracted p-wave which travels, over most of its path, in the lower (higher velocity) layer of the Watkins-Kovach lunar model. It can be seen from Figure 6 that the travel times for the two models are quite similar although the travel-times for the sixth root velocity dependence are uniformly shorter. This implies that the velocity increases more rapidly with depth for the sixth root model than it does for the Watkins-Kovach model. However, no adjustments of the parameter values of Equation (1) have been made in obtaining the travel-times for $T_{1/6}$ in Figure 6. It is surprising that the correspondence between the travel-times of the two models is as good as it is under these circumstances.

Equation (8) shows that the time interval between multiply-reflected waves will increase as the distance increases (i.e., as t_0 increases), but that the time interval between successive arrivals will decrease for any given source. The time intervals between successive arrivals will be large and highly variable initially and will be

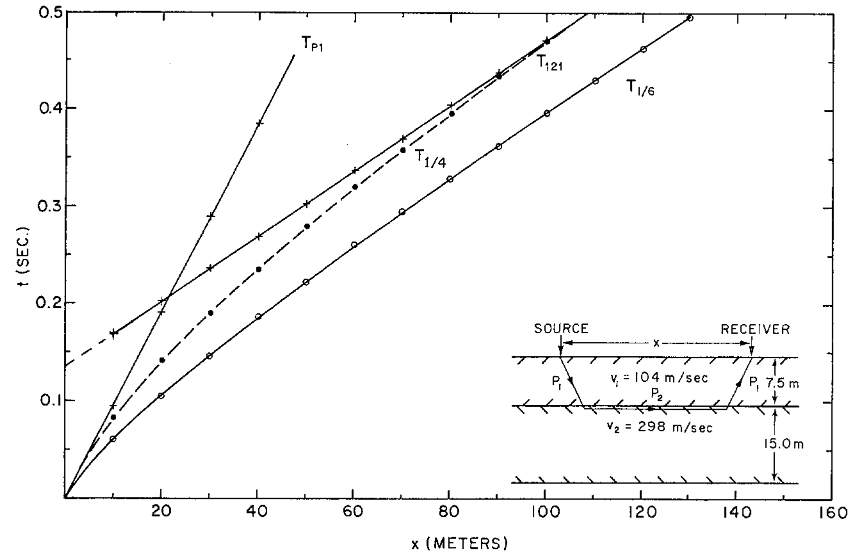

Fig. 6. Travel-time curves for three lunar models. Refs.: Watkins and Kovach (1971) and Carrier (1971).

smaller and more uniform for arrivals that are reflected from the surface many times and arrive later. Note, however, that for a flat surface, an event that arrives ten times later than the direct wave must experience $m = 10^6$ reflections and would be confined to a depth 10^{-6} of the direct ray's maximum penetration depth. Since the durations of the lunar seismic signals are generally longer than ten times the direct ray travel-time, significant scattering of the energy by surface undulations would be necessary. The randomness of the slopes of the surface undulations would lead to a waveform envelope that would be described by the diffusion equation as postulated by Latham *et al.* (1970a).

Recently, Carrier (1971) has suggested that the shear wave velocity on the Moon increases as the fourth root of depth near the surface. From Kaufman (1953) we find that the travel-time relationship for a velocity-depth variation of the form

$$v = v_0 (z/z_0)^{1/4}$$

is given by

$$t = (2\pi z_0/v_0) [2x/3\pi z_0]^{3/4}.$$

By use of this result, a fitted travel-time curve has been calculated and it is shown in Figure 6 as the dashed curve labelled $T_{1/4}$. For this curve we use $z_0 = 1$ km and $v_0 =$ 748 m/s. A very good fit is obtained between this travel-time curve and the one resulting from the Watkins-Kovach model. However, it must be borne in mind that the value of v_0 is chosen to make a good fit.

Comparison of the travel-times of the Watkins-Kovach near surface model with the self-compacting model (sixth-root dependence with depth) indicates that the velocity increases more slowly for $z < 30$ m than is predicted by the self-compacting sphere model. On the other hand, the self-compacting sphere model and the fourth root velocity-depth variation model give velocities that are too low at greater depths. The travel times for the Apollo 13 S-IVB impact at 135 km and for the Apollo 12 LM impact at 75 km were 29.1 s and 23.5 (+2.1, −3.7) s, respectively (Latham *et al.*, 1970b) and these are much lower than the values calculated using the sixth-root velocity model (namely, 161 and 99 s, respectively) and the fourth-root velocity model (namely, 104 and 67 s, respectively). However, these signals would penetrate to depths of 23 km and 12 km, respectively, in the sixth-root velocity model and to depths of 64 km and 35 km, respectively, in the fourth-root velocity model. Thus, the thickness of the powder layer would have to be considerably less than these depths.

4. Conclusions

The long duration of the lunar seismic signal can be explained by multiply-reflected rays in a self-compacting powder layer. The velocity near the surface is very small in this case and leads to rays which are normally incident at the surface. The normal incidence results in large scatter of the rays due to small angle surface undulations.

The variable character of the signal envelope can be explained as being a source-receiver distance effect. This is due to the diffusive nature of the scattering and due to the fact that, for a sixth root velocity model, later (multiply-reflected) arrivals would have travel-times which are a fixed ratio of the direct travel time (see Equation (8))

No correlation of the horizontal and vertical components could be anticipated for the normally incident rays as predicted by a very low near-surface velocity. Also, the random scattering due to surface undulations would eliminate any correlation that might have existed.

The spectrum of the signal varies over the length of the signal because the time intervals between multiply reflected arrivals are larger and more variable for the early part of the record (see Equation (8)). Also, the later parts of the signals are confined to the near surface (see Equation (9)) and the attenuation and scattering would be expected to be larger there.

The variable near surface velocity measurements may be due to the rapid velocity variation predicted by the 'sixth-root' model in the first meter of depth (0 to ~180 m/s).

In conclusion, a self-compacting powder layer for the lunar near surface layer gives a sixth root velocity-depth variation which helps to explain many of the unique characteristics of the lunar seismogram. However, it is necessary to introduce a high degree of scattering to explain, quantitatively, the seismic signal duration.

References

Anderson, O. L., Scholz, C., Soga, N., Warren, N., and Schreiber, E.: 1970, *Proc. Apollo 11 Lunar Science Conference*, vol. **3**, *Geochim. Cosmochim. Acta, Suppl. 1*, pp. 1959–1973.

Carrier, D.: 1971, private communication.

Chang, G. K., Gunther, P., and James, D. B.: 1970, *J. Geophys. Res.* **75**, 7426–7438.

Gassmann, F.: 1951, *Geophys.* **16**, 673–685. See also *Geophys.* **18**, 269.

Gold, T. and Soter, S.: 1970, *Science* **169**, 1071–1075.

Kanamori, H., Nur, A., Chung, D., Wones, D., and Simmons, G.: 1970, *Science* **167**, 732–734.

Kanamori, H., Mizutani, H., and Hamano, Y.: 1971, Informal Proceedings of the 1971 Lunar Science Conference, Jan. 11–14, 1971, vol. II, National Aeronautics and Space Administration, Manned Spacecraft Center, Houston, Texas.

Kaufman, H.: 1953, *Geophys.* **18**, 289–297.

Latham, G. V., Ewing, M., Press, F., Sutton, G., Dorman, J., Nakamura, Y., Toksoz, N., Wiggins, R., Derr, J., and Duennebier, F.: 1970a, *Science* **167**, 455–457. See also *Proc. Apollo 11 Lunar Science Conference*, vol. **3**, *Geochim. Cosmochim. Acta, Suppl. 1*, 2309–2320.

Latham, G. V., Ewing, M., Dorman, J., Press, F., Sutton, G., Meissner, R., Duennebier, F., Nakamura, Y., Kovach, R., and Yates, M.: 1970b, *Science* **170**, 620–626.

Latham, G. V.: 1971, *EOS, Trans. Amer. Geophys. Un.* **52**, 162–165.

Love, A. E. H.: 1944, *A Treatise on the Mathematical Theory of Elasticity*, Dover Publications, Inc., New York, Fourth Edition, pp. 193f.

Mukhamedzhanov, A. K.: 1970, Can. Tech. Sc., Moscow, Defence Scientific Information Service, DRB Canada, Translation *T-542R*, June (1970).

Schreiber, E., Anderson, O., Soga, N., Warren, N., and Scholz, C.: 1970, *Science* **167**, 732–734.

Sutton, G. H. and Duennebier, F. K.: 1970, *J. Geophys. Res.* **75**, 7439–7444.

Watkins, J. and Kovach, R.: 1971, *EOS, Trans. Amer. Geophys. Un.* **52**, 534. See also: Kovach, R., J. Watkins and T. Landers, 'Active Seismic Experiment', Apollo 14 Preliminary Science Report, NASA SP-272, National Aeronautics and Space Administration, Washington, D.C., pp. 163–174 (1971).

White, J. E. and Sengbush, R. L.: 1953, *Geophys.* **18**, 54–69.