A ONE-DIMENSIONAL GASDYNAMICAL MODEL OF MAGNETOSPHERIC CONVECTION

M. V. SAMOKHIN
Moscow Radiotechnical Institute, U.S.S.R. Academy of Sciences, Moscow U.S.S.R.

(Received 3 November, 1977)

Abstract. The plasma flow in the equatorial plane of the magnetosphere is examined within the framework of a one-dimensional model in which all quantities are supposed to depend only on the distance along the Sun–Earth axis. The following models are considered: (1) the gasdynamical model in which the Ampère force is ignored, (2) the magnetohydrodynamical model in which the normal component of the Ampère force on the magnetopause is taken into account. The flow regime is calculated in the region including two regions: (1) the layer of the return flow where flow velocity is directed from the Sun, (2) the region of convection where the velocity is directed toward the Sun – on the assumption that the form of the magnetopause and the distribution of the solar wind pressure on the magnetopause are known.

The following physical mechanisms are taken into account: (1) the appearance of a centrifugal force owing to the magnetopause curvature, the centrifugal force partly compensating for the solar wind pressure; (2) the existence of the critical point which is analogous to the point of transition through the local sound velocity in the Laval nozzle or in the Parker model of the solar corona. The thickness of the layer of the return flow and the velocity of convection in the magnetosphere are calculated; and the following peculiarities are found: (1) in the gasdynamical model the convection regime is only possible with high velocities corresponding to the substorm, (2) in the magnetohydrodynamic model the convection velocity and the thickness of the layer of the return flow are reduced; the reduction being connected to the fact that the pressure of the solar wind is partially compensated for by the jump of the magnetic pressure on the magnetopause.

Introduction

According to modern ideas, the plasma moves toward the Sun in the principal volume of the magnetosphere (Axford, 1969) and the plasma moves in the opposite direction in a thin layer near the magnetopause (Freeman *et al.*, 1968; Bogott and Mozer, 1971). These flows differ from each other in the magnitude of their velocity; the first is known as the convection and the latter as the return flow. The existence of such flow patterns in the magnetosphere is supported by the data of Vela satellites (Hones *et al.*, 1972, 1973), by investigations of the plasma mantle (Rosenbauer *et al.*, 1975), by the electric drift measurements in the tail (Villante and Lazarus, 1975; Walker *et al.*, 1975). The convection creates the electric field in the equatorial plane directed from dawn to dusk (Obayashi and Nishida, 1968; Chappell, 1974; Roederer, 1974).

The cause of the existence of convection is not finally established; the dissipative mechanisms of viscous-like friction (Axford and Hines, 1961) or of field line merging (Dungey, 1961; Levy *et al.*, 1964) have been proposed and a nondissipative mechanism, in which the existence of the return flow is the condition of equilibrium of the particular tangential discontinuity – magnetopause – under the solar wind pressure changing along the magnetopause, is offered (Samokhin, 1976). In the latter case, in contrast with the Chapman-Ferraro model, the solar wind pressure on the magnetospheric boundary is partly compensated for by the Ampère force, and partly by the gas pressure of magnetospheric plasma which is redistributed because of the plasma flow.

The purpose of the paper is to develop further the last concept. With the aid of such supplementary physical ideas as (1) the appearance of the inertia force owing to the curvature of the magnetopause and (2) the existence of the critical point, analogous to that of transition through the local sound velocity in the Laval nozzle, or in the Parker model of the solar corona, one succeeds in finding the velocity of convection in the magnetospheric tail and the thickness of the return flow. Two models are discussed: (1) the gasdynamical model in which the Ampère force is omitted and (2) the magnetohydrodynamical model in which the component of the Ampère force normal to the magnetopause is taken into account.

1. The Formulation of the Problem in the Case of the Gasdynamical Model

The pattern of convection and of the return flow in the equatorial plane according to the modern ideas is shown in Figure 1. Here 0 is the region of the solar wind, 1 is the

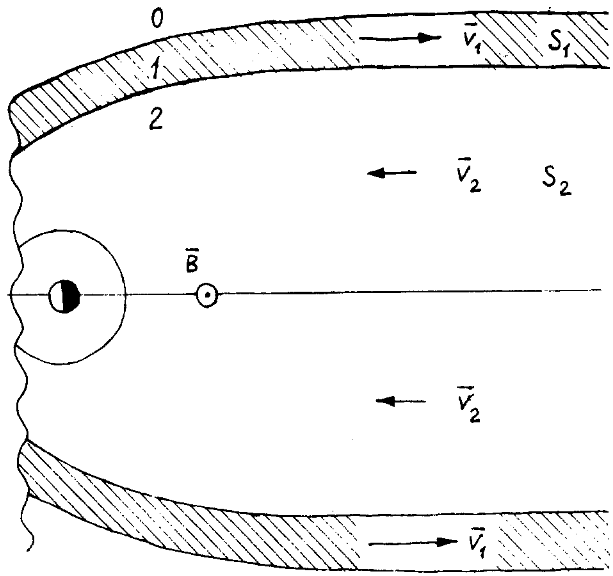

Fig. 1. The assumed pattern of convection and the return flow in the equatorial plane of the magnetosphere. The layer of the return flow 1 is marked by strokes, the magnetic field in the neutral sheet 2 (in the region of convection) is oriented to the north, v_1, v_2 are the velocities of the return flow and convection, S_1 is the thickness of the layer of the return flow adjacent to the magnetopause, S_2 is the halfwidth of the region of convection. The form of the magnetospheric boundary and the distribution of the solar wind pressure 0 on the boundary are considered to be known, the plasmosphere of the set form is excluded from the cross-section of the region of convection 2. The calculations result in the position of the boundary between the regions 1 and 2. The wavy line separates the region of the magnetosphere on the day side where the one-dimensional model becomes invalid (see the text).

layer of the return flow, 2 is the region of convection. The arrows point the direction of the plasma flow, S_1 and S_2 are cross-sections of the corresponding regions perpendicular to the Sun–Earth axis. The plasmasphere in which the plasma taking part in the convection flow does not penetrate is excluded from the region 2. Magnetic field is directed upwards from the plane of the figure.

The calculation of the magnetospheric plasma flow pattern in stationary conditions represents a sufficiently complex problem even in the framework of the continuous-medium approach, when the magnetospheric plasma is supposed to be an ideally conducting gas with isotropic pressure, and is supposed to obey the one-fluid magnetohydrodynamic equations

$$\begin{gathered} \varrho(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \frac{1}{c}[\mathbf{jB}], \qquad \operatorname{div}\mathbf{B} = 0, \qquad \operatorname{div}\varrho\mathbf{v} = 0, \\ \operatorname{rot}\mathbf{B} = \frac{4\pi}{c}\mathbf{j}, \qquad \operatorname{rot}[\mathbf{vB}] = 0, \qquad (\mathbf{v}\nabla)\frac{p}{\varrho^{\gamma}} = 0, \end{gathered} \tag{1}$$

where $\varrho, p, \mathbf{v}, \mathbf{B}, \mathbf{j}, \gamma, c$ are, respectively, the density, the pressure, the velocity, the magnetic field, the current, the adiabatic power law, and the velocity of light. Correspondingly it seems to be reasonable to consider the convection model in which (1) simplification should result from the geometry of the problem, (2) the fundamental physical processes should be taken into account. The geometrical simplifications must be based on the fact that the magnetospheric tail is pulled out for a considerable geocentric distance; therefore all physical values are considered to depend only on the distance along the Sun–Earth axis (the one-dimensional problem).

Let us examine the part of individual physical forces. The first cause of the appearance of the return flow seems to be a decrease of the solar wind pressure along the magnetopause in the direction from the Sun. According to the momentum equation (1), the force $-\nabla p$ directed from the Sun drives the plasma to move in the same direction. Besides, since the thickness of the layer of the return flow S_1 is not zero, a component of the current arises across the layer $j_n \sim S_1 j/R$, where R is the characteristic scale of length, j is the fundamental current in the layer to compensate for the solar wind pressure. It is easy to see that, in the equatorial cross-section of the layer, the magnetic field is oriented to the north and decreases in the direction from the Sun; therefore, the component j_n is directed from dawn to dusk, as is the corresponding component of the Ampère force from the Sun. The component of the Ampère force is the second cause which makes the plasma in the layer move in a direction away from the Sun.

In Equations (1) one can distinguish the terms of different orders in the flow velocity. In the zero order (i.e. in the absence of the plasma movement) the currents must dissipate for some seconds, because of the absence of sources and because of the final ionospheric conductivity. Then the pressure gradients must disappear. The only kind of motion inside the magnetosphere which is driven by the outer source (the solar wind) is known to be convection. Therefore, the Ampère force $[\mathbf{jB}]/c$ and the pressure gradient ∇p have the same source – the plasma flow – and, generally speaking, are of the same order of magnitude as the inertial force $\varrho(\mathbf{v}\nabla)\mathbf{v}$. The components of the Ampère force can scarcely be well approximated in the framework of a one-dimensional model. Therefore, if one does not seek an exact solution but only an order of magnitude evaluation it is reasonable to omit the Ampère force in the momentum equation, and reduce the problem to a gasdynamical one.

From this point of view it is easy to ascertain the time of the arrangement of the stationary return flow T. So long as $\varrho v/T \sim \Delta p/R$, where Δp is the pressure fall bound to the plasma flow, i.e. $\Delta p \sim \varrho v^2$, R is the characteristic scale, ϱ is the density, v is the plasma velocity, we obtain $T \sim R/v$ finally. For $v \sim 200 \text{ km s}^{-1}$ and $R \sim 10R_E$ the time is $T \sim 5$ min.

Let us, therefore, summarize the basic simplifications: (1) the equatorial cross-section of the magnetosphere is considered, (2) all values are supposed to depend only on the distance along the Sun–Earth axis, (3) the Ampère force is omitted, (4) the magnetosphere is supposed to consist of the layer of the return flow near the magnetopause, which has an unknown thickness, and the region of convection from which the plasmopause is expelled, (5) the form of the magnetopause and the form of the plasmapause are considered to be given, the solar wind parameters (the density, the temperature, the velocity), the solar wind pressure distribution on the magnetopause and the values of the plasma density in the region of convection in the infinity and in the layer of the return flow at infinity (i.e. in the distant tail) are supposed to be known, and (6) the power in the adiabatic law is considered to be 2.

The first simplification goes back to the fact that the corresponding experimental data concern mainly the region of the magnetosphere near the equatorial plane. The second simplification can be justified by the fact that the magnetosphere is elongated considerably in the direction from the Sun. However, on the dayside of the magnetosphere the curvature of the current lines of liquid is comparable with the characteristic scale and the assumption becomes invalid. The third assumption is justified by the fact that, in the real magnetosphere, the Ampère force and the pressure gradient are generally of the same order of magnitude. Therefore, for an estimate of the orders of magnitude, one term from the two can be omitted only if the vectors $-\nabla p$ and $[\mathbf{jB}]/c$ are not in opposite directions. This can be verified by comparison of the results of the framework calculations of the model with observational data. The fourth simplification is based on the up-to-date ideas that the magnetospheric boundary is a tangential discontinuity and the plasmapause is an equipotential surface; and, for this simplification, the convection patterns which are the result of the appearance of the equivalent ionospheric current systems corresponding to the periodic variations, the data of satellites with geostationary orbits, the satellites Vela, the measurements of the electric fields in the ionosphere with the help of balloons, etc., are used. The fifth simplification is connected with the one-dimensionality, and is essential for the purpose of this paper, which is to evaluate the convection velocity in the magnetosphere by use of the solar-wind parameters. The sixth simplification is not essential, and has been introduced only to simplify the equations.

In the scope of these simplifications, Equations (1) assume the forms

$$\varrho_i v_i S_i = Q, \qquad \frac{2p_i}{\varrho_i} + \frac{v_i^2}{2} = \frac{2p_\infty}{\varrho_{i\infty}} + \frac{v_{i\infty}^2}{2},$$
$$\frac{p_i}{\varrho_i^2} = \frac{p_\infty}{\varrho_{i\infty}^2}, \qquad i = 1, 2, \qquad S = \sum_{i=1}^{2} S_i, \tag{2}$$

where $Q = \text{const}$ (the subscript ∞ means that the corresponding values are taken at great geocentrical distances), S is the set cross-section of the magnetosphere in the equatorial plane – i.e. the distance from the Sun–Earth axis to the magnetopause of set form (see below), the cross-section of the plasmasphere being extracted. At great distances the cross-section S is finite and the pressures in the layer of the return flow ($i = 1$) in the region of convection ($i = 2$) and in the solar wind are not equal to zero and are equal to p_∞.

Because of the curvature of the magnetopause in the layer of the return flow, the centrifugal force appears as

$$f = \frac{\varrho_1 v_1^2}{R} S_1, \tag{3}$$

which partly compensates the solar-wind pressure on the boundary, where R is the local radius of curvature. Then in the region of convection the pressure is $p_2 = p - f$, where p is the solar-wind pressure on the magnetospheric boundary.

Let us introduce the system of co-ordinates with the centre in the stagnation (subsolar) point of the magnetopause and the axis x, y, z, directed toward the dusk side, from the Sun and to the north. Let us approximate the form of the magnetopause in the equatorial plane with the aid of an equation which follows as the solution of the problem of flow around a wire with current (Zhigulev, 1959; Hurley, 1961) that

$$y = l \ln \frac{2}{1 + \cos x/l}, \tag{4}$$

the value $h = l\pi/2$ being equal to the distance from the subsolar point to the geomagnetic dipole, the value $S_\infty = l\pi$ being the half-width of the tail cross-section at the great geocentrical distances. It is easy to be convinced that, in spite of its simplicity, Equation (4) is applicable to this purpose for $l\pi \sim 20R_E$.

Let us approximate the solar-wind pressure on the magnetopause with the aid of the equation

$$p = p_0 \cos^2 \chi + p_\infty, \tag{5}$$

where p_0 is the solar-wind pressure at the subsolar point without p_∞, χ is the angle between the normal to the magnetospheric boundary and the velocity of the undisturbed solar wind, the velocity being directed along the axis y, p_∞ is the chaotic pressure of the solar wind in the distant tail. The plasmasphere will be considered to be the circle of the radius a with the centre in the point $x = 0, y = h$. Then

$$S = \begin{cases} x, & |y - h| > a, \\ x - \sqrt{a^2 - (y-h)^2}, & |y - h| < a, \end{cases} \tag{6}$$

where the value of x is a function of y, and obeys Equation (4). The values R and $\cos\chi$ can be written in the form

$$R = \frac{\mathrm{d}^2 y/\mathrm{d}x^2}{[1 + (\mathrm{d}y/\mathrm{d}x)^2]^{3/2}}, \qquad \cos^2\chi = \frac{1}{1 + (\mathrm{d}y/\mathrm{d}x)^2}. \tag{7}$$

If we introduce the nondimensional values $M_i^2 = \varrho_{i\infty} v_{i\infty}^2/(2p_\infty)$, $i = 1, 2$, $u = \varrho_1/\varrho_{1\infty}$, $w = \varrho_2/\varrho_{2\infty}$, $\tau = S_{1\infty}/S_\infty$, $u_m = 1 + M_1^2/2$, $w_m = 1 + M_2^2/2$, $\zeta = p/p_\infty$, $\eta = p_0/p_\infty$, $\xi = \varrho_{1\infty}/(2\varrho_{2\infty})$ and take account of Equations (3)–(5), Equations (2) can be rewritten in the form

$$\begin{aligned} &F = \frac{1}{u\sqrt{u_m - u}} + \sqrt{2\xi}\,\frac{1}{w\sqrt{w_m - w}} - \frac{\sqrt{2}\,S}{M_1 S_\infty \tau} = 0, \\ &w^2 = \zeta - \frac{2\sqrt{2}\,M_1 S_\infty \tau}{R}\sqrt{u_m - u}, \\ &\zeta = 1 + \frac{\eta - 1}{2}\left(1 + \cos\frac{x}{l}\right), \qquad w_m = 1 + M_1^2\xi\left(\frac{\tau}{1-\tau}\right)^2. \end{aligned} \tag{8}$$

Therefore, the problem reduces to finding the value of u as a function of y which is determined implicitly by Equations (8) and (4)–(6). Then the pressures and the velocities of flow in the corresponding regions and the thickness of the layer of the return flows obeys the equations

$$\begin{aligned} &p_1 = p_\infty u^2, \qquad p_2 = p_\infty w^2, \qquad v_1 = v_{1\infty}\sqrt{1 + \frac{2}{M_1^2}(1 - u)}, \\ &v_2 = v_{2\infty}\sqrt{1 + \frac{2}{M_2^2}(1 - w)}, \qquad S_1 = \frac{M_1 S_\infty \tau}{\sqrt{2}\,u\sqrt{u_m - u}}. \end{aligned} \tag{9}$$

In the layer of the return flow the plasma velocity increases from zero (near the stagnation point) to the value of the order of the solar-wind velocity (at great geocentrical distances), passing through the local sound velocity. Therefore, by analogy with the Laval nozzle or Parker's model of the expansion of the solar corona (Parker, 1963), the single solution can be realized which passes through the critical point where $\partial F/\partial u = 0$, $\partial F/\partial x = 0$ (it is more convenient to use the changeable value x instead of y). These additional equations result in the position of the critical point and the value w_m, i.e. the convection velocity at great geocentrical distances.

2. The Method of Solution*

The position of the critical point and the value of w_m obey Equations (8) – i.e.

$$\frac{\partial F}{\partial u} = -\frac{2u_m - 3u}{2u^2(u_m - u)^{3/2}} - \sqrt{\xi}\,\frac{M_1 S_\infty \tau (2w_m - 3w)}{2R\sqrt{u_m - u}\, w^3 (w_m - w)^{3/2}} = 0, \tag{10}$$

* This section is of the nature of an appendix, and can be omitted on a first reading.

$$\frac{\partial F}{\partial x} = -\sqrt{\xi}\,\frac{2w_m - 3w}{2\sqrt{2}\,w^3(w_m - w)^{3/2}}\left(\frac{\mathrm{d}\zeta}{\mathrm{d}x} + \frac{\zeta - w^2}{R}\frac{\mathrm{d}R}{\mathrm{d}x}\right) - \frac{\sqrt{2}}{M_1 S_\infty \tau}\frac{\mathrm{d}S}{\mathrm{d}x} = 0. \tag{11}$$

The layer of the return flow is located between the region of convection and the magnetosheath. The velocity in that layer is about the same as in the magnetosheath, and greater than the velocity of convection; one can also expect that the density in the layer of the return flow to be greater than in the region of convection, and the thickness of the layer to be smaller in comparison with the cross-section of the tail; while the dynamical pressure of the solar wind will be large in comparison with the random one. Therefore, in the zero-order approximation, one can consider that $\xi \gg 1$, $\eta \gg 1$, $\tau \ll 1$, $S_1 \ll S$; and if so, Equations (8), (10) and (11) will assume the forms

$$\begin{gathered} w_m = \frac{\zeta^{3/2} - S_\infty^2/S^2}{\zeta - S_\infty^2/S^2}, \qquad w = \sqrt{\zeta}, \\ \frac{(S_\infty/S)^2(\zeta^{3/2} - 2) - \zeta^{3/2}}{4\zeta(\sqrt{\zeta} - 1)}\left(\frac{S}{S_\infty}\right)^2\frac{\mathrm{d}\zeta}{\mathrm{d}x} + \frac{1}{S_\infty}\frac{\mathrm{d}S}{\mathrm{d}x} = 0, \\ \frac{2u_m - 3u}{u^2(u_m - u)} = -\mu, \qquad \mu = \frac{S_\infty}{R}\,\frac{(S_\infty/S)^2(\zeta^{3/2} - 2) - \zeta^{3/2}}{\zeta(\sqrt{\zeta} - 1)}\left(\frac{S}{S_\infty}\right)^3. \end{gathered} \tag{12}$$

The system of Equations (12) was solved in the following way. At first the solution $x = x_0$ of the second Equation (12) was found graphically; then its value specified numerically by dividing the interval in half. The values w_m, w and u were determined from the remaining equations, the method of successive approximations was applied for finding the value u as

$$\begin{aligned} &u^{(0)} = 0, \qquad u^{(n+1)} = \frac{\sqrt{9 - 8u_m\mu(u_m - u^{(n)})} - 3}{-2\mu(u_m - u^{(n)})} && \text{for } \mu < 0, \quad n = 0, 1, \ldots, \\ &u^{(0)} = u_m, \qquad u^{(n+1)} = u_m\frac{2 + \mu u^{(n)2}}{3 + \mu u^{(n)2}} && \text{for } \mu > 0, \quad n = 0, 1, \ldots. \end{aligned} \tag{13}$$

These equations can easily be completed if the left-hand side of the last but one equation (12) is represented graphically.

After the solution of Equations (12) has been found, further specification was made by successive approximations with the aid of the equations deduced from Equations (8), (10) and (12); in these equations the assumption that $\xi \gg 1, \eta \gg 1, \tau \ll 1$ was dropped; i.e.,

$$\begin{gathered} w = \sqrt{\zeta} - \alpha, \\ \alpha = \sqrt{\zeta} - \sqrt{\zeta - \sqrt{\frac{8S_\infty^2(w_m - 1)(u_m - u)}{\xi R^2}\,\frac{1}{(1 + \sqrt{w_m - 1}/M_1\sqrt{\xi})^2}}}, \\ \nu = \left[\frac{S}{S_\infty}\left(1 + \frac{\sqrt{w_m - 1}}{M_1\sqrt{\xi}}\right) - \frac{1}{\sqrt{2\xi}}\,\frac{\sqrt{w_m - 1}}{u\sqrt{u_m - u}}\right]^{-1}, \\ w_m = \frac{w^3 - \nu^2}{w^2 - \nu^2}, \qquad \mu = \frac{S_\infty}{R}\,\frac{\nu^2(3w - 2) - w^3}{\nu^3 w^2(w - 1)}\,\frac{1}{1 + \sqrt{w_m - 1}/M_1\sqrt{\xi}}, \end{gathered} \tag{14}$$

the value u being determined by the above-mentioned way from the equation

$$\frac{2u_m - 3u}{u^2(u_m - u)} = -\mu, \tag{15}$$

and the location of the critical point being found with the help of the equation

$$\frac{1}{1 + \sqrt{w_m - 1}/M_1\sqrt{\xi}} \frac{\nu^2(3w - 2) - w^3}{4w^2\nu^3(w - 1)} \times$$
$$\times \left(\frac{\mathrm{d}\zeta}{\mathrm{d}x} + \frac{\zeta - w^2}{R}\frac{\mathrm{d}R}{\mathrm{d}x}\right) + \frac{1}{S_\infty}\frac{\mathrm{d}S}{\mathrm{d}x} = 0. \tag{16}$$

On the right-hand sides of Equations (14) the preceding approximation was introduced and the left-hand sides obtained by successive approximation and from the corresponding values of μ, ν.

The dependence of the parameters of the layer of the return flow and of the convection region on the geocentric distance follows the equations $F = 0$ and (8), or

$$f(z) = \frac{1}{\sqrt{2\xi}\,(u_m - z^2)z} + \frac{1}{\sqrt{\zeta - az}\,\sqrt{w_m - \sqrt{\zeta - az}}} = b, \tag{17}$$

where a new variable z is introduced by setting

$$z = \sqrt{u_m - u}, \qquad a = \frac{2S_\infty}{R}\sqrt{\frac{2(w_m - 1)}{\xi}}\,\frac{1}{1 + \sqrt{w_m - 1}/M_1\sqrt{\xi}},$$

$$b = \frac{S}{S_\infty\sqrt{w_m - 1}}\left(1 + \frac{\sqrt{w_m - 1}}{M_1\sqrt{\xi}}\right).$$

It is easy to see from Equation (17) that there must be a value of z at the same time in the intervals

$$0 < z < \sqrt{u_m}, \qquad \frac{\zeta - w_m^2}{a} < z < \frac{\zeta}{a}. \tag{18}$$

The value z can be evaluated numerically in the following way. Let us find the minimum of the function (17) $f_{\min} = f(z_{\min})$. If $f_{\min} < b$ then two solutions of Equation (17) exist which can be evaluated by the method of successive approximations. The expressions for the first and the second solutions in four cases shown in Figure 2 are given below (the first formula is the zero-order approximation; and with the aid of the second formula the next approximation can be evaluated if the preceding approximation is set into the right-hand side).

The case 1, $0 < z < \zeta/a$:

(1) $$z = 0, \qquad z = (2\xi)^{-1/2}(u_m - z^2)^{-1} \times$$
$$\times\, [b - (\zeta - az)^{-1/2}(w_m - \sqrt{\zeta - az})^{-1/2}]^{-1}, \tag{19}$$

(2) $$z = \zeta/a, \qquad z = \{\zeta - (w_m - \sqrt{\zeta - az})^{-1} \times$$
$$\times\, [b - (2\xi)^{-1/2}(u_m - z^2)^{-1}z^{-1}]^{-2}\}/a. \tag{20}$$

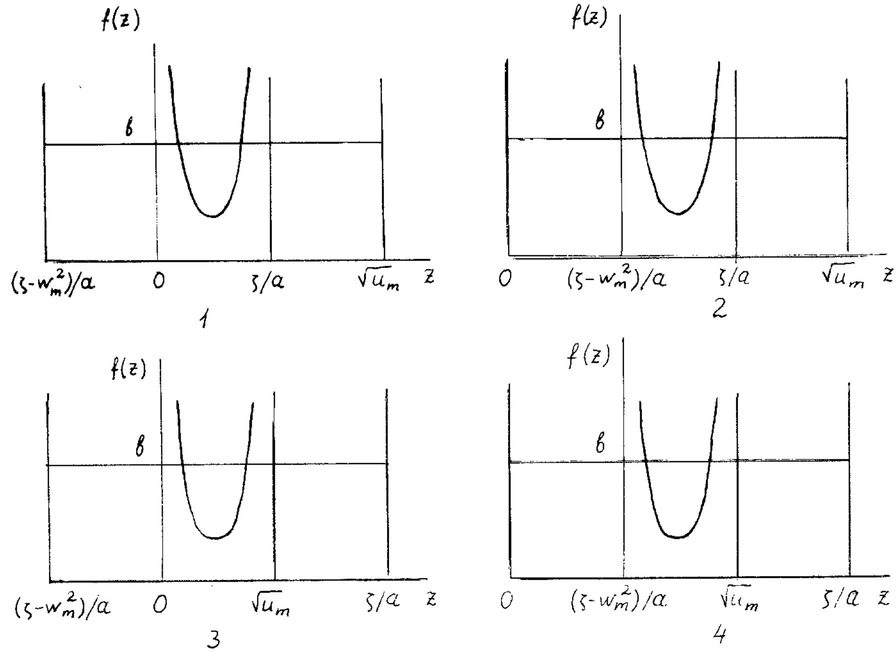

Fig. 2. The qualitative analysis of the dependence of the left part of Equation (17) on z for different parameters of the problem (the notations are explained in the text). In the case 1 $(\zeta - w_m^2)/a < 0 < \zeta/a < \sqrt{u_m}$ we have $0 < z < \zeta/a$, in the case 2 $0 < (\zeta - w_m^2)/a < \zeta/a < \sqrt{u_m}$ we have $(\zeta - w_m^2)/a < z < \zeta/a$, in the case 3 $(\zeta - w_m^2)/a < 0 < \sqrt{u_m} < \zeta/a$ we have $0 \leq z < \sqrt{u_m}$, in the case 4 $0 < (\zeta - w_m^2)/a < \sqrt{u_m} < \zeta/a$ we have $(\zeta - w_m^2)/a < z < \sqrt{u_m}$.

The case 2, $(\zeta - w_m^2)/a < z < \zeta/a$:

(1) $z = (\zeta - w_m^2)/a, \quad z = \{\zeta - \{w_m - (\zeta - az)^{-1} \times$

$$\times [b - (2\xi)^{-1/2}(u_m - z^2)^{-1}z^{-1}]^{-2}\}^2\}/a, \tag{21}$$

(2) the value z is found by use of Equations (20).

The case 3, $0 < z < u_m$:

(1) the value z is found by use of Equations (19),

(2) $z = \sqrt{u_m}, \quad z = \{u_m - (2\xi)^{-1/2}z^{-1} \times$

$$\times [b - (\zeta - az)^{-1/2}(w_m - \sqrt{\zeta - az})^{-1/2}]^{-1}\}^{1/2}. \tag{22}$$

The case 4, $(\zeta - w_m^2)/a < z < \sqrt{u_m}$:

(1) the value z is found by use of Equations (21),

(2) the value z is found by use of Equations (22).

3. The Formulation of the Problem in the Case of the MHD-model

In the case of the MHD-model the component of the Ampère force, which is perpendicular to the magnetopause, can be taken into account. Let us consider that (1) the current layer coincides with the layer of the return flow; and (2) the current layer is locally flat. The assumption (1) is justified by the fact that, according to the above-mentioned discussion, the Ampère force and the pressure drop are of the order ϱv^2; therefore, the current is more intense where the flow velocity is greater. The assumption (2) is justified by the fact that the layer is sufficiently thin and taking into account the curvature results in the correction of the order of the relation of the layer thickness to the radius of curvature. Then, since the flat current layer results in magnetic field on two sides which has the same values and opposite directions,

$$B_g + B = B_m, \qquad B_g - B = B_i, \tag{23}$$

where B_g is the field of the sources which there are inside the magnetosphere (the dipole and the neutral sheet), B is the field which is generated by the boundary current sheet, B_m is the field on the inward boundary of the layer of the return flow, B_i is the field on the outward boundary (out of the magnetosphere). Hence,

$$B = B_g - B_i, \qquad B_m = 2B_g - B_i. \tag{24}$$

Then the component of the magnetic pressure which is normal to the boundary is given by the equations

$$p_m = \frac{1}{c} IB_1, \qquad I = \frac{c}{2\pi} B, \qquad B_1 = \frac{B_m + B_i}{2} = B_g, \tag{25}$$

where I is the current which currents through the cross-section of the layer of the return flow, B_1 – the mean field in the layer.

So that the equatorial cross-section is considered, the field B_g can be approximated by the equation

$$B_g = \frac{M_g}{(x^2 + y^2)^{3/2}} + B_0, \tag{26}$$

where M_g is the moment of the geomagnetic dipole, B_0 is the constant field in the neutral sheet of the tail which is perpendicular to the neutral sheet. Equations (2) then assume the forms

$$\varrho_i v_i S_i = Q_0, \qquad \frac{2p_i}{\varrho_i} + \frac{v_i^2}{2} = \frac{2p'_\infty}{\varrho_{i\infty}} + \frac{v_{i\infty}^2}{2}, \qquad \frac{p_i}{\varrho_i^2} = \frac{p'_\infty}{\varrho_{i\infty}^2},$$
$$i = 1, 2, \quad S = \sum_{i=1}^{2} S_i, \tag{27}$$

where

$$p'_\infty = p_\infty - \frac{(B_0 - B_i)B_0}{2\pi}, \tag{28}$$

p_∞ is the gas pressure in the solar wind at infinity, it standing $p_{2\infty} = p_{1\infty}$ because of the absence of the magnetic field jump between the regions 1 and 2. Analogously,

$$p_2 = p - f - p_M, \qquad M_i^2 = \frac{\varrho_{i\infty} v_{i\infty}^2}{2p'_\infty}, \qquad i = 1, 2; \tag{29}$$

and, finally, the basic equations take the form (8) as in the case of a gasdynamical model if the value ζ is substituted for

$$\zeta_1 = (1 + \gamma)\zeta - \zeta_M, \tag{30}$$

where

$$\gamma = \frac{(B_0 - B_i)B_0}{2\pi p'_\infty}, \qquad \zeta_M = \frac{p_M}{p'_\infty}, \tag{31}$$

or

$$\begin{aligned} \zeta_M &= \frac{\alpha S_\infty^6}{(x^2 + y^2)^3} + \frac{\beta S_\infty^3}{(x^2 + y^2)^{3/2}} + \gamma, \\ \alpha &= \frac{M_g^2}{2\pi p'_\infty S_\infty^6}, \qquad \beta = \frac{M_g(2B_0 - B_i)}{2\pi p'_\infty S_\infty^3}. \end{aligned} \tag{32}$$

4. The Results of Calculations

As a preliminary, it is necessary to find the connection between the solar-wind parameters and the parameters of the return current layer. If the magnetospheric boundary is considered to be a tangential discontinuity, then the fact that the value ϱv^2 must be of the same order of magnitude on two sides of the boundary (Samokhin, 1970) ensures the equality of the complete (gas and magnetic) pressure on two sides of the discontinuity, and the above-mentioned concept that the movement of the plasma causes a diminution of the gas and magnetic pressures. Let us denote the parameters of the undisturbed solar wind with the subscript u, the solar-wind parameters at the stagnation point with the subscript 0, and at infinity with the subscript ∞. Then for large Mach numbers and the power 2 of the adiabatic law we have

$$p_0 = \frac{27}{32}\varrho_u v_u^2, \qquad \varrho_0 = \frac{27}{8}\varrho_u. \tag{33}$$

Since the magnetopause is the current line of liquid and the pressure obeys Equation (5), the equations

$$v_\infty = 2\sqrt{p_0/\varrho_0}\,\sqrt{1 - 1/\sqrt{\eta}}, \qquad \varrho_\infty = \varrho_0/\sqrt{\eta}, \qquad p_\infty = p_0/\eta \tag{34}$$

follow the Bernoulli equation and the adiabatic law. Then, in accordance with the approximate equality $\varrho_\infty v_\infty^2 \sim \varrho_{1\infty} v_{1\infty}^2$, we conclude finally that

$$M_1 = \sqrt{2(\sqrt{\eta} - 1)}, \qquad u_m = \sqrt{\eta}. \tag{35}$$

Figure 3 shows the results of the numerical calculation of the form and the size of the layer of the return flow and the other parameters of plasma in the magnetosphere with the above-mentioned method. The following values of the initial parameters have been adopted: the half-width of the tail at infinity $\pi l = 20R_{\mathrm{E}}$, the distance from the centre of the Earth to the subsolar point $h = 10R_{\mathrm{E}}$, the size of the plasmosphere $a = 6R_{\mathrm{E}}$, the relation of the densities at infinity in the layer of the return flow and in

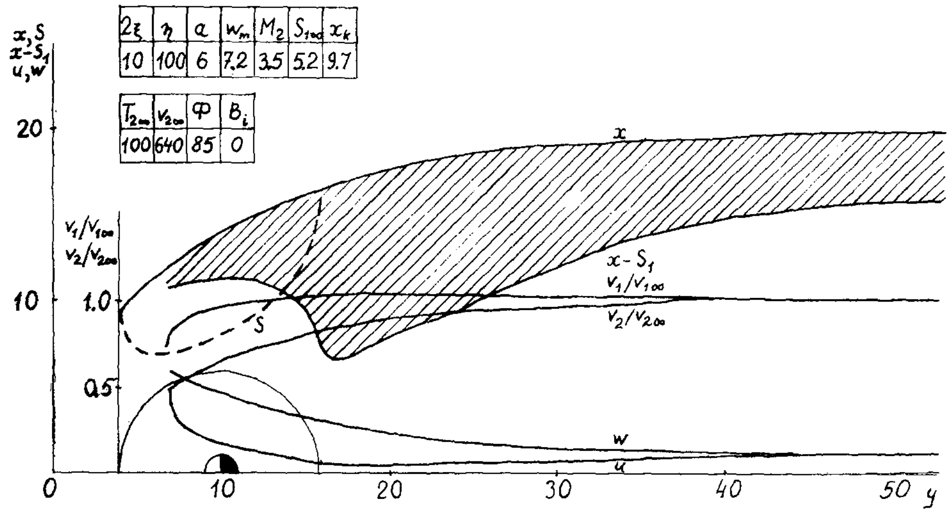

Fig. 3. The change of the thickness of the layer of the return flow, the velocities and the densities in the layer of the return flow and in the region of convection with the distance from the subsolar point. The equatorial plane of the magnetosphere is shown, the layer of the return flow is marked by strokes. The function $x(y)$ gives the change of the cross-section of the magnetosphere with the distance from the subsolar point y in the Earth radii accordingly to Equation (4), S_1 is the cross-section of the layer of the return flow adjacent to the magnetopause, $v_1/v_{1\infty}$ is the relation of the velocity of the return flow at a given point to the velocity of this flow at infinity (for $y \to \infty$), i.e. in the distant tail u is the relation of the density in the layer of the return flow at a given point to the density at infinity, $v_2/v_{2\infty}$ is the relation of the convection velocity at a given point to the convection velocity at infinity, w is the relation of the density at the point in question in the region of convection to the density at infinity in the region of convection. In the first table 2ξ is the relation of the densities at infinity in the layer of the return flow and in the region of convection, η is the relation of the dynamical and chaotic pressures in the solar wind, a is the radius of the plasmosphere, M_2 is the Mach number at infinity in the region of convection, $w_m = 1 + M_2^2/2$, $S_{1\infty}$ is the value of the thickness of the layer of the return flow at infinity, x_k is the co-ordinate x of the critical point, i.e. the point analogous to the point of transition through the local velocity of sound in the Laval nozzle or in the Parker model of extension of the solar corona. In the second table $T_{2\infty}$ is the supposed temperature of plasma in eV in the region of convection at infinity, φ is the potential difference in the tail in kV which is bound to the convection, B_i is the north component of the interplanetary magnetic field in γ (the velocity $v_{2\infty}$ is in km s^{-1}).

the region of convection $2\xi = 10$, the relation of the dynamical and statical pressures in the solar wind $\eta = 100$; the model is gasdynamical (i.e. the interplanetary magnetic field is zero, $B_i = 0$, the magnetic pressure in the magnetosphere is not taken into account). The first line of the upper part of Figure 3 shows the calculated values of the Mach number in the region of convection M_2 (and w_m), the thickness of the layer of the return flow $S_{1\infty}$ at infinity and the co-ordinate of the critical point x_k are pointed out. If the temperature of the plasma sheet is considered to be $T_{2\infty} = 100$ eV at infinity, then by use of the Mach number M_2 the velocity of convection at infinity $v_{2\infty}$ and the potential difference φ across the tail in the real magnetosphere (with the normal component $B_0 = 1\gamma$ in the neutral sheet) which corresponds to the velocity of convection $v_{2\infty}$ can be evaluated. These values at the second line of the upper part of

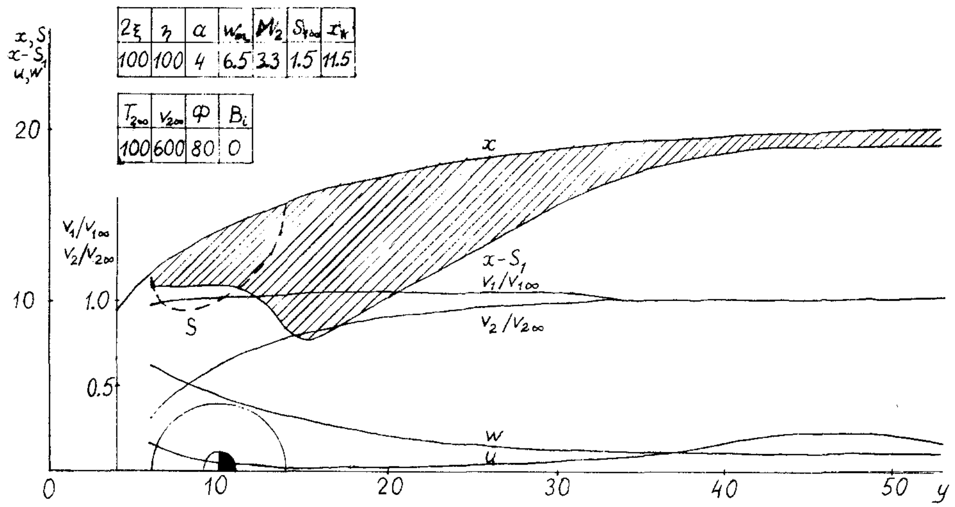

Fig. 4. The parameter 2ξ is ten times greater than on Figure 3, the value a is decreased to $4R_{\mathrm{E}}$ (see the caption of Figure 3).

Figure 3 are given. Along the horizontal axis the distance from the subsolar point in R_{E} is marked, along the vertical axis the following values are marked: (1) the position of the magnetospheric boundary $x = x(y)$, (2) the inward boundary of the layer of the return flow $x - S_1$ (the layer of the return flow is marked by strokes), (3) the cross-section of the region of convection (the broken line), (4) the density in the layer of the return flow u which is divided by the density at infinity, (5) the dimensionless density in the region of convection w, (6) the dimensionless velocity in the layer of the return flow $v_1/v_{1\infty}$, (7) the dimensionless velocity of convection $v_2/v_{2\infty}$. It is seen that the layer of the return flow becomes very thin near the critical point at the dawn and dusk sides of the magnetosphere, has the maximum thickness in the cross-section of the tail on the geocentrical distance $\sim 10R_{\mathrm{E}}$ and with the further increase of the geocentrical distance the thickness of it decreases approximately to $5R_{\mathrm{E}}$. In Figure 4 the analogous results for $2\xi = 100$, $\eta = 100$ and $a = 4$ are shown (the rest parameters are the same). Increasing ξ results in decreasing the thickness of the layer of the return flow to $1.5R_{\mathrm{E}}$ on the great distances which is found to coincide with the data of Pioneer 7 (Intriligator and Wolfe, 1972) and somewhat reduces the velocity of convection. In Figure 5 the variant of the model is shown in which the contribution of the normal component of the Ampère force (the jump of the magnetic field) in supporting the equilibrium of the layer of the return flow is taken into account. The interplanetary magnetic field is supposed to have the north component $B_i = 2\gamma$, the normal component of the geomagnetic field in the neutral sheet at large geocentrical distances is supposed to be $B_0 = 1\gamma$. The rest of the initial parameters are the same as for Figure 4. It is seen that taking into account the jump of the geomagnetic field on the magnetopause results in a decrease of the thickness of the layer of the return flow, the velocity of convection and the potential difference in the tail as one could expect. It is worth

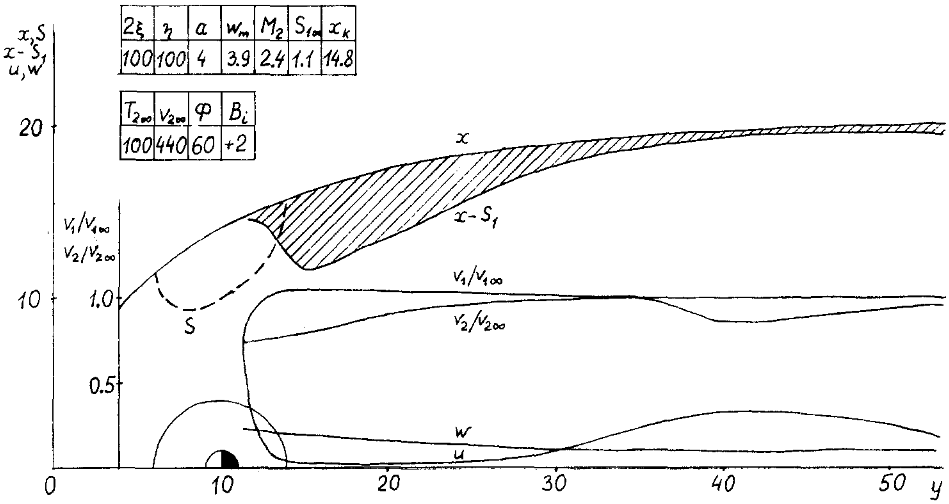

Fig. 5. The normal component of the Ampère force on the magnetopause is taken into account (the MHD model), the interplanetary magnetic field $B_i = 2\gamma$ is directed to the north, the rest parameters are the same as for Figure 4 (see the caption of Figure 3).

noting that the method of calculation employed does not allow to continue the solution to the region occupied by the interval between the stagnation point and the critical point; besides, in the latter region the one-dimensional model becomes invalid since the characteristic length along the axis x and y turn out to be comparable.

A comparison of the results of theoretical calculation with the data provided by the Vela satellites (Hones *et al.*, 1972, 1973) demonstrates that the considered mechanism is sufficient to explain high velocities of convection (up to ~1000 km s^{-1}) directed to the Earth and registered as substorms near the neutral sheet of the tail.

Conclusions

For investigation of the plasma flow in the equatorial plane of the magnetosphere taking into account the real configuration of the magnetosphere, the following one-dimensional models can be offered: (1) the gasdynamical model in which the Ampère force is ignored and the solar-wind pressure on the layer of the return flow near the magnetopause is compensated by the centrifugal force, which is bound to the curvature of the layer, and the plasma pressure inside the magnetosphere in the region of convection and (2) the MHD-model, in which the normal component of the Ampère force on the magnetopause is considered. The following physical mechanisms are essential: (1) the appearance of the centrifugal force because of the curvature of the magnetopause and because of the relatively rapid motion of plasma in the layer of the return flow which partly compensate the solar-wind pressure for, (2) the existence of the critical point analogous to that of transition through the velocity of sound in the Laval nozzle, or in the model of the solar corona of Parker, which makes it possible to obtain a single solution. The values of the thickness of the layer of the return flow, the velocity of convection and the potential difference across the tail which were calculated in the framework of these models agree in orders of magnitude with the observational data. In the gasdynamical model the convection regime is possible only for high flow velocities corresponding to substorms. In the MHD-model, by selecting the initial parameters of the problem, the velocity of convection can be decreased to the values corresponding to the undisturbed geomagnetic conditions; then the jump of the magnetic pressure on the magnetopause is the principal cause of support of the equilibrium layer of the return flow.

References

Axford, W. I.: 1969, *Rev. Geophys.* **7**, 421.
Axford, W. I. and Hines, C. O.: 1961, *Can. J. Phys.* **39**, 1433.
Bogott, F. and Mozer, F. S.: 1971, *J. Geophys. Res.* **76**, 892.
Chappell, C. R.: 1974, in *Correl. Interplanetary and Magnetospheric Observ.*, Proc. 7th Eslab. Symp. Saulgau, 1973; Dordrecht–Boston, 1974, p. 277.
Dungey, J. W.: 1961, *Phys. Rev. Lett.* **6**, 47.
Freeman, J. W., Warren, C. S. and MacGuire, J. J.: 1968, *J. Geophys. Res.* **73**, 5719.
Hones, E. W., Asbridge, J. R., Bame, S. J., Montgomery, M. D., Singer, S. and Akasofu, S.-I.: 1972, *J. Geophys. Res.* **77**, 5503.
Hurley, J.: 1961, *Phys. Fluids* **4**, 109.
Intriligator, D. S. and Wolfe, J. H.: 1972, *J. Geophys. Res.* **77**, 5480.
Levy, R. H., Petscheck, H. E. and Siscoe, G. L.: 1964, *AIAA J.* **2**, 2065.
McCoy, J. E., Lin, R. P., McGuire, R. E., Chase, L. M. and Anderson, K. A.: 1975, *J. Geophys. Res.* **80**, 3217.
Obayashi, T. and Nishida, A.: 1968, *Space Sci. Rev.* **8**, 3.
Parker, E. N.: *Interplanetary Dynamical Processes*, New York–London, Interscience Publishers (a division of John Wiley and Sons).
Roederer, J. G.: 1974, in *Correl. Interplanetary and Magnetospheric Observ.*, Proc. 7th Eslab. Symp. Saulgau, 1973; Dordrecht–Boston, 1974, p. 257.
Rosenbauer, H., Grünwaldt, H., Montgomery, M. D., Pashmann, G. and Sckopke, N.: 1975, *J. Geophys. Res.* **80**, 2723.
Samokhin, M. V.: 1970, *Kosmich. Issled.* **8**, 790.
Samokhin, M. V.: 1976, *Astrophys. Space Sci.* **44**, 13.
Villante, U. and Lazarus, A. J.: 1975, *J. Geophys. Res.* **80**, 1245.
Walker, R. C., Villante, U. and Lazarus, A. J.: 1975, *J. Geophys. Res.* **80**, 1238.
Zhigulev, V. N.: 1959, *Dokl. U.S.S.R. Acad. Sci.* **124**, 1001.