THE LUNAR CAPTURE HYPOTHESIS REVISITED

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Abstract. Recent work on planetary formation processes have suggested that ancient planetary bodies could have been warmer and, therefore, more easily deformable soon after formation than at present. By use of the estimates for the elastic parameters believed to be appropriate for a warm ancient Moon and Earth, it is shown that the energy of deformation of the planetary bodies during a close gravitational encounter was sufficient to effect capture.

The lunar capture hypothesis has been promoted for various reasons by Urey (1952), Alfvén (1954), Cloud (1968, 1972), Singer (1968, 1970), Gerstenkorn (1969), Alfvén and Arrhenius (1972, 1976) and others. However, Kaula and Harris (1973) have shown that capture of a lunar-sized body with the physical properties of the present nearly *rigid* Moon is implausible. In this paper we show that capture of a warm (*deformable*) lunar-sized body is possible using plausible assumptions for the values of the interaction parameters.

Information from the Apollo missions has led some investigators to favor a warm ancient Moon (Smith *et al.*, 1970; Wood, 1972). This idea has been most recently amplified by Wood (1975) and Walker *et al.* (1975) who suggest that the petrology of the ancient anorthositic lunar crust can be explained as having resulted from crystal fractionation and gravitational separation in a well-stirred global subcrustal magma chamber referred to by Wood (1975) as a magma ocean. The high temperature origin of lunar mare rocks also attests to a warm Moon at least a billion years after its formation.

Several investigators have attempted to relate mare formation to tidal interactions with Earth (Kopal, 1966; Cloud, 1968, 1972; Alfvén and Arrhenius, 1969, 1976; Stuart-Alexander and Howard, 1970; Hartung, 1976; Friedlander and Smith, 1977). Lipskiy *et al.* (1966) and Stuart-Alexander and Howard (1970) describe the maria as lying in a crude global belt mainly on the lunar front side. More recently, Malcuit *et al.* (1975) reported that several of the large circular maria (Orientale, Imbrium, Serenitatis, Crisium, and Smythii) are very nearly distributed along a lunar great circle. Two other trends associated with these particular maria are that the mean diameter of each decreases from Imbrium to the east and that the mean elevation of the mare surfaces decreases in the same direction (Wollenhaupt and Sjorgen, 1972). Malcuit *et al.* (1975) suggest that this approximate great-circle pattern of large circular maria and associated trends may be the signature of a very close encounter with Earth.

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TABLE I

Energy to dissipate for lunar capture within sphere of influence of Earth

V_∞ (km/sec)	Apogee of Capture Orbit (Perigee = $1.4\,R_e$) (earth radii)	Energy to dissipate (10^{35} ergs)
0.0	270	1.69
0.2	270	1.84
0.4	270	2.28
0.6	270	3.02
0.8	270	4.04
1.0	270	5.37
0.0	540	0.85
0.2	540	1.00
0.4	540	1.44
0.6	540	2.17
0.8	540	3.20
1.0	540	4.52

For stable capture, enough orbital energy must be dissipated within the interacting bodies during one, or a few encounters, to change the lunar heliocentric orbit into a geocentric orbit within the sphere of influence (Roy, 1965; Öpik, 1976) of Earth. We assume with Öpik that $270\,R_e$ (Earth radii) is a reasonable logarithmic mean value for the radius of the Earth's sphere of influence. However, there is a transition zone ($\leqslant 540\,R_e$) in which capture can occur (Öpik, 1976).

Table I shows the minimum amount of orbital energy that must be dissipated for capture for various encounter speeds (V_∞) and for two values for apogee (both within the Earth's sphere of influence). The table shows that capture from a nearly Earth-coincident orbit entails dissipation of about 10^{35} ergs, the exact value depending on the encounter speed and the eccentricity of the capture orbit. The purpose of this paper is to show that, given certain narrowly defined orbital and elastic parameters, gravitational capture of a lunar-sized body is *possible*. Although the conditions on the orbital and elastic parameters are stringent, they are physically realistic conditions for planetary bodies in the early history of the solar system.

Munk and MacDonald (1960) show that the rate at which work is done in elastically deforming a body can be represented as the sum of two integrals

$$\frac{\Delta E}{\Delta t} = -\int_{\text{vol}} \frac{p_0}{\rho}\frac{dp}{dt}\,dV + \int_{\text{vol}} \tau_{ij}\frac{d\epsilon_{ij}}{dt}\,dV. \tag{1}$$

The first integral represents the effects of compression, the second the effects of the disturbing stresses ϵ_{ij}. Ignoring the effects of compression, Kaula and Harris show that the second integral can be written (for a satellite approaching the Earth to perigee distance r_p) as

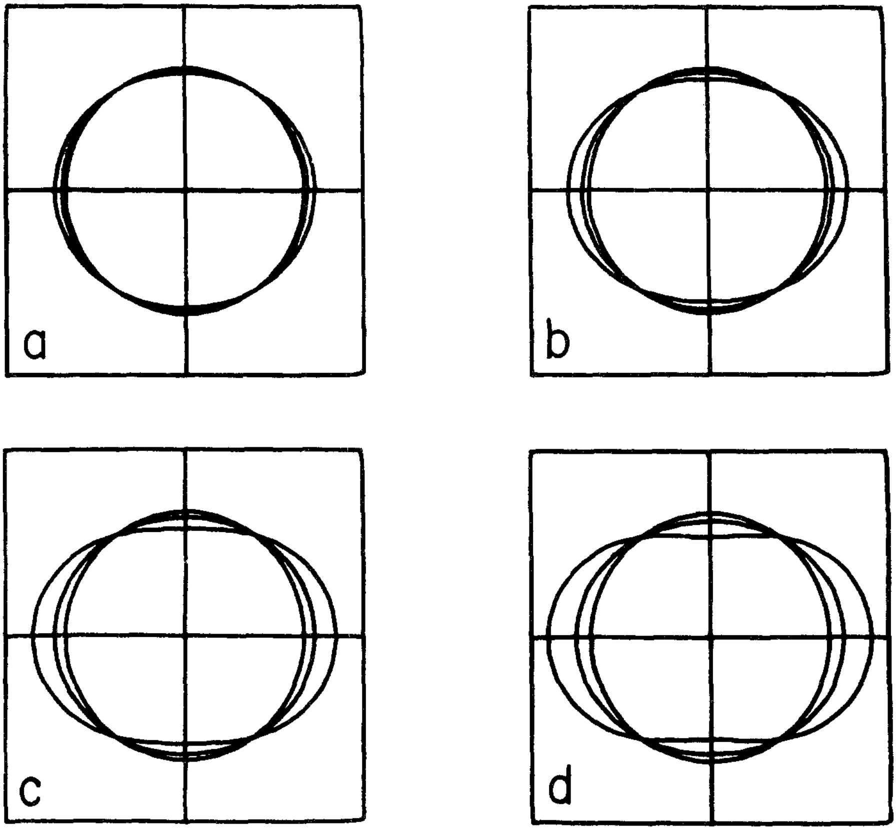

Fig. 1. Scale diagrams for a homogeneous lunar model showing the relationship between the displacement and potential Love numbers, h and k, and body deformation at various perigee distances, r_p. In all cases (a.–d.), $r_p = 1.4\,R_e$ (Earth radii), $2.0\,R_e$, and $10\,R_e$. (a) $h = 0.15, k = 0.08$, maximum radial displacement (mrd) at $1.4\,R_e = 9.8\%$; (b) $h = 0.30$, $k = 0.16$, mrd = 20%; (c) $h = 0.45$, $k = 0.24$, mrd = 30%; (d) $h = 0.60, k = 0.32$, mrd = 39%.

$$\Delta E_{\text{stored}} = -9h(1+k)GM_e^2R_m^5 \int_{r_p}^{\infty} \frac{dr}{r^7} \int_0^{\pi} d\theta \sin\theta\, P_{20}^2(\theta), \tag{2}$$

where h and k are the Love numbers appropriate for the satellite, M_e is the mass of the Earth, R_m is the radius of the satellite and P_{20} is the $l = 2$, $m = 0$ associated Legendre polynomial. According to Munk and MacDonald, the first integral in Equation (1) could result in an amount of stored energy equal to the second integral. However, in this paper we ignore the first integral. With this formulation it can be shown that the energy which can be stored in the body by elastic deformation is given by

$$\Delta E_{\text{stored}} = \frac{3h(1+k)GM_e^2R_m^5}{5r_p^6}. \tag{3}$$

Thus, the energy that can be stored in a deformed body can be described in terms of

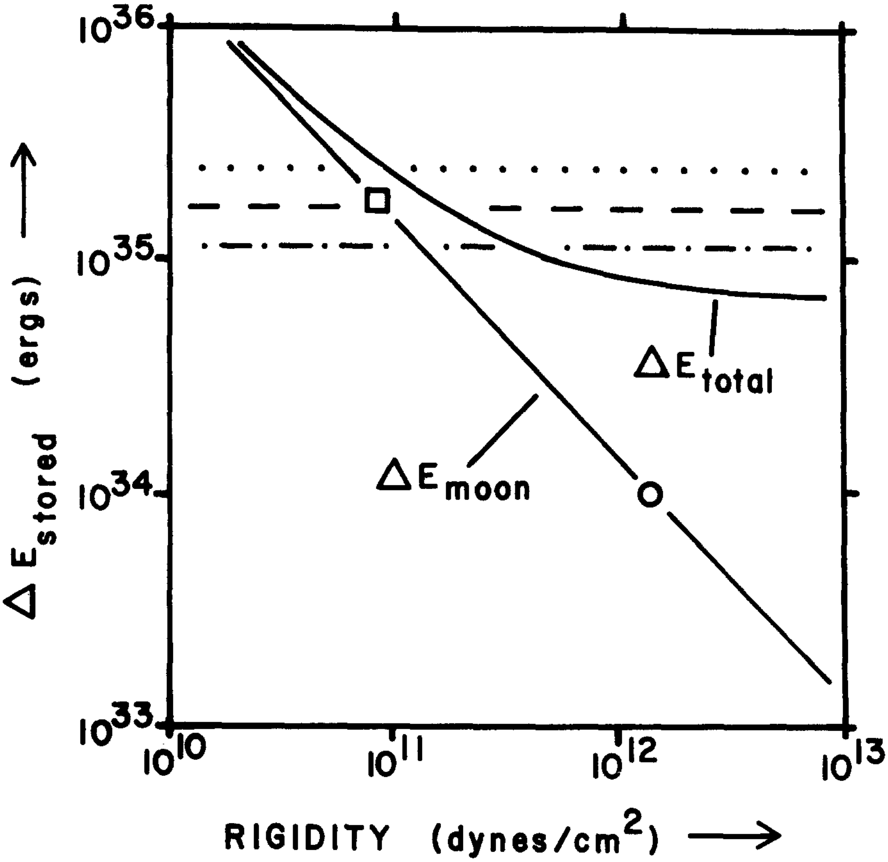

Fig. 2. Plot of stored energy (ΔE_{stored}) vs rigidity (μ) for various lunar models (lower curve labeled ΔE_{moon}) and for combined Earth and Moon (upper curve labeled ΔE_{total}). In both cases $r_p = 1.36\,R_e$. For the lunar model $\lambda/\mu = 1.25$; for the earth model, $h = 0.9$, $k = 0.5$. Dotted line represents energy to be dissipated for capture of a lunar-sized body with $V_\infty = 0.5$ km/sec into a geocentric orbit with apogee of $270\,R_e$. Dashed line represents energy to be dissipated for capture of a lunar-sized body with $V_\infty = 0.1$ km/sec into a geocentric orbit with apogee of $270\,R_e$. Dot-dashed line represents energy to be dissipated for capture of a lunar-sized body with $V_\infty = 0.1$ km/sec into a geocentric orbit with apogee of $400\,R_e$. ○ = Kaula and Harris (1973) estimate of the energy that could be stored in the present Moon during a close encounter. □ = energy that could be stored in Harrison's (1963) inhomogeneous lunar model 7, case 3 ($h = 0.33$; $k = 0.19$).

its Love numbers: h, the displacement Love number, and k, the potential Love number, and perigee. Kaula and Harris (1973) used Love numbers that are appropriate for the present cold, rigid Moon – $h = 0.033$ and $k = 0.020$. Their argument is relatively simple and shows that only about 10^{34} ergs could be stored in the deformed body and only a fraction of that energy would be dissipated. According to their analysis, the stored energy is much too small for capture in one pass, and Kaula and Harris conclude that capture by a gravitational encounter is implausible. Their beautifully simple argument is compelling.

But, when one looks at the alternatives to gravitational capture, one finds them no less implausible, e.g., one finds *ad hoc.* suggestions such as multibody breakup and subsequent reformation (e.g., Mitler, 1975; Smith, 1976).

Hence, we have re-examined the Kaula and Harris model in an attempt to make the model more nearly realistic by using elastic properties appropriate for a warm, deformable ancient Moon. Since the Love numbers contain all information about the elastic properties of the deformed body, it is these numbers which need to be recalculated. We use the results of a model of planetary tidal deformation which was developed by Love (1911). The model treats the deformation of a compressible self-gravitating body which is characterized by a rigidity μ and the first Lamé constant λ. For the present work, we assume the tidal deformation to be due to a potential of the second harmonic and, therefore, the radial deformation can be described by the associated Legendre polynomial P_{20}.

Equations for calculation of the Love numbers are

$$h = \frac{5}{2} - \frac{5}{6}\frac{3\alpha^2+\beta^2}{\alpha^2-\beta^2}A_2\psi_2 - \frac{1}{3}A_2\psi_1 + \frac{5}{6}\frac{\alpha^2+3\beta^2}{\alpha^2-\beta^2}B_2\chi_2 + \frac{1}{3}B_2\chi_1 \tag{4}$$

$$k = A_2\psi_2 + B_2\chi_2 + C_2 - 1. \tag{5}$$

The functions α, β, ψ, χ, etc. depend on the boundary conditions and elastic properties of the deformed body and are defined in Love's work.

Figure 1 shows the deformation of the lunar body with various Love numbers as the Moon approaches the Earth. The simplifying assumption here is that the Moon is homogeneous. The maximum radial deformation for such a lunar model is given in the caption of Figure 1. Note that for the case where $h = 0.60$, at $1.4R_e$, the characteristic P_{20} deformation is extreme and the Moon is beginning to show signs of fissioning. Clearly, there are constraints on the values for the physical parameters.

Figure 2 shows the energy of deformation, ΔE_{stored}, for various lunar body models. We have also considered the energy that can be deposited in a model ancient Earth. This Earth model is somewhat more deformable than the present Earth with Love number values approximately 50% larger than those for the present Earth. The total energy stored in the two interacting bodies is given by the curve labeled ΔE_{total}. Note that for the lunar rigidities $\mu \approx 5 \times 10^{10}$ dynes/cm^2 and $\lambda/\mu = 1.25$, approximately 4×10^{35} ergs are stored in the bodies. These values for the elastic parameters correspond to the lunar model shown in Figure 1c and are consistent with Harrison's (1963) inhomogeneous lunar model 7, case 3 (see caption of Figure 2). The dotted, dashed and dot-dashed lines indicate the energy required for capture of a lunar-sized body under various conditions. Thus, if about one-fourth of the energy of deformation is dissipated, then gravitational capture of a lunar-sized body with reasonable elastic properties and a low V_∞ is possible in even a single close encounter.

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