OBSERVATIONS OF BAILY'S BEADS FROM NEAR THE NORTHERN LIMIT OF THE TOTAL SOLAR ECLIPSE OF JUNE 20, 1974

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Abstract. Baily beads were observed from near the northern limit of totality during the eclipse of June 20, 1974; no attempt of identification was made at the time of observation. The observations were analysed to identify the features of the lunar profile which gave rise to the beads observed. They were then analysed to derive corrections to the relative positions of the Sun and Moon, and to investigate certain features of the lunar profile.

1. Introduction

With the continual improvement in our knowledge of the Moon's shape and motion, it is becoming more feasible to use the Moon as a reference body for determining the positions of other celestial objects. The observation of solar eclipses have often been used in the past to yield precise measures of the relative positions of the Sun and Moon by a variety of techniques. Kristenson (1951) and Mori and Kubo (1971) photographed the flash spectrum at intervals near the times of second and third contacts. Banachiewicz and Kordylewski (1932) took direct photographs near the times of second and third contacts. All of these experiments were performed close to the central line of the eclipse ... the method of Kristenson is, by its nature, not suitable at any great distance from the central line. Since these experiments were made near the central line of the eclipse, one would not expect them to yield a determination of the relative latitude of the Sun and Moon to a high order of accuracy, since most of the Baily beads that will be seen will occur at the east and west limbs ie. where the dependence upon latitude is least. Thus, for example, the results obtained by Kristenson at two different stations agree to better than 0".01 in long. but differ by 0".46 in lat.

It has recently been suggested (Dunham and Dunham, 1973) that a more accurate determination of the relative latitudes of the Sun and Moon may be obtained near (but inside) the limits of an eclipse. The prime reason for this is that near the limits, the duration of the eclipse is highly sensitive to any change in relative latitude. A change in duration of 1 s. corresponding to a change in relative latitude of 0".04 or less is typical. A second feature of observations made near the limits is that the edge of the Moon where Baily beads occur can be measured to a high degree of accuracy through the observation of grazing occultations. One of the prime problems in the observation of Baily beads is the limited accuracy in our knowledge of the lunar profile. Grazing occultations can improve the accuracy of the profile by an order of magnitude, but only at the northern and southern limbs of the Moon. Hence, by observing the Baily beads from near the path limits, it should be possible to analyse the results using a "correct" profile, and thus overcome many previous problems.

Here I report observations of Baily beads associated with the total solar eclipse of June 20, 1974, made near the northern limit of totality. The results were analysed using a profile which was not corrected from the results of grazing occultations. Further, the results were obtained by visual means alone, with no absolute identification of the beads. The lunar features responsible for each of the beads were identified in the course of the analysis. The observations are used to investigate the relative positions of the Sun and Moon, and also to investigate some macroscopic features of the lunar profile.

2. Location of the Observing Site

The observation was made some 6 km south of the northern limit of totality as computed from the Besselian elements published in the Astronomical Ephemeris for 1974. The coordinates of the site were measured from a 1:100000 survey map entitled 'Busselton' (sheet 1930) drawn to the Australian Geodetic Datum 1966, published by the Australian Government in 1969. The site was located 5 m north, and 10 m west of the intersection of the Caves Road, and another running generally south-east, at Quininup, Western Australia. The measured coordinates are:

$$\lambda = -\,115°01'50''.6 \pm 0''.8,$$
$$\phi = -\;\;33°14'26''.6 \pm 0''.8,$$
$$h = 100\ \text{m}.$$

3. Equipment Used

The Baily beads were observed visually by projecting an image of the Sun onto a white screen. A 60 mm 'Unitron' refractor was used, and the diameter of the projected image was approximately 10 cm. An abbreviated commentary of the bead events was recorded on a portable tape recorder, together with time signals broadcast by VNG (Lyndhurst, Victoria). Subsequent replay of the tape enabled the time of each Bead event to be determined to ±0.3 s (m.e.). (It should be noted that the observe was well versed in this method of data recording through experience with grazing occultations.)

An attempt was made to obtain a photographic record of the Baily beads. However this failed, and it thus became necessary to identify lunar features solely from the times and other comments recorded on the tape recorder.

4. The Observations

The eclipse was viewed under reasonable conditions. Some high altitude cloud was present during the total time of observation. However this cloud was quite thin, and no effect could be discerned on the projected solar image.

From the tape recording of the eclipse, 72 timings were obtained of events associated with the Baily beads. Prior to totality, the events relate to the breaking of the solar crescent by a lunar mountain to form a bead, and to the subsequent disappearance of that bead. After totality, the events relate to the appearance of the solar limb at the bottom of a lunar valley, and the subsequent merging of that bead with the solar crescent. In general, there was only one bead present at any time, and the positions of the beads followed a regular progression along the lunar profile. The exception to this was within about 15 s of totality, when a large number of beads were present. Occasionally, more than one event did occur at substantially the same time. When this occurred, the commentary on the tape was generally sufficient to establish the number of features involved. Thus the commentary on the tape was generally able to be kept to a limited statement such as 'bead, top end', signifying the formation of a bead at a particular end of the solar crescent, followed by 'gone'. By this method, almost every Baily bead that occurred was recorded.

Table I, lists the times of the events recorded (0.4 s having been removed as an allowance for personal equation), and the type of event that was seen, as deduced from the commentary. In the second column, *n* and *s* refer to the northern and southern ends of the solar crescent, f=formation, g=disappearance, d=division into two,

TABLE I

Time U.T.	Description of event	Identified feature *WA* height ° ″	Δr
5ʰ04ᵐ06ˢ.2	*s f*		
04 16.0	*s g*		
10 46.2	*s f*	121.20+1.35	−0.19
48.1	*s g*	121.60+0.55	+0.08
49.5	*s f*	120.75+1.62	+0.52
10 49.8	*s g*	120.80+1.40	+0.45
11 02.3	*s f*	118.00+0.65	+0.26
03.6	*s g*	118.33−0.05	+0.40
48.1	*s f*	102.15+1.38	−0.14
49.3	*s d* (2) (*a* and *b*)	103.05−0.05	−0.34
51.5	*s g* (a)	102.80−0.07	+0.22
51.7	*s g* (*b*)	103.65−0.75	+0.40
55.9	*s f*	99.67+1.33	+0.38
11 59.4	*s g*	100.25−0.73	+0.03
12 04.3	*s f*	94.83+1.38	−0.67
06.8	*s g*	95.60+0.28	−0.27
07.7	*s f*	92.70+2.07	−0.42
10.6	*s g*	93.13+1.17	+0.03
17.8	*s f*		
18.3	*s g*		
22.5	*s g*		
25.7	*s f*		
26.7	*s g*		
12 27.7	*s f*	83.90+1.05	−0.65

Table I (Continued)

Time U.T.		Description of event	Identified feature *WA* height ° ″	*Δr*
	29.7	*s g*	84.10+0.44	−0.45
	31.7	*s f*	82.05+0.95	−0.56
	33.0	*s g*	82.50+0.27	−0.53
	37.1	*n f*	19.25+0.07	−0.47
	37.3	*s f*	78.35+0.83	−0.99
	39.9	*s g*	78.70+0.28	−0.47
	43.2	*n g*	19.00−0.17	−0.41
	45.3	*s f* (3)	75.80+0.25	−0.34
12	50.7	*s f*	71.80+0.40	−0.53
13	03.7	*s f* (2)	65.95−0.55	−0.25
	05.4	*s f*	63.20−0.17	−0.52
	06.5	*s g*	63.45−0.58	−0.53
	09.5	*s f* (2)		
	10.7	*s g* (2)		
	11.7	*n f*	22.40+0.20	−0.47
	12.5	*n g*	22.01−0.07	−0.59
	18.9	*s f* (large bead)	52.40+0.03	−0.37
	19.9	*s g* (large bead)	52.60−0.42	−0.55
	21.8	*s f* (2 or 3)		
	24.3	*s g* (2 or 3)		
	25.5	break in crescent	40.70+0.53	−0.75
	29.7	*s f* (2 or 3)		
	31.0	*n f* (several)		
	33.1	6 in centre	38.10−0.26	−0.49
			36.80+0.03	−0.29
			35.55+0.14	−0.25
			35.00+0.11	−0.31
			32.40+0.23	−0.23
13	36.7	all beads gone	32.00−0.08	−0.13
			37.35−0.46	−0.21
15	04.6	third contact	357.10−0.87	−0.41
	09.3	5 or 6 formed	346.40−0.76	−0.04
			347.90−0.57	−0.11
			351.00−0.17	−0.01
			352.70−0.20	−0.14
			0.60−0.47	−0.24
	13.0	5 or 6 merge	359.70−0.02	−0.15
			349.20+0.22	+0.01
			347.00 0.00	0.00
	14.0	*s a*	343.40−0.73	−0.40
	16.9	*s m*	344.95+0.35	−0.03
	18.0	*s a*	338.90−0.40	+0.10
	19.0	*s a* (far out)	337 ?	
	20.4	*s m*	340.20+0.62	+0.40
	22.7	*s a* (*j*)	334.10−0.30	+0.47
	22.8	*s a* (*k*)	332.00−1.20	+0.12
	22.9	*s a* (*l*)	330.90−1.70	−0.06
	26.2	*s m* (*k*+*l*)	331.60−0.87	−0.24
	28.9	*s m* (*j*+*k*+*l*)	333.40+0.62	+0.12

Table I (Continued)

Time U.T.	Description of even	Identified feature *WA* height ° ″	Δr
30.5	*s a*		
31.6	*s m*		
37.7	*n a* (2)		
40.4	*s a*	321.60−0.80?	−0.30
42.7	*s m*	321.70−0.13?	−0.33
51.4	*n m*		
54.4	*s m*	315.40+1.33?	+0.51
56.7	*n m*		
59.3	*s m* (2)	310.60+0.62?	+0.65
16 02.0	*n m*		
17 00	end observation.		

TABLE IIA

Geocentric position of the Sun

U.T.	R.A.	Decl.
$5^h04^m00^s$	$5^h53^m29\overset{s}{.}173$	$+23°26'00\overset{''}{.}81$
08 00	5 53 29.867	+23 26 00.92
12 00	5 53 30.560	+23 26 01.03
16 00	5 53 31.254	+23 26 01.13

Horizontal Parallax = 8″.653.

a = appearance, and m = merging, of a bead. The numbers indicate the number of beads involved, when the commentary indicated the presence of a plurality of beads.

5. The Position of the Sun and Moon

The positions of the Sun and Moon for the duration of the eclipse are required for both the identification of the lunar features which caused the Baily beads, and for the reduction of the observations.

The position of the Sun was calculated using the ecliptic latitude and longitude data given in the Astronomical Ephemeris for 1974, to a precision of 0″.01, and including the necessary corrections for the IAU System of Astronomical Constants. The solar semidiameter was calculated using the value 15′59″.63 for the semidiameter of the Sun at unit distance, exclusive of irradiation. The geocentric position of the Sun is given in Table IIA.

The position of the Moon is from the Lunar Ephemeris $j=2$. Table IIB gives the geocentric position of the Moon. In deriving the positions of the Sun and Moon from the Astronomical Ephemeris, the solar ephemeris was entered using $\Delta T = 45.04$ s, with the corresponding value $\Delta T = 43.70$ s in the Lunar Ephemeris.

TABLE IIB

Geocentric position of the Moon

U.T.	R.A.	Decl.	Hor. parallax.
5ʰ04ᵐ00ˢ	5ʰ53ᵐ52ˢ.540	+22°35′32″.11	60′37″.04
5 08 00	5 54 03.264	+22 35 20.12	60 37.10
5 12 00	5 54 13.987	+22 35 08.09	60 37.17
5 16 00	5 54 24.711	+22 34 56.02	60 37.23

TABLE III

Position of the Moon relative to the Sun during the eclipse

U.T.	x	y	Moon's radius R_m	Position angles of extremities of Solar crescent		Relative limb inclination at extremities q
5ʰ04ᵐ00ˢ	−238″.99	− 9″.76	1000″.48	157°.33	17°.99	13°.74
04 30	−228.00	−11.75	1000.48	156.39	17.71	13.07
10 30	− 95.83	−36.14	1000.43	123.65	15.02	5.05
11 00	− 84.80	−38.20	1000.43	116.52	14.98	4.38
11 30	− 73.76	−40.28	1000.42	107.69	15.03	3.69
12 00	− 62.72	−42.36	1000.42	96.69	15.25	3.00
12 30	− 51.68	−44.45	1000.42	82.83	15.76	2.29
13 00	− 40.64	−46.54	1000.41	65.20	17.03	1.53
13 30	− 29.58	−48.66	1000.41	41.07	21.52	0.59
14 00	− 18.53	−50.74	1000.41	–	–	–
14 30	− 7.48	−52.86	1000.40	–	–	–
15 00	+ 3.58	−54.97	1000.40	–	–	–
15 15	+ 9.11	−56.03	1000.40	342.04	359.49	0.52
15 30	+ 14.64	−57.09	1000.39	328.20	3.03	1.07
16 00	+ 25.70	−59.22	1000.39	307.76	5.32	1.89

Topocentric semidiameter of Sun, R_s=944.33.

The geocentric positions were used to compute the corresponding topocentric positions of the Sun and Moon, using the standard formulae. The topocentric semidiameters of both the Sun and Moon were also calculated. To place the topocentric positions into a usable form, the position of the Moon with reference to the position of the Sun is formed, using rectangular coordinates centered on the Sun, y – axis pointing north, x – axis pointing east. Table III lists the values of x and y at intervals during the eclipse, and the topocentric semidiameters of the Moon (R_m) and the Sun.

6. The Identification of the Lunar Features

Since no photographic record of the Baily beads was obtained, it is necessary to identify the lunar features which gave rise to the beads solely from the relative positions of the Sun and Moon and from the knowledge of the lunar profile. It is important to note here the reason for the occurrence of Baily beads, viz. when the solar limb is near coincident with the lunar limb, the peaks of the lunar break the solar limb leaving a separated portion of the solar limb visible through an adjacent valley. Since the maximum deviation of the actual lunar limb from the mean is of the order of $\pm 2''$, Baily beads can only be formed where the solar limb is within about $\pm 2''$ of the mean lunar limb. This means that, except for times very close to totality, beads will be confined to the region at about the intersection of the solar and lunar limbs. Hence by calculating the positions of the intersections of the limbs, on the lunar profile, for each recorded event, it is possible to identify the lunar feature which caused the event.

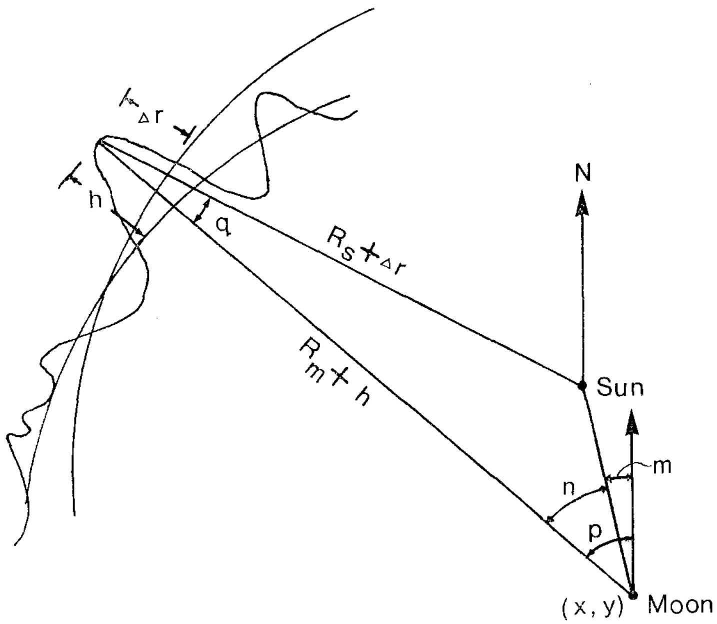

Fig. 1a.

Referring to Figure 1a which illustrates the geometry of the eclipse, the identification of the lunar features requires the determination of angle p. For the purpose of calculating the position angle of the extremities of the solar crescent, the quantities h and Δr in Figure 1a are assumed to be zero. This is equivalent to using a smooth lunar profile. From the topocentric positions, the coordinates of the Moon relative to the Sun (x, y) are known (Table III). The distance between the Sun and Moon, and the angle m, are known: also known are the distances R_m and R_s, the lunar and solar semidiameters. Hence, the position angle of the extremity of the solar crescent can be readily derived. Note that there are two solutions, $m+n$ and $m-n$. In the course of evaluating p, it is advantageous to also compute the angle q. This angle represents subtended between the solar and lunar limbs, and is most useful in the identification of the lunar features. Table III lists the position angles of the extremities of the solar crescent, and the angle q, at intervals during the eclipse.

It is implicit in the reduction of this observation that an accurate lunar profile is available. The most accurate and convenient profile available at present is that prepared by Watts (1963). Watts charts were entered with the following topocentric librations calculated for mid-eclipse:

$$l = -3\overset{\circ}{.}05, \qquad b = +0\overset{\circ}{.}23, \qquad C = -2\overset{\circ}{.}16.$$

The values obtained from Watts charts were plotted using a vertical scale of 1 in. = 2″, and a horizontal scale of 1 in. = 2° of Watts angle. The resulting whole Moon profile was 15 ft long.

By use of the profile so obtained, and the data in Table III, it is now possible to identify the lunar features which gave rise to the beads observed. For convenience, graphs of the position angle of the solar crescent extremities, and the angle q can be formed as a function of time. It must also be noted that the Watts angle is related to the position angle by $WA = p - C$, where p is the position angle, and C is the topocentric position angle of the lunar pole, given above.

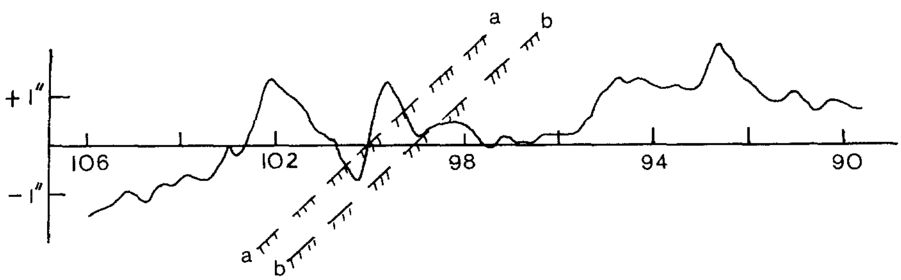

Fig. 1b.

Figure 1b illustrates the process of identification. At time **a**, a bead was seen to form, and at time **b**, that bead was seen to disappear. The angle p, and hence the Watts angle through $WA = p + 2\overset{\circ}{.}16$, is found for the times **a** and **b** from the graph of p. Likewise the angle q was obtained from the graph of q. On the profile, a line is drawn for each of these times, intersecting the mean lunar limb at the Watts angle as found above, and inclined to the mean lunar limb at the angle q, remembering to allow for any magnification of scale in the vertical. Figure 1b shows the resulting lines, a and b, corresponding to times **a** and **b**. The inclination of these lines to the lunar profile, given by the angle q, is about 5°. In Figure 1b, the vertical scale is magnified by about 17:1.

By repeating this process for each recorded event, it is possible to identify the lunar feature associated therewith. For example, in Figure 1b the bead was caused by the valley centred on Watts angle 100°.2. Although Figure 1b illustrates a relatively easy identification, it was found possible to identify lunar features for 75% of the timings, having regard for the following desiderata:

(i) Deep valleys with steep sides will most likely form a bead. Hence identification should be made with such a feature in preference to a small shallow feature.

(ii) The difference between the position of the line drawn on the profile (such as lines *a* and *b* in Figure 1b), and the edge of the profile feature being considered should not be large. Preferably it should be less than 1″.

(iii) There must be continuity in the identifications, i.e., each event must be associated with a different lunar feature, and close events should have a similar difference between the position of the line drawn on the profile and the respective lunar features being considered.

(iv) When the commentary indicates the existence of more than one bead, the identified features should give rise to the number of beads indicated by the commentary.

Table I in the third column gives the identification of the lunar feature, found using the above precepts, for each of the events. The feature is identified by its Watts angle, and the height of the feature at mean lunar distance. The identification corresponds to that part of the profile in the immediate vicinity of the identified feature whose tangent is parallel to the solar limb. In those cases where no identification was made, the lack of identification is due almost entirely to the lack of significant lunar features at the appropriate region of the profile, any valleys in that region being less than 0″.3 deep according to the profile derived from Watts charts.

7. Reduction of the Timings

Having identified the lunar features corresponding to each recorded event, the timings can be analysed to obtain the position of the actual Sun with respect to the ephemeris position of the Sun.

The significance of the timings is that at each recorded time, the solar limb was coincident with the lunar feature which has been identified for that particular time. If we treat the timings as a whole, the position of each identified feature at its corresponding recorded time maps out the location of the actual solar limb with respect to the assumed lunar position. By comparing the thus derived location of the solar limb with that obtained from the ephemeris position of the Sun, the correction to be applied to the ephemeris position to obtain the actual position of the Sun can be obtained, assuming the position of the Moon obtained from the ephemeris to be a true representation of the lunar position.

Figure 1a shows the configuration of the Sun and Moon during the eclipse, and illustrates the quantities needed in the reduction of the timings. For each event, the corresponding Lunar feature has been identified, and is listed in Table I. Hence, in the notation of Figure 1a, position angle p, and the height h is known. Further, the ephemeris position of the Moon, relative to the ephemeris position of the Sun is known, together with the solar and lunar semidiameters (R_s and R_m). These are given in Table III. Thus by interpolating in Table III, the quantities (x, y), R_m and R_s can be obtained at the time of each event.

If the ephemeris positions of the Sun and the Moon were a true representation of the positions of these bodies, then at the time of each recorded event the solar limb would be coincident with the identified lunar feature. However, this is not true, and hence in general the distance Δr in Figure 1a is non-zero. Hence the quantity Δr represents the combined displacements of the Sun and Moon, and this is the quantity required for the analysis of the timings.

Before we proceed with the evaluation of Δr for each of the events, two corrections must be allowed for. Firstly, the lunar profile which was drawn direct from Watts charts is drawn for a Moon at mean distance when the Moon's semidiameter is 932″.6. Thus the height of each identified feature taken directly from Watts charts must be multiplied by 1.073 to obtain the apparent height of the feature during the eclipse, 1.073 being the ratio of the apparent to mean semidiameters. Secondly, 0°.25 is subtracted from the Watts angle of each feature to remove a systematic error in the system of Watts angles, as found by Morrison (1970)

The method used for the determination of Δr was as follows:

For each event, x and y were obtained by interpolating in Table III.

From Table I, using the Watts angle of the feature corresponding to the event, the position angle (p) of the feature was obtained from $p = WA - 2°.41$, the angle $-2°.41$ being the sum of the correction to the system of Watts angles and the topocentric position angle of the lunar pole (C).

The height of each feature taken from Table I was multiplied by 1.073 and added to R_m (Table III) to give $R_m + h$. The quantities $F = x + (R_m + h) \sin p$ and $G = y + + (R_m + h) \cos p$ were formed. From this it follows that $\Delta r = (F^2 + G^2)^{1/2} - R_s$, R_s being given in Table III. The fourth column of Table I lists the value of Δr obtained for each of the events where an identification was made. The values of Δr are plotted as a function of Watts angle in Figure 2.

In Figure 2, the points corresponding to second and third contacts are marked with an asterisk. In the case of second contact, it was not possible to distinguish between two features, and, hence, both are marked. When referring to Figure 2 it is important to understand the physical significance of the quantity Δr. The quantity $R_s + \Delta r$ represents the distance of the observed solar limb from the Sun's centre. Since R_s is the known solar semidiameter, the quantity Δr is indicative of the displacement of the

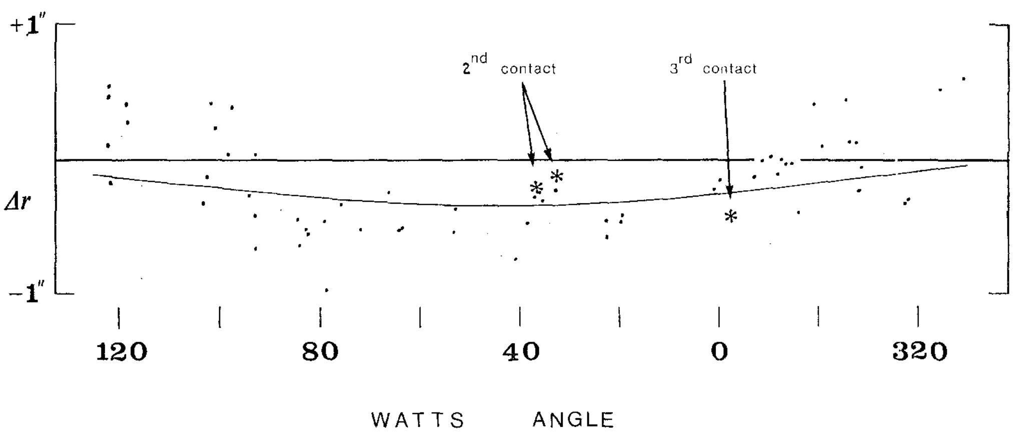

Fig. 2.

observed solar limb relative to the ephemeris location of the limb. Thus when Δr is positive, the observed limb is further from the ephemeris position of the Sun than is the ephemeris position. Conversely, a negative Δr means that the observed limb is closer to the Sun than is the ephemeris limb. By analysing the dependence of Δr on Watts angle, the actual position of the Sun, relative to the ephemeris position, and the semidiameter of the Sun, can be ascertained.

In analysing the values of Δr, use was made of the small value of the angle q (Figure 1a). In every case being considered, q is less than 5°. Hence, $\sec q$ is less than 1.004, and hence the analysis of Δr can be made with reference to the centre of the Moon, rather than the Sun, using the angle p which has already been determined as the angular coordinate. (With reference to the centre of the Moon, the limb displacement is $\Delta r \sec q$. With $\sec q$ very nearly 1, the displacement is effectively Δr. The maximum error occasioned is less than 0''.003.)

It is desired in this analysis to refer the shift in the position of the Sun to ecliptic latitude and longitude. Since the ecliptic is inclined at approx. 23° to the equator, it is generally inclined at some angle θ to the parallels of declination. Since ecliptic latitude and longitude are referred to the ecliptic, it is necessary to evaluate θ. For this eclipse, θ was computed to be +0°.63. Thus to refer position angles to the ecliptic system of coordinates, it is necessary to add 0°.63 to the position angles. Thus, in order to refer the Watts angles of the lunar features to the ecliptic system, it is necessary to add −1°.78 this being the sum of the lunar libration C, the correction to the system of Watts angles, and the rotation of the system to refer the angles to the ecliptic.

The analysis of Δr was made with respect to three quantities, it being assumed that these will be the main contributors to Δr. Specifically, the values of Δr were analysed by the method of least squares to evaluate the shift of the Sun in ecliptic latitude and longitude, and the radius of the Sun, referred to the ephemeris position of the Sun. Putting Δl, Δb, ΔR_s as the shift in the Sun's ecliptic longitude, ecliptic latitude, and semidiameter, and noting the implicit assumption with regard to the position of the Moon, we find that

$$\Delta l = -0''.19 \pm ''.13 \text{ (m.e.)}, \qquad \Delta b = -0''.21 \pm ''.10 \text{ (m.e.)},$$
$$\Delta R_s = -0''.05 \pm .17 \text{ (m.e.)}.$$

The line of best fit, as computed from these values for Δl, Δb, and ΔR_s is plotted in Figure 2.

8. Errors

Before discussing the results of the last section, it is important to investigate the sources of potential errors and how they will affect the quantity Δr. The three sources of potential error are:

(i) The prime source of error is in the accuracy of the adopted lunar profile, particularly in respect of the height of each of the identified features. Morrison (1970), in discussing the results obtained from grazing occultations, points out that for features of angular extent greater than 0°.5, the mean deviation of the actual profile from the

Watts profile is of the order 0″.2. Furthermore, if the feature is much narrower than 0°.5, the Watts profile is likely to be in greater error, and if a feature is less than 0°.3 in angular extent, it is likely to be entirely absent from the Watts profile. Thus it is to be expected that there will be a spread in the values of Δr of at least $\pm 0''.2$.

(ii) A second source of error arises from the method of recording the observations of the Baily beads. As was pointed out earlier, the timing error of any event when extracted from the tape recording is estimated to be ± 0.3 s. Figure 3 demonstrates the dependence in Watts angle of the resulting error in Δr.

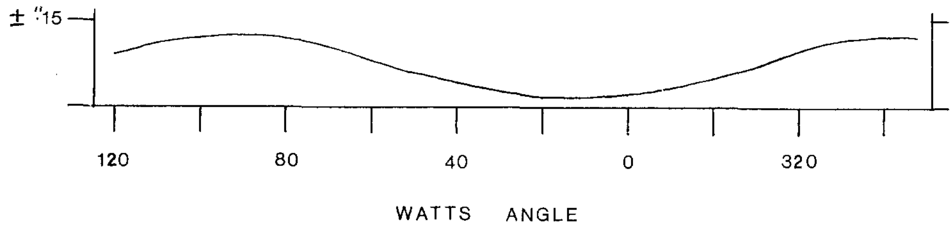

Fig. 3.

(iii) The third source of error arises out of the method of observation. The events recorded represent the times at which the intensity of light from the Sun at given points on the Lunar profile is essentially zero. However, in practice this condition is not achieved. In general, the resolution capabilities of the telescope used is insufficient to resolve the lunar features in much detail. Hence, as seen with the telescope, the formation and disappearance of the beads will be evidenced by the change of illuminaton at that point. Since the background illumination whilst observing the beads is substantial, the time at which the light level from the Sun appears to drop to zero will not correspond to the time at which the solar limb is coincident with the lunar limb. Rather the solar limb will be some small distance above the lunar limb, resulting in a value of Δr which will be too negative. Furthermore, since the background illumination can be expected to be greater at times further away from totality, it is to be expected that the magnitude of the error will be greater at times further away from totality.

When the event corresponds to the Sun being visible at the bottom of a valley, the above effect is the only one apparent. However, when the event corresponds to the top of a mountain being in coincidence with the solar limb, the situation becomes more complex, primarily through the effects of irradiation. Specifically, events caused by the tops of mountains are timed when a break is seen to form, or cease, in the solar crescent. The observation of this break is complicated by the presence of the solar limb on either side of the break. First, because of irradiation, some of the light from the exposed solar crescent will 'spill' into the region of the break, thus impairing the visibility of the break at the time of formation. Secondly, and more importantly, irradiation will still act on the exposed Sun on either side if the break, increasing the width of the solar crescent. The effect of this is to form a 'valley' on the edge of the solar limb opposite the mountain when the mountain is near coincident with the solar limb. Third, the width of the break formed by the mountain must be sufficient to be resolved by the telescope.

The effect of all these is complicated and beyond computation. It is apparent from the above discussion that the values of Δr for events formed by valleys will, most likely, be somewhat different to the values for events formed by adjacent mountains. Inspection of the results in Table I shows that Δr for mountain tops is often (but by no means always) more positive than for valleys. The mean difference is $+0\overset{''}{.}11$. Since this difference is less than the expected error in the profile, it was assumed for th analysis that the events at mountain tops and at valleys gave an equally accurate determination of the location of the solar limb.

As to the effects occurring at both mountain tops and valley bottoms, it is to be expected that the resulting error in Δr will be very nearly the same throughout the period of observation. This is equivalent to saying that zero light level is recorded when in fact the Sun is a small, very nearly constant, distance above the lunar feature. As such, it is to be expected that the analysis of the observations will give a result indicating that the Sun's semidiameter requires diminution to satisfy the observations.

9. Known Corrections

The results obtained in Section 7 give the corrections to be applied to the position of the Sun assuming the position of the Moon as given by the Lunar Ephemeris is a true representation of the lunar position. However this is not the case. Various investigations of the lunar motion have shown that substantial differences exist between the Ephemeris and actual lunar position.

Morrison and Sadler (1969) have investigated lunar occultations during the years 1960 to 1969, reducing the results using the Lunar Ephemeris $j=1$, taking into account lunar limb profile irregularities as derived from the charts by Watts (1963). To satisfy the observations, they derived a set of 5 periodic terms to be applied to the ephemeris position of the Moon to obtain the actual position. Evaluating these terms for the time of this eclipse, we find the correction to be applied to the position of the Moon to be

$$\Delta l = +0\overset{''}{.}05 \pm 0\overset{''}{.}01, \qquad \Delta b = +0\overset{''}{.}25 \pm 0\overset{''}{.}03.$$

A further correction for which allowance must be made is for a change in the assumed value of ΔT. The preceding analysis has used a value of $\Delta T = 43.70$ s in the Lunar Ephemeris, with the corresponding value of 45.04 in the Solar Ephemeris. Since ΔT is not evaluated to full precision until several years after any given time, allowance must be made for any resulting changes in the value of ΔT.

The allowance is most easily computed by considering the relative topocentric motions of the Sun and Moon, as given in Table III, when referred to the ecliptic. The computed allowances are

$$\Delta l = +0\overset{''}{.}37\, \delta\Delta T, \qquad \Delta b = -0\overset{''}{.}07\, \delta\Delta T,$$

where $\delta\Delta T$ is the change of Ephemeris Time from that assumed herein.

Applying these corrections, we obtain the following expressions for the corrections to be applied to the Ephemeris Solar position to obtain the actual position of the Sun:

$$l = -0\overset{''}{.}15 + 0\overset{''}{.}37\,\delta\Delta T \pm 0\overset{''}{.}14 \text{ m.e.}$$
$$b = +0\overset{''}{.}04 - 0\overset{''}{.}07\,\delta\Delta T \pm 0.13 \text{ m.e.}$$
$$R_s = -0\overset{''}{.}05 \qquad \pm 0.17 \text{ m.e.}$$

10. Discussion

In the preceding sections, the corrections to be applied to the ephemeris position of the Sun to obtain the observed position have been derived using the position of the Moon.

The quantity ΔR_s, the observed change required in the Solar radius was found to be $-0\overset{''}{.}05 \pm 0\overset{''}{.}17$. But as discussed previously, the method of observation will most likely result in a value of ΔR_s that is too negative by an indeterminable amount. The relatively large mean error in the value for ΔR_s would seem to bear this out. The derived value of ΔR_s being so close to zero tends to indicate that, if anything, the assumed value of R_s is too small. However this observation is not very suitable for investigating the solar radius because of the method used.

Referring to Figure 2, we plot the line of best fit derived from the least-squares analysis for Δl, Δb, ΔR_s (Section 7). This line is representative of the actual position of the solar limb, and equivalently the ephemeris location of the lunar limb, and the horizontal axis is representative of the ephemeris location of the solar limb. Each of the points indicate the ephemeris location of the lunar limb for each of the recorded events: thus when a point is located above the line of best fit, the event was recorded when the actual solar limb was below the assumed lunar profile, and vice-versa.

Inspection of Figure 2 shows that, without regard to the position of the line of best fit, the spread of the plotted points is generally less than $\pm 0\overset{''}{.}3$, the exception being at the ends of the plot where the spread is somewhat greater. In this regard, it is significant to note that the standard error for a 'well defined feature' is about $\pm 0\overset{''}{.}2$ (Morrison, 1970), and for other features is larger. Further, from Figure 3 the uncertainty resulting from the timing method is, on average, about $\pm 0\overset{''}{.}1$. Hence it would appear that much, if not most, of the scatter evident in Figure 2 is due to a combination of the uncertainties in the timing of the events, and in the height of the profile obtained from the charts by Watts. (It is in this regard that the results obtained from grazing occultations assume considerable importance, since a profile derived from a grazing occultation can have a standard error as low as $0\overset{''}{.}02$, an order of magnitude better than Watts charts. The use of a profile derived from grazing occultations will thus remove the largest cause of scatter in the results. However, since grazing occultations can only be observed against the lunar profile in the vicinity of the lunar poles, advantage can only be taken if the eclipse is viewed from near the path limits.)

The line plotted on Figure 2 represents the position of the actual position of the solar limb on the assumption that the values of Δr are caused solely by a change in the values of the relative longitude, latitude, and semidiameter of the Sun and the Moon. However, this line does not fit the points as well as might be expected. In particular, there appears to be a considerable excess of points above the line at either end of the plot, with the greatest deviation being for Watts angles greater than 95°. It is apparent from the discussion of the possible errors that this deviation can not be explained in terms of the method of observation, nor by errors in the recording of the times of the events. Since many of the features in this region are 'well defined' in the sense of Morrison (1970), it would not appear that a satisfactory explanation lies in the accuracy of the detail in the Watts profile. The only satisfactory explanation would seem to lie in the presence of a discontinuity in the vertical datum, at about Watts Angle 95°, of the lunar profile.

Before discussing this point further, it must be noted that the points on Figure 2 for Watts angles greater than 95° lie, almost without exception, above the position of the plotted line. This means that at the recorded time of the events concerned, the actual position of the solar limb (as herein derived) lay behind the lunar limb, and thus the solar limb should have neen invisible for a considerable time prior to the recorded time of the event. However, the method of observation, with particular reference to events caused by the bottoms of valleys on the lunar profile, makes it certain that the solar limb was visible up until at least the recorded time of the event. It is thus apparent that this positive deviation of Δr must be a real effect, caused by errors in the lunar profile. Of course, one obvious potential source of error lies in incorrect identification of the features of the lunar profile. However, using the precepts previously set out, for the identification of the features, it is believed that no errors of this nature exist. For example, Figure 1b illustrates the identification of a bead, the identification being the valley centred on *WA* 100°.2. In Table II, the events occur at $5^h11^m55^s.9$, and $59^s.4$, respectively. The derived values of Δr are $+0''.38$ and $+0''.03$ respectively. Both of these points lie well above the plotted line in Figure 2. If this positive excess is ascribed to a misidentification, then a feature giving rise to a value of Δr of approximation $-0''.2$ (cf., Figure 2) should be identifiable. i.e., there should be a feature approximation $0''.3$ below the lines **a** and **b** in Figure 1b, which can form a suitable bead. No such feature is present in Figure 1b, and it must therefore be concluded that this identification is correct. Similar considerations apply to all of the events.

The presence of a discontinuity in the Watts datum is not without precedent. It is a quite common result in grazing occultations to find large systematic vertical errors in the Watts profile. However, grazing occultations are limited to the profile near the lunar poles, and cannot afford any independent confirmation of a discontinuity in the vertical datum at Watts angles such as 95°. Inspection of the plot of Figure 2 indicates that the profile for Watts angles greater than 95° is systematically heigher than the profile for Watts angles less than 95° by approx. $0''.4$.

Some comment is appropriate for those events in Table I for which no identification was made. In nearly every instance, non-identification was due to the lunar profile, as taken from Watts charts, being too smooth to identify a suitable feature with any degree of confidence. In these instances, the duration of the observed bead was very short, indicating that the feature was in fact fairly small. Additionally, the lack of a suitable feature on the Watts profile indicates in itself the uncertainties and limitations inherent in the use of the charts.

One bead event, however, does not fall into the category of a small feature. The first bead observed in the eclipse, some 9½ min prior to totality, could not be identified on the Watts profile. At the time of the event, the Watts angle of the solar limb extremity was about 159°.2, and the Solar limb was inclined at an angle of 13°.4 to the lunar limb. This bead was clearly seen by both observers present for some 10 s. When attempting an identification, the only possible feature is at Watts angle 161°.0. This is 1°.8 in Watts angle from the expected position of the feature and is clearly not the correct feature. At the expected position of the feature on the lunar profile is a large mountain plateau some 2″ above the mean lunar limb, extending 2°.5 in Watts angle. The only tenable conclusion is the presence of a deep and narrow valley in the middle of this plateau, at Watts angle 159°.2.

11. Summary

During the total solar eclipse of June 20, 1974, Baily beads were observed from near the northern limit of totality. From the times at which the associated events occurred, the lunar profile features which gave rise to the beads were identified by use of the charts prepared by Watts. By use of these identifications, the times were analysed to derive corrections to the ecliptic longitude, latitude, and semidiameter of the Sun, on the assumption that the ephemeris position of the Moon is representative of the actual lunar position. The corrections obtained are:

$$\Delta l = -0''.19 \pm ''.13 \text{ m.e.},$$
$$\Delta b = -0''.21 \pm ''.10 \text{ m.e.},$$
$$\Delta R_s = -0''.05 \pm ''.17 \text{ m.e.}.$$

On making allowance for these corrections, the results were further investigated. It was found that a discontinuity in the vertical datum exists in the Watts charts at about 95° Watts angle. The profile derived from the Watts charts is systematically higher by approximately 0″.4 for Watts angles greater than 95°, compared with the profile for angles less than 95°. Also, one very prominent bead could not be satisfactorily identified and it is concluded that a deep valley exists at the appropriate part of the profile.

It is apparent from the results obtained, and their errors, that the observation of Baily beads from near the limits of a total eclipse over the whole period in which they are observable, making no attempt at a definite identification at the time of observation, can result in a determination of the relative positions of the Sun and the Moon to an acceptable order of accuracy, and it is to be expected that the accuracy will be improved markedly upon the availability of profiles derived from grazing occultations. This type of observation is also useful for investigating the consistency of the Watts charts over large angular distances, and in particular, in regions not capable of being investigated by way of grazing occultations.

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