THE RESIDUAL PERMANENT MAGNETIC DIPOLE MOMENT OF THE MOON

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Abstract. The residual dipole moment of the outer spherical shell of the Moon, magnetized in the field of an internal dipole is calculated for the case when the permeability of the shell differs from unity. It is shown that, using an average value of surface magnetization from returned lunar crystalline rock samples and a global figure for the lunar permeability of 1.012, that a residual moment of the order of 10^{15} to 10^{16} Am^2 is expected. This value is some two or three orders of magnitude lower than the moment for a shell magnetized in an external uniform field and is of the same order as the upper limit of the residual moment detected by Russell *et al.* (1974). At present the magnetic data and the thermal state of the Moon are not known with sufficient accuracy to distinguish between a crust magnetized in an internal dipole field of constant polarity and a crust magnetized in the dipole field of a self-reversing core dynamo. Refined measurements of the relevant parameters together with the theory presented in this paper could enable these two possibilities to be distinguished.

1. Introduction

Studies of the natural remanence of returned lunar samples have shown strong evidence for an ancient lunar field which was present when the rocks were formed some some 3.2 to 4.0 *AE* ago. Although such studies can indicate the strength of the field (from 5 to 130 μT, i.e. 0.05 to 1.3 G, Stephenson *et al.*, 1975), they cannot, in the absence of orientated samples from different locations, determine whether the ancient field was of internal or external origin. Information about the present lunar dipole field has been obtained by Russell *et al.* (1974) from magnetic field observations taken by the Apollo 15 subsatellite, and their analysis has shown that the present lunar dipole moment is less than 1.3×10^{15} Am^2 (1.3×10^{18} G cm^3). They conclude that since an ancient lunar dipole (whether produced by an internal core dynamo or by a primeval permanent magnetization) would have to have been stronger than 1.5×10^{20} Am^2, the permanently magnetized rocks (which are responsible for the surface magnetic fields) show no trace of any ancient lunar dipole field.

Runcorn (1975), however, has pointed out that if the outer shell of the Moon became magnetized in the field of an internal dipole, it would not give rise to an external dipole field – provided that the original dipole were removed at a later date. Moreover, in view of the remanence carried by lunar rocks, the negligible present dipole moment is strong evidence for an internal origin of the ancient field since, if the outer shell were magnetized in a uniform external field (e.g., parallel to the rotation axis), a present dipole moment greater than the upper limit set by Russell *et al.* (1974) would be expected.

The two major theories for an internal origin of the ancient lunar field are (a) lunar

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dynamo, (b) primeval permanent magnetization. Primeval permanent magnetization has been suggested by Runcorn and Urey (1973), the Moon accreting 'cold' in the presence of an external field and, hence, acquiring a permanent magnetization which would persist after the external field had decayed. Surface rocks cooling in the dipole field produced by the uniformly magnetized interior would then acquire a thermoremanent magnetization, building up a magnetized crust which would remain after removal of the primeval magnetization of the interior by radioactive heating (see Figure 2). The magnetized shell now remaining would on this hypothesis be indistinguishable from that which would result if the internal mechanism producing an axial dipole field were a nonreversing lunar dynamo which is now inactive.

Palaeointensity measurements on returned lunar samples have, however, indicated equatorial surface lunar fields up to 130 μT (1.3 G) in strength (Stephenson *et al.*, 1975) and to produce such a surface field, a uniformly permanently magnetized sphere would have to be magnetized to an intensity of 310 Am^{-1} (0.31 G). Lunar basalts which commonly contain about 0.1% free iron (Huffman *et al.*, 1975) have a saturation magnetization of about 3 Am^{-1} (3×10^{-3} G) (Pearce *et al.*, 1973). If the iron carrying the primeval magnetization has similar characteristics to that found in lunar basalts then the iron content of the lunar interior (below the magnetized outer shell) would have to be about 10% by weight if saturated, and since fields of the order of hundreds of Gauss are required to produced saturation, either the initial field in which the Moon accreted must have been of this order or the iron concentration below the present day outer magnetized shell must exceed 10%. Both these possibilities seem unlikely so that permanent magnetization seems to be a less attractive alternative than the simpler idea of a lunar dynamo (Runcorn *et al.*, 1970).

It is the purpose of this paper to show that the measurement of the present residual dipole moment of the Moon provides the possibility of distinguishing between an outer shell magnetized in the field of an internal dipole of constant direction (nonreversing dynamo or primeval magnetization) and a reversing dipole field due to a self-reversing dynamo. The argument is based on the fact that Runcorn's result that the external field produced by a shell magnetized in an internal dipole field of constant polarity is zero, neglects permeability effects; this paper shows that if the permeabilities between the inner and outer regions or the outer regions and free space are different, a residual external dipole moment should be observed. From values of the natural remanence of lunar samples and from the lunar global permeability determined by Parkin *et al.* (1974) it is shown that this residual moment is, depending on the present thermal state of the Moon and the thickness of the magnetized shell, of the same order as the upper limit set by Russell *et al.* (1974).

2. The Dipole Source and the Magnetization of the Shell

The simplified model for representing a body which contains an internal dipole is shown in Figure 1. The body is of radius b and has an outer shell of permeability μ_2. Inside the shell which has an inner radius a_s is a region of permeability μ_1, and a

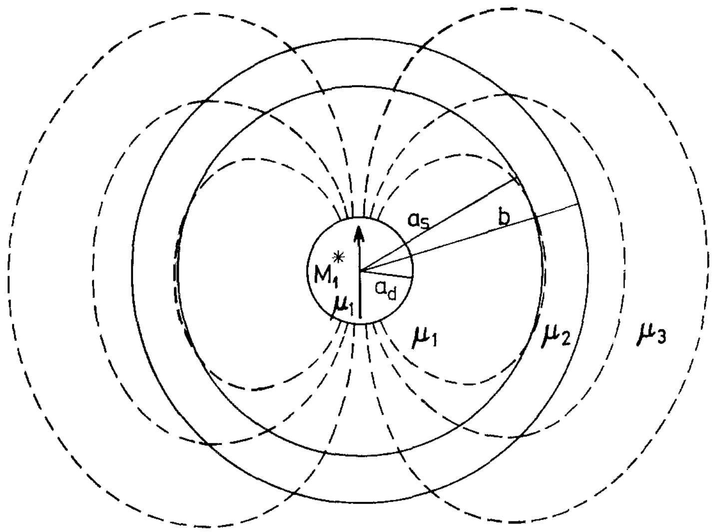

Fig. 1. Initial model just before TRM is acquired. Outer shell is of permeability μ_2, internal dipole is represented by a uniformly magnetized sphere of permeability μ_1, radius a_d and magnetization M^*_1 per unit volume (M^*_1 in absence of demagnetizing field).

central sphere of radius a_d within this region carries a permanent magnetization $M_1^* - H_d[(\mu_1/\mu_0)-1]$ where M_1^* is the magnetization per unit volume in the absence of the uniform internal demagnetizing field H_d.

The solution of the equations for determining the fields in the system may be evaluated from the theory for determining the externally observed moment of a magnet embedded in a sphere of permeable material. This solution was utilised in calculating the observed moment of a magnetized inclusion of high Curie point within a titanomagnetite particle of lower Curie point (Stephenson, 1975) and the same theory applies to the current problem, there being a factor $\sim 10^{30}$ in the volumes of the bodies concerned.

The potentials within the four regions initially are:

$$\text{for} \quad 0 \leqslant r \leqslant a_d\colon \ \phi = - H_{di} r \cos\theta, \tag{1}$$

$$\text{for} \quad a_d \leqslant r \leqslant a_s\colon \phi = - H_{1i} r \cos\theta + a_d^3 H'_{1i} \frac{\cos\theta}{r^2}, \tag{2}$$

$$\text{for} \quad a_s \leqslant r \leqslant b\colon \ \phi = - H_{2i} r \cos\theta + a_s^3 H'_{2i} \frac{\cos\theta}{r^2}, \tag{3}$$

$$\text{for} \quad b \leqslant r\colon \qquad \phi = b^3 H'_{3i} \frac{\cos\theta}{r^2}. \tag{4}$$

In these equations the terms proportional to r arise from uniform fields and those proportional to r^{-2} come from dipole fields. By noting that ϕ and B are continuous at the boundaries, the following six simultaneous equations are derived:

$$\text{at} \quad r=a_d: \quad -H_{di} = -H_{1i} + H'_{1i}, \tag{5}$$

$$\mu_0 M_1^* + \mu H_{di} = \mu_1 H_{1i} + 2\mu_1 H'_{1i}, \tag{6}$$

$$\text{at} \quad r=a_s: \quad -H_{1i} + f_s H'_{1i} = -H_{2i} + H'_{2i}, \tag{7}$$

$$\mu_1 H_{1i} + 2\mu_1 f_s H'_{1i} = \mu_2 H_{2i} + 2\mu_2 H'_{2i}, \tag{8}$$

$$\text{at} \quad r=b: \quad -H_{2i} + \sigma_s H'_{2i} = H'_{3i}, \tag{9}$$

$$\mu_2 H_{2i} + 2\sigma_s \mu_2 H'_{2i} = 2\mu_3 H'_{3i}, \tag{10}$$

where $\sigma_s = (a_s/b)^3$ and $f_s = (a_d/a_s)^3$. The solutions of these equations are

$$H_{di} = -\mu_0 M_1^* \{[\mu_1(1+2f_s) + 2\mu_2(1-f_s)](\mu_2+2\mu_3) + 2\sigma[\mu_1(1+2f_s) - \mu_2(1-f_s)](\mu_2-\mu_3)\}/3\mu_1 D'_i, \tag{11}$$

$$H_{1i} = -2\mu_0 f_s M_1^* [(\mu_1-\mu_2)(\mu_2+2\mu_3) + \sigma_s(2\mu_1+\mu_2)(\mu_2-\mu_3)]/3\mu_1 D'_i, \tag{12}$$

$$H'_{1i} = \mu_0 M_1^*/3\mu_1, \tag{13}$$

$$H_{2i} = -2\mu_0 f_s \sigma_s M_1^* (\mu_2-\mu_3)/D'_i, \tag{14}$$

$$H'_{2i} = \mu_0 f_s M_1^* (\mu_2+2\mu_3)/D'_i, \tag{15}$$

$$H'_{3i} = 3\mu_0 f_s \sigma_s \mu_2 M_1^*/D'_i, \tag{16}$$

where

$$D'_i = (\mu_1+2\mu_2)(\mu_2+2\mu_3) + 2\sigma_s(\mu_1-\mu_2)(\mu_2-\mu_3). \tag{17}$$

Note that the ratio $H_{2i}/H'_{2i} = -2\sigma_s(\mu_2-\mu_3)/(\mu_2+2\mu_3)$ and is independent of μ_1 and the radius a_d of the central permanently magnetized region.

The induced magnetization within the shell is $\chi_2 H_{2ti}$ where the susceptibility $\chi_2 = (\mu_2/\mu_0) - 1$ and the total field in the shell is

$$H_{2ti} = -\frac{\partial\phi}{\partial r} = H_{2i}\cos\theta + \frac{2a_s^3}{r^3} H'_{2i}\cos\theta. \tag{18}$$

If this magnetization suddenly becomes frozen in, which would happen during the acquisition of thermo-remanent magnetization (TRM), then the resultant permanent TRM M_2^* may be written as

$$M_2^* = k\left(H_{2i} + \frac{2a_s^3}{r^3} H'_{2i}\right)\cos\theta, \tag{19}$$

where k is a constant representing the efficiency of the TRM acquisition process. The surface magnetization M_s^* at the magnetic pole ($\cos\theta = 1$, $r = b$) is thus from (14), (15), and (19),

$$M_s^* = 6k\mu_0 f_s \sigma_s \mu_3 M_1^*/D'_i. \tag{20}$$

Putting $m_1 = (\frac{4}{3})\pi a_d^3 M_1^*$ thus gives

$$M_s^* = 9k\mu_0\mu_3 m_1/2\pi b^3 D'_i, \tag{21}$$

M_s^* is thus independent of the radius a_d of the internally permanently magnetized source region and is proportional to the dipole moment m_1 as defined above.

3. The Residual Moment of the Shell

If the original permanent magnetization of the source region is removed, leaving only the magnetized shell as in Figure 2, a similar approach may be used to calculate the residual moment of the system. The outer shell now carries permanent magnetization, is of permeability μ_2 and surrounds an internal sphere of permeability μ_1.

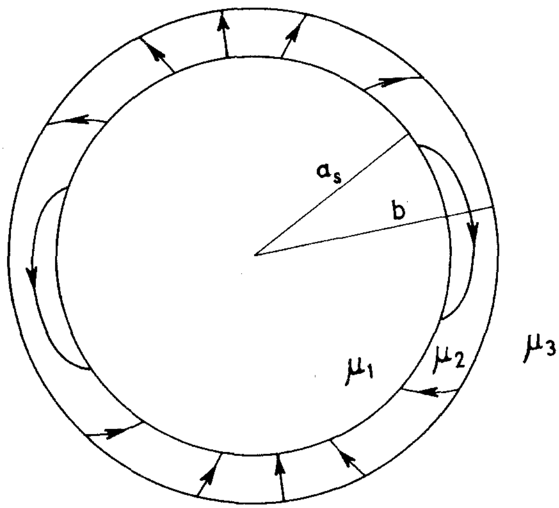

Fig. 2. Magnetized shell of permeability μ_2 surrounding an interior which is of permeability μ_1 and has lost its permanent dipole moment.

The potentials within the system may thus be written, as before,

$$\text{For} \quad 0 \leqslant r \leqslant a_s\colon \ \phi = -H_1 r \cos\theta, \tag{22}$$

$$\text{For} \quad a_s \leqslant r \leqslant b\colon \ \phi = -H_2 r\cos\theta + a_s^3 H_2' \frac{\cos\theta}{r^2}, \tag{23}$$

$$\text{For} \quad b \leqslant r\colon \qquad \phi = b^3 H_3' \frac{\cos\theta}{r^2}. \tag{24}$$

The four simultaneous equations derived by making B and ϕ continuous at $r=a$ and $r=b$ then become

$$H_1 = H_2 - H_2', \tag{25}$$

$$\mu_1 H_1 = \mu_2 H_2 + 2\mu_2 H_2' + \mu_0 k(H_{2i} + 2H_{2i}'), \tag{26}$$

$$H_2 - \sigma_s H_2' = -H_3', \tag{27}$$

$$\mu_0 k(H_{2i} + 2\sigma_s H_{2i}') + \mu_2 H_2 + 2\sigma_s \mu_2 H_2' = 2\mu_3 H_3', \tag{28}$$

The solutions of these equations in terms of the relative permeabilities $\mu_{1r}=\mu_1/\mu_0$ etc. are

$$H_1 = \frac{(1-\sigma_s)\,M_s^*}{3\sigma_s\mu_{3r}D_s}\left\{(\mu_{2r}+2\mu_{3r})^2+2\sigma_s(\mu_{2r}-\mu_{3r})^2\right\}, \tag{29}$$

$$H_2 = \frac{M_s^*}{3\mu_{3r}D_s}\left\{2\mu_{2r}^2-\mu_{3r}(3\mu_{1r}+4\mu_{2r}+4\mu_{3r})\right\}, \tag{30}$$

$$H_2' = -\frac{M_s^*}{3\sigma_s\mu_{3r}D_s}\left\{(\mu_{2r}+2\mu_{3r})^2+\sigma_s(-\mu_{2r}^2+\mu_{3r}[3\mu_{1r}-4\mu_{2r}+2\mu_{3r}])\right\}, \tag{31}$$

$$H_3' = \frac{(1-\sigma_s)\,M_s^*}{\mu_{3r}D_s}\left\{\mu_{1r}\mu_{3r}-\mu_{2r}^2\right\}, \tag{32}$$

where

$$D_s = (\mu_{1r}+2\mu_{2r})(\mu_{2r}+2\mu_{3r})+2\sigma_s(\mu_{1r}-\mu_{2r})(\mu_{2r}-\mu_{3r}). \tag{33}$$

The externally observed dipole moment m is related to the term H_3' by the equation $m=4\pi b^3 H_3'$, since

$$H(r) = -\frac{\partial\phi}{\partial r} = \frac{2b^3}{r^3}H_3'\cos\theta = \frac{2m\cos\theta}{4\pi r^3}; \tag{34}$$

and at $\cos\theta=1$, and $r=b$, $H=2H_3'$, i.e. the surface field at the magnetic pole is $2H_3'$. Thus

$$m = \frac{4\pi b^3 M_s^*(1-\sigma_s)}{\mu_{3r}D_s}\left\{\mu_{1r}\mu_{3r}-\mu_{2r}^2\right\}. \tag{35}$$

It can be seen from this equation that if $\mu_{2r}=\sqrt{\mu_{1r}\mu_{3r}}$ the residual moment is zero.

It is interesting to note that this condition, with $\mu_{3r}=1$, is precisely that required for the observed moment of a magnetized spherical inclusion of permeability μ_1, embedded in a sphere of permeability μ_2, to be a maximum (Stephenson, 1975). The condition $\mu_{2r}=\sqrt{\mu_{1r}\mu_{3r}}$ is also fulfilled if $\mu_{1r}=\mu_{2r}=\mu_{3r}=1$ and this corresponds to the result derived by Runcorn (1975). The implicit assumption of Runcorn that all relative permeabilities are unity can never be strictly correct however, because at the instant when TRM is acquired, the relative permeability of the shell must necessarily exceed unity. Even if the magnetization were then to become 'hard' when the shell cooled, i.e. relative permeability $\mu_{2r}=1$, it is easy to show from Equations (14), (15) and (25)–(28) that a small non-zero moment results.

4. The Residual Lunar Moment

The internal source producing the dipole moment in the above model is represented by a uniformly magnetized central sphere (Figure 1). The above model is thus a direct representation of the primeval magnetization origin of the ancient lunar field if $a_d=a_s$. The model can also however be applied to the case of a lunar dynamo if it is assumed that this produces a dipole moment m_1 which is related to the permanent magnetization M_1^* via the relation $m_1=(\frac{4}{3})\pi a_d^3 M_1^*$, i.e. the dynamo or any other internal source can be represented by a uniformly magnetized sphere of any radius a_d provided of course that $a_d \leqslant a_s$ where a_s is the inner radius of the outer shell.

To explain the magnetization process of the outer shell it is assumed that it consists of a matrix which is paramagnetic at all temperatures under consideration (e.g. olivine, pyroxene) but which contains a small quantity of iron particles which are uniformly distributed. This shell is originally above the iron Curie point (770 °C) and is situated in the magnetic field produced by the internal central magnetic dipole. It then cools and acquires a TRM in the ambient field.

It is further assumed that the iron particles within the outer shell are roughly equidimensional and are greater in size than a few hundred ångstroms and are thus multidomain. This is a reasonable assumption since lunar basalts contain such grains and any smaller single domain particles would not be able to survive for long enough at the temperatures at which the basalts formed (Pearce *et al.*, 1972). Since multidomain iron is of high intrinsic susceptibility χ_i and since such particles have an apparent susceptibility per unit volume of $\chi_a=\chi_i/(1+N\chi_i)$ where N is the demagnetizing factor ($\frac{1}{3}$ for a sphere) then the apparent susceptibility is $1/N$ and is independent of any temperature variation of χ_i provided that the latter remains high. Since the susceptibility of the paramagnetic constituents of lunar basalts below the iron Curie point (770 °C) is invariably smaller than the contribution from the iron particles of volume fraction F, then the permeability of the outer shell $\mu_{2r}=1+\chi_2 \doteqdot 1+F/N$ and can thus be regarded as being constant below 770 °C.

The next simplifying assumption is that the blocking temperature T_B for all iron particles is the same, so that when the shell cools through T_B (where $T_B \leqslant T_C$), the induced magnetization within the shell of permeability μ_2 is frozen in; any further cooling below this temperature then causes a change in intensity of magnetization but not the direction. For a distribution of blocking temperatures, the direction of the local field at the lower temperatures would be modified by the macroscopic demagnetizing field produced by the permanent TRM acquired at the higher temperatures, but this effect in the case of the Moon is small enough to be neglected.

After removal of the original central dipole by, for instance, radioactive heating in the case of primeval magnetization, or by cessation of dynamo activity due to the magnetic Reynolds number falling below its critical value, the only permanent magnetization in the body would then be the TRM in the outer shell as in Figure 2. The boundary between the regions of permeability μ_1 and μ_2 in Figure 2 is therefore taken to be the Curie point isotherm (770 °C), so that μ_2 refers to the permeability (i.e., dB/dH at $H=0$) of the weakly ferromagnetic outer shell which carries the TRM, while μ_1 is the permeability of the paramagnetic interior. Because of these permeability differences the residual moment (Equation (35)) will not be zero. It is also possible that the Curie point isotherm may lie at a radius a_t, below the lower boundary of the outer magnetized shell (at a_s), and this refined model is illustrated in Figure 3. (This situation would apply for instance if the blocking temperature T_B were below the

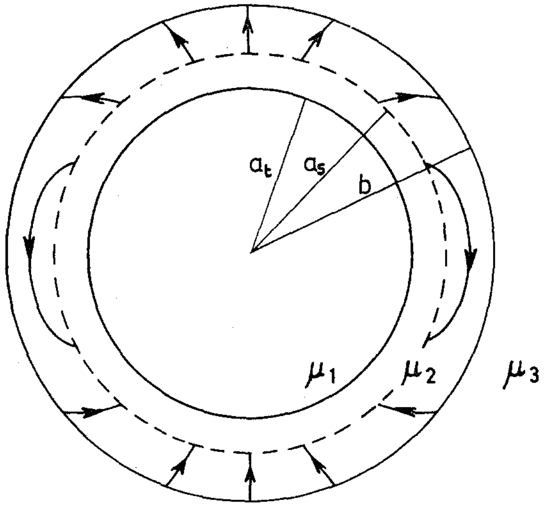

Fig. 3. Refined model where base of magnetized outer crust (dashed line) and Curie point isotherm (solid line) do not necessarily coincide. Curie point isotherm is the boundary between regions of permeability μ_1 and μ_2.

Curie point or if the Curie point isotherm had retreated from the surface after the outer shell had become magnetized). Solution of the six simultaneous equations (similar to Equations (25)–(28)) which can be constructed for this situation leads to the following equation for the residual moment due to the outer shell:

$$m = \frac{4\pi b^3 M_s^* (1-\sigma_s)}{3\mu_{3r} D_t} \left\{ \left(\frac{\sigma_t}{\sigma_s}\right) (\mu_{1r} - \mu_{2r})(\mu_{2r} + 2\mu_{3r}) - (\mu_{1r} + 2\mu_{2r})(\mu_{2r} - \mu_{3r}) \right\}, \tag{36}$$

where

$$D_t = (\mu_{1r} + 2\mu_{2r})(\mu_{2r} + 2\mu_{3r}) + 2\sigma_t(\mu_{1r} - \mu_{2r})(\mu_{2r} - \mu_{3r}) \tag{37}$$

and

$$\sigma_t = (a_t/b)^3 .$$

Note that when $\sigma_t = \sigma_s$, this equation becomes identical to Equation (35).

5. The Residual Moment and Lunar Permeability

The intrinsic global lunar permeability μ_r has been obtained by Parkin *et al.* (1974) from a whole-Moon hysteresis curve constructed from Apollo 12 surface magnetometer and Explorer 35 magnetometer data. The observed whole Moon susceptibility χ_0 is given by the equation

$$\chi_0 = 3\, \frac{(\mu_{1r} + 2\mu_{2r})(\mu_{2r} - 1) + \sigma_t(\mu_{1r} - \mu_{2r})(2\mu_{2r} + 1)}{(\mu_{1r} + 2\mu_{2r})(\mu_{2r} + 2) + 2\sigma_t(\mu_{1r} - \mu_{2r})(\mu_{2r} - 1)} \tag{38}$$

in the present notation (see Stephenson, 1975; Equation (10)), and the average whole Moon permeability μ_r is obtained by setting $\mu_r = \mu_{1r} = \mu_{2r}$ to give

$$\chi_0 = 3\,\frac{\mu_r - 1}{\mu_r + 2}.$$

Parkin *et al.* (1974) have determined μ_r to be 1.012 ± 0.006 (95% confidence limits) and with the error expressed as the standard deviation, $\mu_r = 1.012 \pm 0.003$. These results have recently been revised by Dyal *et al.* (1975) using data from Apollo 12, 15, 16 and Explorer 35 magnetometers. They obtain a solid Moon permeability of 1.012 after taking in to account the effect of a lunar ionosphere. Since μ_r, μ_{1r} and μ_{2r} are close to unity it is possible to simplify Equations (36) and (38) which then become, with $\mu_{3r} = 1$,

$$m \simeq \tfrac{4}{9}\pi b^3 M_s^* (1 - \sigma_s)\left\{\left(\frac{\sigma_t}{\sigma_s}\right)(\mu_{1r} - \mu_{2r}) - (\mu_{2r} - 1)\right\} \tag{40}$$

and

$$\chi_0 \simeq \mu_r - 1 \simeq \sigma_t \chi_1 + (1 - \sigma_t)\,\chi_2, \tag{41}$$

where $\chi_1 = \mu_{1r} - 1$ etc.

Putting the ratio $x = \chi_1/\chi_2$ we get for m the expression

$$m \simeq -\tfrac{8}{9}\pi b^3 M_s^* \chi_0 K, \tag{42}$$

where

$$K = \left(\frac{1 - \sigma_s}{2}\right)\left[\frac{1 + \dfrac{\sigma_t}{\sigma_s}\left(\dfrac{1 - x}{1 - \sigma_t x}\right)}{1 - \sigma_t\left(\dfrac{1 - x}{1 - \sigma_t x}\right)}\right] \tag{43}$$

lying between 0 and 1. The residual moment thus depends on the thickness of the magnetized region and on the depth of the Curie point isotherm. It is also seen from Equation (42) that the direction is negative, i.e. the sense of the residual moment is opposed to that of the original moment which was responsible for magnetizing the outer shell. It can also be shown that a lunar ionosphere 100 km thick, of permeability 0.8 (Dyal *et al.*, 1975) does not significantly affect the expected residual moment.

6. Evaluation of M_s^*

M_s^* is the polar surface magnetization of the rock in the absence of macroscopic demagnetizing effects and since the iron particles carrying the remanence form a dilute assembly within the rock ($\sim$0.1%), the overall susceptibility of the rock is low so that the demagnetizing field of a returned sample $H_d = NM$ (where N is the demagnetizing factor) produces an induced moment $NM\chi_2$ in opposition to M_s^* which is small and can be neglected. Thus $M_s^* \simeq M_s$, where M_s is the magnetization of a polar rock sample. The magnitude of M_s may be estimated from two different sources. The first is from the natural remanence of the returned lunar samples, and the second is from an estimate of the strengths of lunar surface fields.

To obtain the best estimate of the average crustal magnetization, an estimate has been made of the natural remanence of lunar basalts from the results of several investigators. Results from breccias and other metamorphic rocks have been rejected as being unrepresentative of the outer shell material. To estimate the NRM is not straightforward because the samples are sometimes unstable and pick up secondary

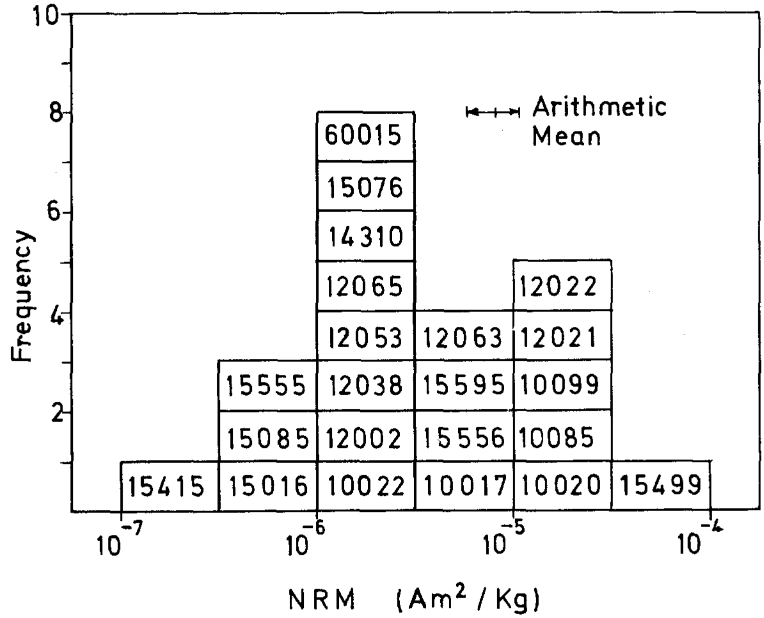

Fig. 4. Histogram of NRM of basalts (see text). (1 Am^2 kg^{-1} = 1 G cm^3 g^{-1}).

components in the Earth's field. From *AF* demagnetization data, the NRM is taken to be the initial measured value if the direction of magnetization remains constant up to peak fields of a few hundredths of a Tesla (few 100 G). In cases where directional data is not given, provided that the demagnetization curve is reasonably smooth, the NRM is taken to be the value of magnetization for a peak demagnetizing field of 3–5 μT (30–50 G) which is typically the field at which secondary magnetizations are removed (Stephenson *et al.*, 1975). Samples which are magnetically unstable or demagnetize in a non-uniform manner have been rejected. Using this analysis, 22 samples have been taken from the literature and a histogram of the estimated natural remanences is shown in Figure 4. The intensities appear to be roughly distributed log-normally and the arithmetic mean of the distribution gives the best estimate of the average surface crustal magnetization. This is 8.3×10^{-6} Am^2 kg^{-1} (where 1 Am^2 kg^{-1} = 1 G cm^3 g^{-1}) with a standard error of $\pm2.4\times10^{-6}$ Am^2 kg^{-1}. Taking an average density of 3.3 gm cm^{-3}, the magnetization/unit volume is thus $(27\pm8)\times$ $\times10^{-3}$ Am^{-1} $(27\times10^{-6}$ G). Since, however, all the Apollo samples come from equatorial regions, the surface polar magnetization will be about twice this value (assuming that the magnetic and geographic poles coincide). This is calculated more exactly by multiplying each natural remanence value by the factor $2/(1+3\sin^2\theta)^{1/2}$ where θ is the latitude of the collection point and then averaging. This factor varies from 2.00 for Apollo 11 samples to 1.61 for Apollo 15 samples. The arithmetic mean of these corrected polar values is then the best estimate of the parameter M_s and is $(51\pm14)\times10^{-3}$ Am^{-1} $(51\times10^{-6}$ G).

The other way of estimating the intensity of magnetization is from the strengths of lunar surface fields. The field produced at the surface of a magnetized body of magnetization M per unit volume may lie anywhere in the region 0 to M depending on the shape of the body. Surface field measurements from the Apollo 12, 14, 15, and 16 sites have average values of 38, 70, 3, and 200 nT (1nT = 1γ) respectively (Dyal *et al.*, 1973) and thus giving each site equal weight gives a mean surface anomaly of 78 ± 43 nT. Thus the minimum possible value of M is $(62\pm34)\times10^{-3}$ Am^{-1} $(62\times10^{-6}$ G) which is of the same order as the average value for the natural remanence of $(27\pm8)\times10^{-3}$ Am^{-1} $(27\times10^{-6}$ G) determined from the igneous rocks. A higher value from the anomalies might be expected because of the presence of breccias which, in general, are more strongly magnetized than the crystalline rocks used to estimate the surface magnetization of the outer shell.

7. Evaluation of the Parameter x

The evaluation of the parameter x in Equation (43) requires a knowledge of the paramagnetic susceptibility of the region below the Curie point isotherm. This may be evaluated from the paramagnetic susceptibility of lunar basalts which from measurements of Pearce *et al.* (1973) give a mean value for 8 samples of $(4.3\pm0.4)\times10^{-7}$ m^3 kg^{-1} $(34\pm3\times10^{-6}$ G cm^3 g^{-1} Oe^{-1}) measured at room temperature. Since the above value is equivalent to a volume susceptibility of 1.41×10^{-3} $(1.12\times10^{-4}$ G Oe^{-1}) then the ratio $x=\chi_1/\chi_0$ is 0.118 at room temperature. Above the Curie point, i.e. at temperatures $>770\,^\circ$C the paramagnetic susceptibility will be a factor of more than 3.5 smaller than 1.41×10^{-3}, i.e. $<0.4\times10^{-3}$. It would therefore seem reasonable to take x as 1/30. The presence of a few percent of paramagnetic free iron would, however, change this value since 1% by volume of free iron, 100° above its Curie point has a susceptibility of about 0.2×10^{-3}. Thus a value of x between about $\frac{1}{10}$ and $\frac{1}{30}$ would be expected. Since the higher the x value, the lower is the function K, a minimum moment may be obtained by using the value $x=0.1$. The function K using $x=0.1$ is plotted in Figure 5 as a function of the Curie point isotherm position specified by a_t/b for various values of the magnetized shell thickness specified by the parameter a_s/b.

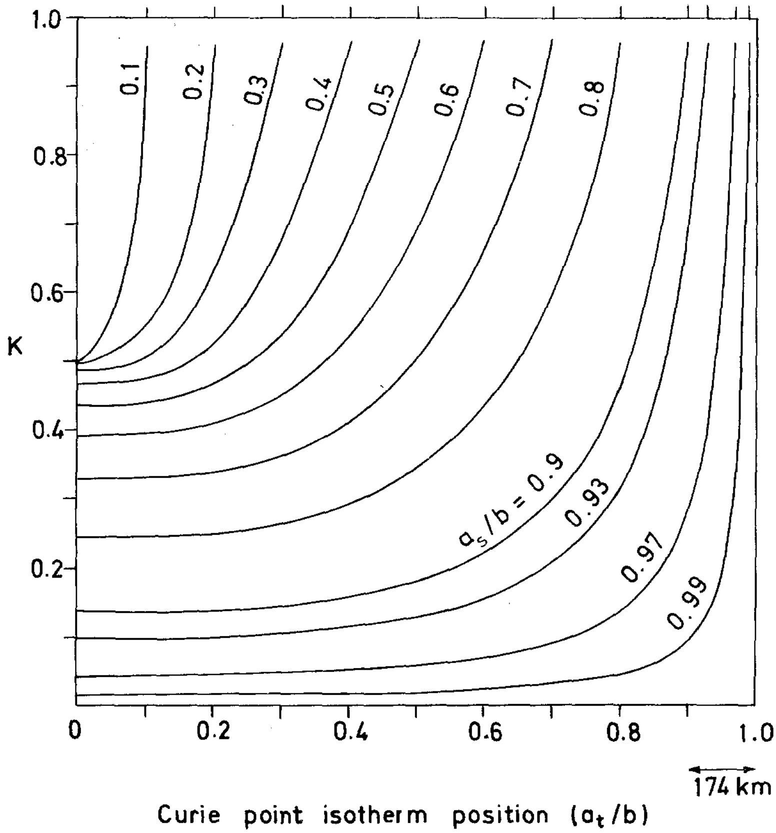

Fig. 5. Plot of function K in Equation (4.3) as a function of normalised Curie point isotherm radius (a_t/b) for various values of normalised magnetized shell radius (a_s/b) (lower boundary). See Figure 3.

8. Evaluation of Residual Moment

Taking $M_s = (51 \pm 14) \times 10^{-3}$ Am^{-1} and $\chi_0 = 0.012 \pm 0.003$ gives a residual moment m from Equation (42) of magnitude

$$m = (9.0 \pm 3.3)\, K \times 10^{15}\ \mathrm{Am}^2. \tag{44}$$

For $K = 1$, the value is almost an order of magnitude higher than the upper limit set by Russell *et al.* (1974) (1.3×10^{15} Am^2). A lower limit to this calculated moment, however, may be obtained from the expression $(m - 1.64\sigma)\, K$ where σ is the standard deviation (3.3×10^{15}) as above. This lower limit represents the level below which the moment has a probability of 0.05 of lying. Representing m_l as the lower limit of the calculated moment

$$m_l = 3.6\, K \times 10^{15}\ \mathrm{Am}^2. \tag{45}$$

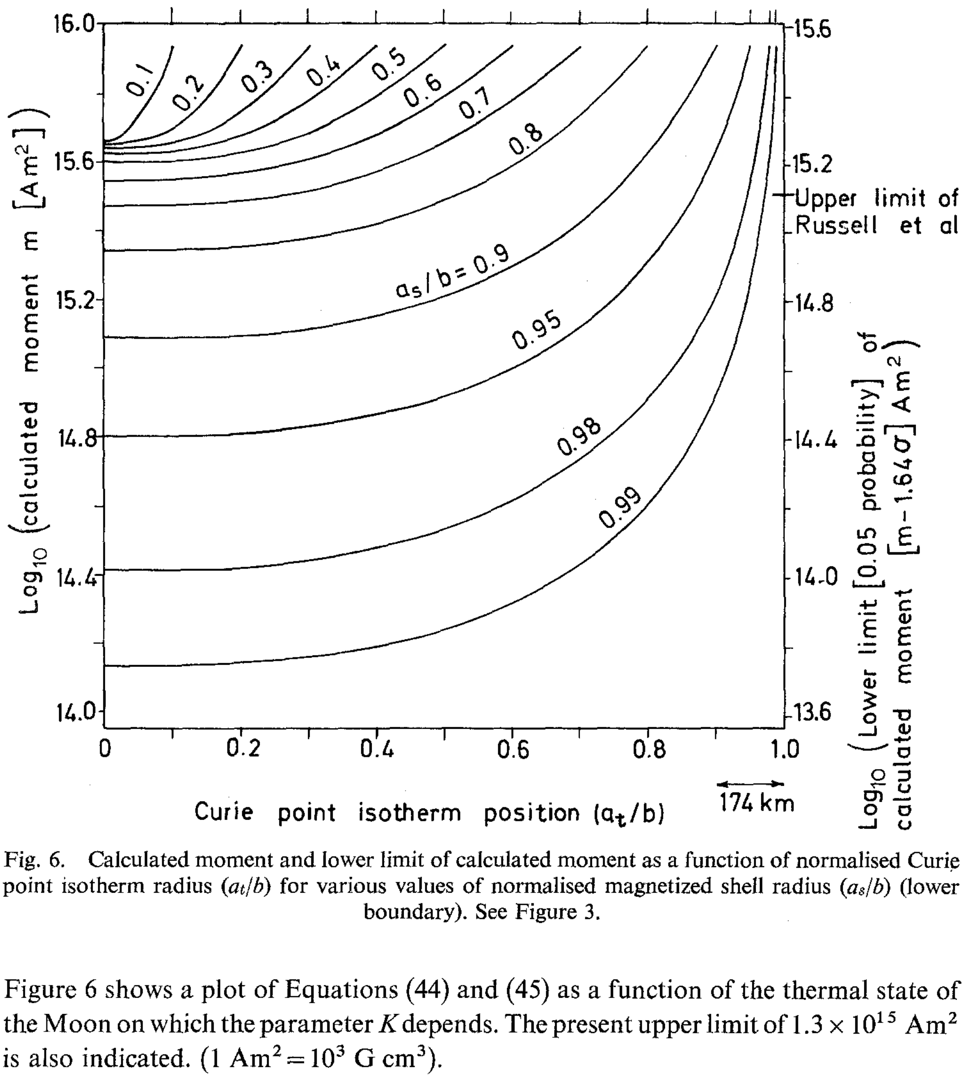

Fig. 6. Calculated moment and lower limit of calculated moment as a function of normalised Curie point isotherm radius (a_t/b) for various values of normalised magnetized shell radius (a_s/b) (lower boundary). See Figure 3.

Figure 6 shows a plot of Equations (44) and (45) as a function of the thermal state of the Moon on which the parameter K depends. The present upper limit of 1.3×10^{15} Am^2 is also indicated. ($1\ Am^2 = 10^3\ G\ cm^3$).

8. Discussion

From Figure 6 it can be seen that if the outer shell is magnetized in the field of an internal dipole of constant direction, those thermal models which give a lower limit to the dipole moment which exceeds the upper limit of Russell *et al.* are ruled out. Thus, for example, if the thickness of the magnetized shell is of the order of 100 km, the Curie point isotherm can not be located at the base of the magnetized crust but must lie at least 150 km below this depth. Unfortunately, the data is not at present good enough to enable more far-reaching conclusions to be made at this stage. This analysis does, however, provide a constraint which has to be met between the present thermal state of the Moon, the thickness of the magnetized shell, and the residual lunar dipole moment, and improved data would make this constraint an important one. A lunar dipole moment less than 10^{14} Am^2 (10^{17} G cm^3) for instance, would mean that the magnetized crust, if it were thicker than 35 km, could not have been magnetized in the field of an internal dipole of constant direction since in this case the observed moment and the calculated moment would not be compatible no matter what the present thermal state of the Moon. In such a case the permanent magnetization hypothesis as suggested by Runcorn and Urey (1973) would find itself in severe difficulties, and a lunar self-reversing dynamo where different parts of the shell would be reversely magnetized and tend to cancel out would then be more likely.

The model used for the preceding calculations is of necessity a simplified one. There are two major assumptions which are not likely to be fulfilled in practice: (a) the ancient field intensity may not have been constant (Stephenson *et al.*, 1975), (b) the surface rocks would not be magnetized simultaneously.

It is not easy to evaluate the perturbing effects of these two factors on the idealized model. The rock samples used to estimate M_s are of varying ages and therefore should give a reasonable estimate of the average magnetization for the lunar rocks even if the ancient field did vary with time. Provided that during each period when the field was of a given strength, magnetization of the crust was built up on a global rather than a local scale, the above idealized model is unlikely to be seriously in error since the rocks of different ages, which will have different average magnetizations (due to a changing internal dipole moment), can be regarded as forming interpenetrating shells whose average value of magnetization is given by the value calculated previously.

If the ancient lunar field were produced by a self-reversing dynamo, the degree to which cancellation of the magnetization of the normal and reversed regions of the shell occurs, would be a crucial factor in determining the present lunar dipole moment. If, for example, the periods of normal and reversed polarity were unequal, or if the magnetization of the shell was mainly acquired during periods of a particular polarity, then the residual moment would be less than that value given by Equation (44) but might still be appreciable. In such a case, the global moment might not be very much smaller than expected from Figure 6. Nevertheless the constraints imposed by this analysis might enable a self-reversing dynamo origin of the ancient lunar field to be distinguished from an internal source of constant polarity, such as a non-reversing dynamo or primeval permanent magnetization if improved knowledge of the various factors involved is obtained. Alternatively if the source of the field were to be identified by other means, the above constraints could help to define the present thermal state of the Moon.

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