# PRE-CAPTURE ORBITS OF THE MOON

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**Abstract.** If the mass of the Earth was not considerably larger than at present, the pre-capture orbit of the Moon was in the range 0.9–1.1 A.U. Capture occurred within several $10^8$ years after formation of the Moon.

Three basically different types of the origin of the Moon have been proposed: (i) fission from the Earth, (ii) capture, and (iii) formation from a swarm of protolunar objects surrounding the Earth. While the fission hypothesis has generally been abandoned, the two other hypotheses possess large circulation in a multitude of variants.

The reasons in favour of a capture origin of the Moon come from several parts, which are summarized briefly, together with their counter-arguments.

(i) The outstanding large mass ratio of about 1 : 80 between the Moon and the Earth in comparison to the ordinary regular satellite systems of Mars, Jupiter, Saturn and Uranus seems to suggest a special origin (capture) of the Moon (Alfvén and Arrhenius, 1972). The captured Moon would destroy any regular satellite system around the Earth (Barricelli and Metcalfe, 1969).

However, if appropriate corrections are introduced for the gas content of the outer planets, the Earth-Moon mass ratio becomes of the same order as the planet-satellite mass ratio of the satellite systems of Jupiter, Saturn and Neptune (Ringwood, 1972).

(ii) The inclination of about 15–20° between the Moon's orbit and the equator of the Earth, when the Moon was at the Roche limit ($\approx$2.9 Earth radii) seems to favour a captured Moon (Öpik, 1972 and references given herein). But the above mentioned inclination can also arise from collisions with planetesimals having less than 1/10 lunar mass (Kaula and Harris, 1973).

(iii) Another argument favouring capture arises if we accept the short tidal time scale of evolution of the lunar orbit from the Roche limit up to its present distance from the Earth ($\approx 2.5 \times 10^9$ yr, Gerstenkorn, 1969; Turcotte *et al.*, 1974). However, this time scale depends on the unknown properties of the Earth's surface and interior over an interval of several $10^9$ yr, (Ruskol, 1966; Öpik, 1972).

(iv) It seems difficult to capture the Moon definitively and to bring it in a nearly circular orbit with semimajor axis $\gtrsim$2.9 e.r. (Ruskol, 1966; Gerstenkorn, 1969; Öpik, 1972; Kaula and Harris, 1973).

(v) The depletion of volatile elements on the Moon by a factor of about 37 (Ganapathy and Anders, 1974) seems to be best accounted for by capture (Singer and Bandermann, 1970). However, Whipple (1973) pointed out that also an Earth-bound Moon could explain the compositional differences.

(vi) The enrichment of the Moon in refractory material (Anderson, 1973) has

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suggested an origin near Mercury (Cameron, 1973), though this is not favoured by the present note.

On the basis of these points I conclude that capture of the Moon is not impossible. It has been shown that capture is possible if there exists

(i) tidal energy dissipation (Öpik, 1972; Kaula and Harris, 1973).

(ii) mass increase of the planet and/or mass loss of the Sun (Ruskol, 1960; Horedt, 1974; Heppenheimer, 1975).

(iii) a resisting medium including collisions with a protosatellite swarm (Aitekeeva, 1968; Bronshten, 1968; Horedt, 1973).

Because it is more easy to handle we have adopted for our calculations energy dissipation by a resisting medium. It produces effects which are similar to tidal energy dissipation, though its dependence on the Earth-Moon distance is not pronounced. We have chosen a resisting medium of constant density which extends up to a distance of $3 \times 10^6$ km from the Earth. Outside this distance the motion of the Moon occurs in vacuum, in order to get a deeper insight into the possible pre-capture orbits.

The approximation of the restricted circular three-body problem is used, because the eccentricity effect of the Earth's orbit is small (Heppenheimer, 1975 and this note). The gravitational constant, the sum of the masses and the Earth-Sun distance are taken as units.

The equations (4) from Horedt (1973) have been integrated with a precision of $10^{-7}$ with $\omega = 1$, i.e. the resisting medium rotates around the Sun with the constant angular velocity of the Earth. We have taken also the same magnitude of the resistance ($k = 0.1$) in order to hold the computational work within reasonable limits (Horedt, 1971, 1973).

In order to obtain pre-capture orbits we started three satellites of inclination with respect to the Earth $i_2 = 0°$, $90°$, $180°$ in a rectangular frame having the centre in the Sun and rotating with the angular velocity of the Earth. The initial conditions correspond to circular orbits on the margin of stability with semimajor axes $a_2 = 7.2 \times 10^5$ km, $1.05 \times 10^6$ km and $1.41 \times 10^6$ km, respectively, (Chebotarev, 1969). These satellites become unstable and leave the vicinity of the Earth after several backward integrated rotations in the resisting medium surrounding the Earth (Horedt, 1971).

After escape the semimajor axes of their orbits around the Sun are approximately $a_1 = 0.96$ and 1.04 A.U. while the eccentricity is $e_1 \approx 0.04$ (Figures 1, 2). The inclination of the orbits around the Sun $i_1$ is evidently zero for the satellites with $i_2 = 0°$ and $180°$, while for the satellite with $i_2 = 90°$ it remains very small: $i_1 \simeq 0\overset{\circ}{.}6$ (cf. Hunter, 1967, for Jupiter).

To check the eccentricity effect of the Earth's orbit, we have integrated the equations of Benest (1971) for the present eccentricity of the Earth. The escaped satellites have essentially the same orbital elements.

It seems not worthwhile to make calculations with initial conditions corresponding to highly eccentric Earth satellites because even for our initial circularly satellites the closest approach of the satellite from the Earth becomes 69000 km for the direct satellite and 18000 km for the retrograde one, corresponding to eccentricities of $e_2 = 0.91$ and 0.99, respectively.

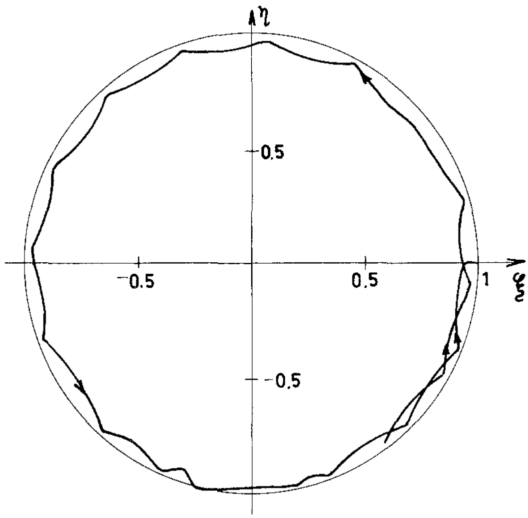


Fig. 1. The pre-capture orbit of a direct satellite of the Earth ($i_2 = 0°$) between −128 and −22 time units. The Earth has the position (1, 0).

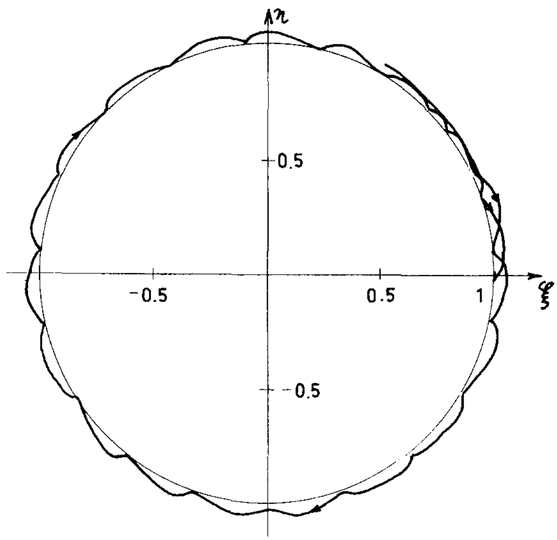


Fig. 2. The pre-capture orbit of a retrograde satellite of the Earth ($i_2 = 180°$) between −161 and −19 time units.

The retrograde satellite escapes outside the Earth orbit (Figure 2) because for orbits with $i_2 \gtrsim 90°$ and/or highly eccentric satellites, Hill's zero velocity curves are open also at the libration point $L_2$ (Stumpff, 1965). This is obvious from the expression of the Jacobi constant of a satellite moving in a moderately perturbed ellipse around the planet (Yegorov, 1959; Heppenheimer, 1975; Horedt, in prep.):

$$C = 3 - 4\mu + \mu/a_2 + 2(\mu a_2(1 - e_2^2))^{1/2} \cos i_2 + r_2^2(3 \cos^2 \theta - 1) + O(\mu).$$

$\mu \ll 1$ is the mass of the planet, $a_2$, $e_2$, $i_2$ the semimajor axis, the eccentricity and the inclination of the orbit of the satellite around the planet, $r_2$ the planet-satellite distance and $\theta$ the angle between the direction Sun-planet and $r_2$. If we take $a_2 = v/2$, $v \simeq (\mu/3)^{1/3}$, ($v/2 \lesssim a_2 \approx v$ for limiting stable satellites, Chebotarev, 1969), $(1 - e_2^2)^{1/2} \cos i_2 \lesssim 0$, $r_2 \lesssim v$, $\cos^2 \theta \simeq 1$ we get

$$C \lesssim 3 + 8v^2 + O(v^3), \quad (v \ll 1),$$

which is smaller than the value for a zero velocity point at $L_2$ (Stumpff, 1965)

$$C = 3 + 9v^2 + O(v^3).$$

If the mass of the Earth would be sometimes considerably larger than its present value $\mu = 3.03 \times 10^{-6}$ there would be possible pre-capture orbits have elements $a_1 \approx 0.75$ A.U and $e_1 \approx 0.2$, (cf. the figures in Horedt, 1971, 1974 for $\mu = 10^{-4} - 10^{-3}$).

If the mass of the Earth never greatly exceeded its present value, the Moon should have been formed in a band between 0.95–1.05 A.U. with low eccentricity and inclination ($e_2 \approx 0.04$, $i_2 \approx 0.^\circ 6$; cf. Lyttleton, 1967; Öpik, 1972, p. 219). Because of these narrow orbital limits it is difficult to see how the Moon could come in a close, nearly circular orbit to the Earth by encounters with other planets. I conclude that if the Moon was captured by the Earth it has been formed in the vicinity of the Earth's orbit. Within several $10^8$ yr after formation it should be captured by the Earth, taking into account the probability of close encounters with the Earth ($\lesssim 5 \times 10^8$ yr, Gerstenkorn, 1969; Öpik, 1972), which would lead to major changes of its initial nearly circular orbital elements.

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