COMMENTS ON THE FIGURE OF THE MOON FROM APOLLO LANDMARK TRACKING

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Abstract. Optical observations were made from the orbiting spacecraft to craters on the lunar surface during Apollo missions 8, 10, 11, 12, 14, and 15. Very accurate selenographic locations for 31 craters have been obtained from these data. The estimated radius values, with respect to the center of mass of the Moon, for the near side maria were smaller than the nominally accepted value of 1738 km. Gross figure of the Moon estimates were obtained for both a sphere and a constrained ellipsoid. These data appear to provide some proof that there is a displacement between the center of figure and the center of mass of the Moon.

1. Introduction

Direct optical observations of relatively small lunar features have been obtained during the first five lunar orbital Apollo missions. Although voluminous data were not gathered, measurements were obtained over a large portion of the lunar surface.

The selenographic positions of the observed lunar features are solved for, or estimated directly from, angular measurements made from the orbiting spacecraft to the landmark, using least-squares techniques. This determination is accomplished in a two-part procedure. The first part (orbit determination) involves determining the spacecraft position at a specified time using Earth-based radar tracking data provided by the Manned Spaceflight Network (MSFN). When the spacecraft orbit has been determined, the position of the spacecraft at each landmark-observation time is obtained simply by integrating the trajectory from the orbit epoch to the time of interest. The second part of the procedure involves processing only the landmark angular measurements to solve for the selenographic-position parameters, the spacecraft positions being held fixed. The accuracy limitations associated with the estimated selenographic positions are dominated by errors in the mathematical model used to describe the lunar gravitational effect. These errors affect the MSFN orbit determination process.The errors caused by the optical instrument are on the order of a magnitude smaller than the gravity model errors.

In this paper, the landmarks will be referenced using a notation system that involves the landmark-identification number and the Apollo mission on which the landmark was observed. For example, H-1/A-12 refers to a landmark identified as H-1 observed on the Apollo 12 mission. The identification numbers are numbers assigned by the Manned Spacecraft Center for use only in flight operations.

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2. Description of Lunar Landmark Data

In all, 31 lunar landmarks were tracked optically from the command module on the Apollo 8, 10, 11, 12, 14 and 15 missions during the respective lunar orbital phases. These landmarks, relatively small craters near the lunar ground track, ranged from 100 to 1500 m in diameter (except for the named features). Two optical devices were available for the crew's use: a 28-power sextant that had a 1.8° field of view, and a 1-power scanning telescope that had a 60° field of view.

The tracking of each landmark consists of at least five sequential sightings, made through one of the instruments, as the spacecraft passes over the landmark. The sightings are usually equally spaced with respect to the landmark nadir. The first sighting is made when the spacecraft is approximately 35° above the landmark local horizon. The third sighting is made at approximately the landmark nadir, and the fifth sighting is made when the spacecraft is again at 35°. The data for each sighting consist of two angular measurements (from the spacecraft to the landmark with respect to an inertial platform) and the time of the sighting.

During a mission, the inertial platform is realigned periodically, and the platform drift between alignments is recorded. In addition, the onboard clock is re-synchronized with the ground clock whenever the time drift exceeds specified limits. The total instrument errors relative to the estimated landmark position are approximately 66 m (1σ) in each position component.

3. Technique Used to Determine Selenographic Positions

The MSFN radar tracking stations obtain Doppler frequency-shift measurements by tracking the spacecraft whenever the spacecraft is in 'view' of the station. The location of the tracking stations and the Earth orbital geometry are such that the spacecraft, when not occulted by the Moon, is in simultaneous view of at least two stations. The Doppler data are processed using a weighted least-squares technique to determine the selenocentric Cartesian components of the spacecraft orbit, with respect to the center of mass, at a specified time*.

Each set of sightings on a particular landmark is then processed using a least-squares technique to determine the selenographic location of the landmark. For the processing, it is assumed that the position of the spacecraft at each sighting time is known from the pertinent MSFN orbit solution. Then, the selenographic latitude, longitude, and radius of the landmark are adjusted simultaneously so that the angular residuals are minimized in a least-squares sense**. Compensation is made for all

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* A lunar gravitational potential model, known as the L1 model, is used for MSFN orbit determination and trajectory prediction. Coefficients for this model are as follows:

$J20 = 2.07108 \times 10^{-4}$
$J30 = -0.21 \times 10^{-4}$
$C22 = 0.20716 \times 10^{-4}$
$C31 = 0.34 \times 10^{-4}$
$C33 = 0.02583 \times 10^{-4}$.

** Residuals are defined as measured angular values minus computed values for each sighting time.

known systematic errors (such as onboard clock errors and inertial platform misalignment and drift) when the landmark data are processed.

A rotating, Moon-referenced coordinate system is used to define the selenographic location of the landmark craters. In this system, the prime meridian passes through the mean center of the apparent disk, which is the 0°-latitude, 0°-longitude point. The latitude is the angle defined by the intersection of the selenocentric radius vector to the landmark and the true lunar equatorial plane; latitude is measured positive north (toward Mare Serenitatis) of, and negative south of, the true lunar equator. The longitude is the angle measured along the lunar equator from the prime meridian to the meridian containing the landmark; longitude is measured positive east (toward Mare Crisium) of, and negative west of, the prime meridian.

The estimated 1σ MSFN orbit-solution uncertainties relative to the selenographic-position components are 500, 660, and 330 m for longitude, latitude, and radius, respectively. These values include errors in physical libration. The total 1σ uncertainties in the Apollo landmark locations caused by all error sources are estimated to be approximately 600, 700, and 400 m for longitude, latitude, and radius, respectively. In an attempt to obtain a better understanding of the orbit determination problem, and thereby reduce the estimated position uncertainties, measurements were made on the same feature on different lunar revolutions during the same mission. Additionally, measurements were made on the same feature during different missions. The rationale being that if the orbit determination errors could be reduced, then the mission-to-mission position estimates would yield information on the physical libration.

4. Results and Discussion

An example of the results obtained from analyzing the sets of position measurements made on the same feature during different lunar revolutions of the same mission are presented in Table I. Each set of data were processed independently in that an orbit solution was obtained for each revolution of Doppler data. These solutions were then used to process the pertinent set of landmark measurements. The maximum differences in any of the position component estimates was less than 100 m.

TABLE I

Apollo landmark location consistency

Landmark identification	Lunar revolution observed	Longitude (DEG)	Latitude (DEG)	Radius (km)
130′/A-10	24	1.280	23.667	1735.43
	25	1.278	23.668	1735.38
	26	1.282	23.669	1735.40
	27	1.282	23.670	1735.41
14-1/A-14	13	15.600	−4.028	1737.07
	15	15.603	−4.027	1737.03

The crater B-1 was surveyed on Apollo missions 8 and 10, and the crater FM-1 was surveyed on Apollo missions 12 and 14. The results of processing these data with Hayn's, Koziel's, and Eckhardt's physical libration models are summarized in Table II. The nominal value of the inclination of the lunar equator with respect to the ecliptic was used for these computations. The mission-to-mission differences in estimated longitude position ranged from 0.0° (for Eckhardt's model) to 0.016° (for Hayn's model). The latitude differences appeared to be biased by approximately −0.01° for the B-1 crater and approximately 0.02° for the FM-1 crater. The inclination values, for the lunar equator with respect to the ecliptic, were then adjusted in an attempt to minimize the mission-to-mission latitude differences. It was found that an inclination correction of 135 arc sec would minimize the overall mission-to-mission differences for both craters. These results are summarized in Table III. In general, Eckhardt's model appeared to provide the best consistency in the mission-to-mission position estimates; therefore, it was used for processing all landmark data. The numerical values for the 31 craters are presented in Table IV. The inclination correction in the physical libration computations was not applied for these data. Therefore, the uncertainties in the estimated crater locations, discussed in the previous section, are still applicable.

TABLE II

Feature locations based on nominal libration inclinations[a]

Feature	Apollo mission	Libration model	Estimated position coordinates			
			Latitude (DEG)	Latitude difference[b] (DEG)	Longitude (DEG)	Longitude difference (DEG)
B-1	8	Hayn	2.577		35.010	
	10		2.562	−0.015	35.026	0.016
	8	Koziel	2.579		35.013	
	10		2.567	−0.012	35.021	0.008
	8	Eckhardt	2.580		35.013	
	10		2.567	−0.013	35.013	0.000
FM-1	12	Hayn	−3.255		−17.321	
	14		−3.230	0.025	−17.329	−0.008
	12	Koziel	−3.254		−17.317	
	14		−3.235	0.019	−17.325	−0.008
	12	Eckhardt	−3.251		−17.325	
	14		−3.231	0.020	−17.326	−0.001

[a] The values used for the inclination of the lunar equator with respect to the ecliptic for the various physical libration models are:

Hayn = 5526.0″
Koziel = 5540.0″
Eckhardt = 5521.5″.

[b] The differences are defined as Apollo 10 minus Apollo 8 or Apollo 14 minus Apollo 12, respectively.

TABLE III

Feature locations based on adjusted libration inclinations[a]

Feature	Apollo mission	Libration model	Estimated position coordinaties			
			Latitude (DEG)	Latitude difference[b] (DEG)	Longitude (DEG)	Longitude difference (DEG)
B-1	8	Hayn	2.573		35.011	
	10		2.571	−0.002	35.024	0.013
	8	Koziel	2.575		35.015	
	10		2.576	0.001	35.019	0.004
	8	Eckhardt	2.577		35.014	
	10		2.577	0.000	35.012	−0.002
FM-1	12	Hayn	−3.268		−17.323	
	14		−3.268	0.000	−17.328	−0.005
	12	Koziel	−3.267		−17.319	
	14		−3.273	−0.006	−17.325	−0.006
	12	Eckhardt	−3.264		−17.327	
	14		−3.269	−0.005	−17.325	0.002

[a] The adjusted values used for the inclination of the lunar equator with respect to the ecliptic for the various physical libration models are:

Hayn = 5661.0″
Koziel = 5675.0″
Eckhardt = 5656.5″.

[b] The differences are defined as Apollo 10 minus Apollo 8 or Apollo 14 minus Apollo 12, respectively.

The near side maria appear to be depressed with the deepest depression, or smallest radius from center of mass, in Mare Smythii. The landmark radius of 1733.01 km in Mare Smythii agrees almost exactly with the value obtained by Watts (as cited by Baldwin, (1963)) from limb-outline measurements using U.S. Naval Observatory photographic plates. This was the only instance where there was good agreement between Apollo observed values and values obtained from earth-based optical observations. Where comparison points were available, the Apollo radii were at least 1 km less than the Earth-based optical values (Table V). A similar disagreement with the earth-based values was noted in the Ranger impact points (Sjogren *et al.*, 1967), Surveyor landing points (Thornton *et al.*, 1968; Labrum *et al.*, 1968) and Lunar Orbiter data (Michael and Tolson, 1967; Compton and Wells, 1969).

In Table IV it can be seen that the near side radius values are on the average lower than the far side values. This strongly indicates the possibility of a displacement between the center of figure and center of mass of the Moon. This is certainly not a new hypothesis since it was used as a possible explanation for the disagreement between the Earth-based optical and Ranger, Surveyor, and Lunar Orbiter radius values. The Apollo data, however, represent the first direct position measurements made on far side features whose accuracy is better than the suspected magnitude of the displacement.

TABLE IV

Apollo landmark crater locations

Landmark identification	Longitude (DEG)	Latitude (DEG)	Radius (km)	Location
		Lunar Near Side		
Encke E/A-14	−40.11	0.49	1736.27	Oceanus Procellarum
Lansberg A/A-12	−31.13	0.14	1736.83	Oceanus Procellarum
193/A-12	−23.23	−3.50	1735.91	Oceanus Procellarum
FM-1/A-14	−17.33	−3.23	1736.80	Hills north of Fra Mauro
14-1/A-14	−15.60	−4.03	1737.05	Hills north of Fra Mauro
H-1/A-12	−15.24	−1.50	1736.09	Near IAU Crater 2922 (Turner)
Mösting A/A-14	−4.96	−3.35	1737.96	
150′/A-10	−1.53	−0.01	1736.50	Sinus Medii
15-1/A-15	3.68	26.09	1735.61	Palus Putredinis
J-1/A-15	10.58	25.91	1734.59	Mare Serenitatis
DE-1/A-12	15.51	−8.93	1737.74	Highlands near center of near side
Dollond E/A-14	15.79	−10.20	1738.04	Highlands near center of near side
DE-2/A-14	19.65	−9.49	1738.87	Highlands near center of near side
130″/A-11	23.66	1.26	1735.34	Mare Tranquillitatis
130′/A-10	23.67	1.28	1735.40	Mare Tranquillitatis
Daguerre 66/A-14	33.15	−11.83	1734.81	Mare Nectaris
B-1/A-10	35.01	2.57	1736.59	Mare Tranquillitatis
CP-2/A-12	56.11	−10.54	1736.39	Mare Fecunditatis
A-1/A-11	65.06	1.80	1735.49	Mare Spumans
Ansgarius N/A-14	81.27	−11.78	1738.44	Highlands south of Mare Smythii
F-1/A-10	88.24	1.89	1733.01	Mare Smythii
		Lunar Far Side		
CP-3/A-8	96.89	−8.90	1735.37	In highlands southeast of IAU Crater 266
RP-5/A-14	99.35	−10.85	1741.39	Highlands west of Pasteur
12-1/A-14	112.31	−5.73	1738.23	
RP-4/A-14	120.59	−6.03	1740.27	
CP-2/A-10	127.95	0.59	1742.28	Approximately 1° northeast of IAU Crater 282
RP-3/A-14	131.91	−3.69	1740.85	
RP-2/A-14	141.31	−0.29	1742.18	South of Mendeleev
CP-2/A-8	163.25	−9.71	1736.95	Within IAU Crater 302
CP-1/A-10	170.13	0.86	1739.07	Near IAU Crater 225
CP-1/A-8	158.04	−6.30	1740.30	North-northwest of IAU Crater 313

The estimated landmark positions were then processed by Wollenhaupt and Sjogren (1972) (JPL) in a small least-squares computer program which simultaneously solves for the parameters of an ellipsoid and the location of the center of figure. Two solutions were obtained for an ellipsoid whose z-axis (parallel to the Moon's polar axis) was constrained to have a fixed value of 1738.0 km. The first solution used all landmark points, and the second used only those landmark points in an equatorial band of 4° latitude. No attempt was made to obtain solutions differentiating between highland and maria landmarks because there was not enough geometry to obtain even gross

TABLE V

Selenographic locations derived from Earth-based sources

Landmark identification	Source	Longitude (DEG)	Latitude (DEG)	Radius (km)	ΔR[a] (km)
Lansberg A	b	−31.073	0.19	1738.43	−1.60
	c	−31.050	0.21	1740.10	−3.27
	d	−31.060	0.19	1738.68	−1.85
Mösting A	b	−5.157	−3.18	1739.34	−1.38
	c	−5.160	−3.18	1739.40	−1.44
	d	−5.160	−3.18	1739.95	−1.99
Daguerre 66	b	33.103	−11.746	1735.81	−1.00
Dollond E	b	15.695	−10.215	1739.74	−1.70
Encke E	b	−40.091	0.364	1737.95	−1.68

[a] ΔR = Apollo landmark radius *minus* source radius.
[b] Meyer and Ruffin (1968).
[c] Schrutka-Rechtenstamm (1958).
[d] Mills and Sudbury (1968).

estimates of the figure. The numerical values for these solutions are summarized in Table VI. The all data solution yielded an estimate of approximately a 1-km difference between the x-axis (along the Earth-Moon line) and the y-axis (normal to x-axis in lunar equatorial plane), with the x-axis having the largest value. It was estimated that the center of the figure was 2.2 km behind and 0.1 km to the left of the center of mass when viewing the Moon from the Earth. Sixteen of the 31 available landmarks were used for the equatorial data only estimate. This solution indicated that the Moon is a sphere in the equatorial plane having an average radius of 1738.8 km. It was estimated that the center of figure was 2.8 km behind and 0.2 km to the left of the center of mass.

TABLE VI

Figure of Moon solutions based on landmark data

	Principal axes[a], km			Center of figure to center of mass displacements[b], km		
Figure	x	y	z	Δx	Δy	Δz
Constrained ellipsoid[d]	1738.6	1737.5	1738.0[c]	−2.2	−0.1	0[c]
Constrained ellipsoid[e]	1738.9	1738.7	1738.0[c]	−2.8	−0.2	0[c]

[a] Principal axis orientation: x along Earth-Moon line; y normal to Earth-Moon line in lunar equatorial plane; z normal to x, y plane coincident with lunar polar axis.
[b] Center of figure (c.f.) to center of mass (c.m.) displacements: Δx same orientation as x. Negative means c.f. is behind c.m. when viewing Moon from Earth; Δy same orientation as y. Negative means c.f. is left of c.m. when viewing Moon from Earth.
[c] Parameter held fixed to indicated value during computations.
[d] All data included in solution.
[e] Only equatorial data included in solution.

The gross figures obtained in these solutions are perhaps not too meaningful primarily due to insufficient data. The same comment can be made regarding the magnitude of the displacement between center of figure and center of mass.

5. Concluding Remarks

The Apollo landmark tracking data represent the first accurate, direct position measurements made on small lunar features. This is particularly true for the lunar far side where the first observations were made during the Apollo 8 mission in December 1968. Unfortunately, it was not possible to obtain a large amount of data via this method primarily because of mission operational constraints. A further constraint was that the feature be on the sunlight portion of the Moon very near the subvehicle point. Consequently, no data were obtained for the region extending from approximately 40° west to 180° west. Therefore, not too much significance can be attached to the gross figure solutions. A similar comment can be made on the magnitude of the estimated center of figure to center of mass displacements, particularly for the ΔY displacement. This estimate may be very erroneous because of lack of western hemisphere data. The estimated ΔX displacement is probably much better, although its value is still questionable. It is interesting to note that the landmark derived values are within the 1.5 to 3.8 km estimates based on the laser altimeter data (Wollenhaupt and Sjogren, 1972).

On the basis of the data presented in this paper, it appears that the radius values derived from the Apollo landmark data provide some proof of the existence of a displacement between the center of figure and center of mass of the Moon along the Earth-Moon line. In addition, all three components of the estimated crater locations should be very useful toward establishing a selenodetic reference system for interpreting or reducing Earth-based observation data.

Analyses of the position estimates for craters surveyed on different missions indicate that a rather large correction to the inclination of the lunar equator with respect to the ecliptic is required to minimize the mission to mission differences. This is based on the assumption that Koziel's values for the principal moments of inertia of the Moon are correct. This may not be a good assumption, but better values are currently not available. It can perhaps be argued that these analyses reflect nothing more than removing biases in the latitude or orbit plane estimates in the orbit determination computations. However, this would require a very fortunate set of circumstances such that a single correction, to the inclination of the lunar equator to the ecliptic, would explain the differences in position estimates of craters surveyed during four different missions.

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