ON THE SECOND AND THIRD MODES OF THE MOON'S FREE LIBRATION

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Abstract. The probable semi-amplitudes of the second and third modes of the Moon's free librations are inferred from the observed semi-amplitude of its first mode, using the statistical relations between the exciting actions and the amplitudes of the free librations, derived in the author's previous paper (Sekiguchi, 1970). It is likely that the semi-amplitudes of these librations exceed some second of arc.

1. Introduction

In the author's previous paper (Sekiguchi, 1970, referred to as (I)), he derived some expressions concerning the relations between the possible amplitudes of the Moon's free librations for three modes and its exciting random perturbations, under the assumption that the random perturbations are completely isotropic and stationary. After finishing the work, he was informed that Professor K. Kozieł of Kracow determined unambiguously the phase and amplitude of the first mode of the Moon's free librations (Kozieł, 1967). According to it, the semi-amplitude A and phase factor a are*:

$$A = 18''.7 \pm 4''.7,$$
$$a = 334^\circ.3 \pm 15^\circ.7\,(1800.0).$$

In the present paper, the first mode means the free libration in longitude whose period is 2.84 years, and the second and the third modes are nutations of figure axis about the rotational axis, whose periods are 27.3 days and 75 years respectively.

From the observed amplitude of the first mode of free libration, we can infer the probable amplitudes of its second and third modes, whose observational verification has not been carried out so far, under some appropriate assumptions.

2. Fundamental Relations

The expressions of (7.7), (7.8) and (7.9) in paper (I) state that

$$\langle x_1^2 \rangle = \frac{\sigma_x^2}{2\Delta n^2}\left\{\frac{1}{K_{\mathrm{II}}} + \frac{1}{2}\left(1 + \frac{4\beta}{\alpha}\right)\frac{1}{K_{\mathrm{III}}}\right\}, \tag{1}$$

* By private communication, he recommended the more favorable values $A = 17'' \pm 3''.7$, $a = 333^\circ.0 \pm 12^\circ.9$ (1800.0). But owing to consistency of adopted values of constants, the author adopted the values in the paper of 1967.

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$$\langle x_2^2 \rangle = \frac{\sigma_x^2}{2\Delta n^2} \left\{ \frac{1}{K_{\text{II}}} + \frac{1}{2}\left(1 + \frac{\alpha}{4\beta}\right)\frac{1}{K_{\text{III}}} \right\}, \tag{2}$$

$$\langle x_3^2 \rangle = \frac{\sigma_x^2}{12\gamma n^2 K_{\text{I}} \Delta}, \tag{3}$$

where meanings of the notations in these expressions are as follows.

We take a rectangular coordinate system (q_1', q_2', q_3'), which coincides with the Moon's principal axes of inertia. The q_1' axis points towards the Earth, the q_2' axis towaıds the Mare Smythii, and the q_3' axis towards the North Pole of the Moon. The principal moments of inertia around the q_1', q_2' and q_3' axes are A, B and C respectively, being supposed $A < B < C$. We put

$$\alpha = \frac{C - B}{A}, \quad \beta = \frac{C - A}{B}, \quad \gamma = \frac{B - A}{C}.$$

Let x_1, x_2 and x_3 be rotation angle around the q_1', q_2' and q_3' respectively, being supposed small. The precise implications of these parameters are mentioned in paper (I).

We consider any aıbitrary set of functions, whose realization is $R(t)$. If there exists a mean value of the functions $R(t)$ for special values of t, we write it as $\langle R(t) \rangle$, which is generally a function of t. Therefore $\langle x_i \rangle$ $(i = 1, 2, 3)$ are mean values taken from the set of functions $x_i(t)$ for special t. In the present case, we assume $\langle x_i \rangle = 0$, hence $\langle x_i^2 \rangle$ are equivalent to the squares of dispersions of the functions $x_i(t)$ for special t.

Let $X_i(t)$ $(i = 1, 2, 3)$ be three components of the exciting function. We assume $\langle X_i \rangle = 0$ and X_i are stationary, hence $\langle X_i^2 \rangle$ are constants. Furthermore, we assume the excitation is isotropic, hence we have

$$\langle X_1^2 \rangle = \langle X_2^2 \rangle = \langle X_3^2 \rangle,$$

and we put these quantities as σ_x^2.

The K_{I}, K_{II} and K_{III} are damping coefficients of the first, second and third modes of free librations respectively. The n is speed of revolution of the Moon and Δ is mean time intervals between two successive shocks given to the Moon.

3. The Probable Amplitudes of Each Component of Free Librations

If we attach suffixes II and III to the components of the second and third modes such that the expression of x_i $(i = 1, 2)$ can be written as

$$\begin{aligned} x_1 &= {}_{\text{II}}x_1 + {}_{\text{III}}x_1, \\ x_2 &= {}_{\text{II}}x_2 + {}_{\text{III}}x_2. \end{aligned} \tag{4}$$

As the two parts of the right-hand sides of these equations are statistically independent, we can easily see that

$$\begin{aligned} \langle x_1^2 \rangle &= \langle {}_{\text{II}}x_1^2 \rangle + \langle {}_{\text{III}}x_1^2 \rangle, \\ \langle x_2^2 \rangle &= \langle {}_{\text{II}}x_2^2 \rangle + \langle {}_{\text{III}}x_2^2 \rangle, \end{aligned}$$

and, from expression (4.7) of paper (I), we have

$$\langle {}_{\mathrm{II}}x_1^2 \rangle = \langle {}_{\mathrm{II}}x_2^2 \rangle = \frac{\sigma_x^2}{2\Delta n^2 K_{\mathrm{II}}}, \tag{5}$$

$$\langle {}_{\mathrm{III}}x_1^2 \rangle = \frac{1}{4}\left(1 + \frac{4\beta}{\alpha}\right)\frac{\sigma_x^2}{\Delta n^2 K_{\mathrm{III}}}, \tag{6}$$

$$\langle {}_{\mathrm{III}}x_2^2 \rangle = \frac{1}{4}\left(1 + \frac{\alpha}{4\beta}\right)\frac{\sigma_x^2}{\Delta n^2 K_{\mathrm{III}}}, \tag{7}$$

using these relations, we can estimate magnitudes of dispersions of various quantities, assuming that the value of $\langle x_3^2 \rangle$ known.

After Kozieł (1967), we adopt the following values of α, β and γ.

$$\left.\begin{aligned} \alpha &= 0.000\,398\,4\,, \\ \beta &= 0.000\,629\,4\,, \\ \gamma &= 0.000\,231\,0\,. \end{aligned}\right\} \tag{8}$$

In the course of the present estimations, the values of K_{I}, K_{II} and K_{III} are most ambiguous. Here, the author adopted a system of these values after Jeffreys and Crampin (1961).

$$\left.\begin{aligned} \frac{1}{K_{\mathrm{I}}} &= 4.1 \times 10^4 \text{ years}\,, \\ \frac{1}{K_{\mathrm{II}}} &= 7.6 \times 10^5 \text{ years}\,, \\ \frac{1}{K_{\mathrm{III}}} &= 2.5 \times 10^5 \text{ years}\,. \end{aligned}\right\} \tag{9}$$

We assume that the dispersion of the first mode libration is equal to the observed semi-amplitude of this mode. Hence we can put

$$\sqrt{\langle x_3^2 \rangle} = 18''\!.7\,.$$

From (3), we have

$$\frac{\sigma_x^2}{n^2\Delta} = 2.364 \times 10^{-5}\,,$$

where the unit is (second of arc)^2 (year)$^{-1}$. Using this value, we can calculate the following values from (1), (2), (5), (6) and (7).

$$\begin{aligned} \sqrt{\langle x_1^2 \rangle} &= 4''\!.8\,, \\ \sqrt{\langle x_2^2 \rangle} &= 3''\!.4\,, \\ \sqrt{\langle {}_{\mathrm{I}}x_1^2 \rangle} &= \sqrt{\langle {}_{\mathrm{I}}x_1^2 \rangle} = 3''\!.0\,, \\ \sqrt{\langle {}_{\mathrm{II}}x_1^2 \rangle} &= 3''\!.8\,, \\ \sqrt{\langle {}_{\mathrm{II}}x_2^2 \rangle} &= 1''\!.5\,. \end{aligned}$$

4. Discussion

The above results show that there are possibilities to find out the second and third modes of the Moon's free libration from observations, though their magnitudes have still large uncertainties, owing to ambiguity in the damping coefficients. The quantities $\sqrt{\langle x_1^2 \rangle}$ and $\sqrt{\langle x_2^2 \rangle}$ indicate the dispersion of deviation from a normal position. If we find the second and third modes of the free libration by harmonic analyses, the quantities $\sqrt{\langle {}_{\text{I}}x_1^2 \rangle}$ ... indicate the measure of probable semi-amplitudes of these modes. The above calculations show that these quantities may attain to observable magnitudes.

Recently the author was informed from Dr. L. V. Morrison of the Royal Greenwich Observatory by a private communication that he investigated the rotation of the Moon's disk about the line of sight using the grazing occultations. The author expects that such investigations contribute to any information about the rotational libration of the Moon's disk.

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