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On the Structure of Certain Excursions of a Markov Process

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Summary. Let X be a Markov process and M a homogeneous random set. For $t \geqq 0$, we set $G_t = \operatorname{Sup}\{s \leqq t: s \in M\}$. The stochastic dependence between the past and the future of G_T is investigated for certain stopping times T. This gives some insight to recent results of Getoor concerning the excursion straddling t and the first excursion exceeding a in length.

I. Introduction

Let $(\Omega, \mathscr{F}, \mathscr{F}_t, X_t, \theta_t, P^x)$ be a strong Markov process and let M be a homogeneous closed well measurable set (for instance $M = \overline{\{t: X_t \in B\}}$, where B is a Borel subset of the state space E). Set

$$G_t = \operatorname{Sup}\{s \leqq t: s \in M\}, \qquad A_t = t - G_t, \quad t \geqq 0.$$

We will establish that if T is a stopping time of the family $(\mathscr{F}_{G_t})$, the past and the future relative to G_T are conditionally independent given (A_T, X_{G_T}). More precisely, there exists a family $(P^{a,x})$ of probabilities on Ω such that $P^{0,x} = P^x$ and

$$E^{\cdot}[f \circ \theta_{G_T} | A_T, \mathscr{F}_{G_T}] = P^{A_T, X_{G_T}}(f) \qquad \text{a.s. on } \{T < \infty\}$$

for all $(\mathscr{F}_{G_t})$-stopping times T and all positive measurable functions f on Ω ($P^{a,x}(f)$ denotes $\int f \, dP^{a,x}$).

In particular if we set

$$T^a = \operatorname{Inf}\{t: A_t > a\}, \qquad G^a = G_{T^a}, \qquad a > 0,$$

one has $A_{T^a} = a$ on $\{G^a < \infty\}$ and the preceding formula specializes as follows

$$E^{\cdot}[f \circ \theta_{G^a} | \mathscr{F}_{G^a}] = P^{a, X_{G^a}}(f) \qquad \text{a.s. on } \{G^a < +\infty\}.$$

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Therefore the past and the future of G^a are conditionally independent given X_{G^a}.

These formulas generalize and make more understandable the following result recently established by Getoor [1]: the distribution of the excursion straddling t, conditionally on $A_t=a$, $X_{G_t}=x$, is the same as the distribution of the first excursion exceeding a in length, conditionally on $X_{G^a}=x$.

We shall also give analogous results for conditioning on $\mathscr{F}_{G_{T-}}$, under somewhat stronger assumptions. The reader will find a lot of nice applications and explicit computations for the excursions from a point in Getoor's paper [1].

II. Notations and Preliminary Results

We shall use the notations and assumptions of [3]. $(\Omega, \mathscr{F}, \mathscr{F}_t, X_t, \theta_t, P^x)$ is the canonical right continuous realization of a Markov semi-group (P_t) satisfying the "hypothèses droites" of Meyer. The state space E is Lusin; in E we distinguish a point δ. We assume that δ is absorbing and that the lifetime $\zeta=\inf\{t: X_t=\delta\}$ is infinite P^x-a.s. for each $x \neq \delta$.

M is a closed random subset of $\mathbb{R}_+$, well measurable (or optional) and homogeneous in $]0,\infty[$:

$$(M-t)\cap]0,\infty[= M\circ\theta_t \cap]0,\infty[, \qquad t \geqq 0.$$

We associate with M the following notations:

$$\begin{aligned}
R &= \inf\{t>0: t\in M\}, \qquad R_t = R\circ\theta_t, \quad D_t = t+R_t, \quad t\geqq 0,\\
G_t &= \sup\{s\leqq t: s\in M\}, \qquad 0\leqq t\leqq\infty,\\
A_t &= t-G_t, \qquad t\geqq 0,\ A_\infty=\infty,\\
G &= \{t>0: R_{t-}=0,\ R_t>0\},\\
F &= \{x\in E: P^x\{R=0\}=1\}.
\end{aligned}$$

The set $G(\omega)$ is the set of the left endpoints in $]0,\infty[$ of the intervals that are contiguous to $M(\omega)$.

R is assumed to be $\mathscr{F}^*$-measurable $-\mathscr{F}^*$ denotes the universal completion of $\mathscr{F}^0=\sigma\{X_s, s\geqq 0\}$. Then one has the following result (see [3]).

Theorem 1. *There exists an "exit system" $(B,\hat{P})$: B is a well measurable random measure, $\hat{P}$ is a kernel from $(E,\mathscr{E}^*)$ to $(\Omega,\mathscr{F}^*)$ such that*

$$E^{\cdot}\Big[\sum_{s\in G} Z_s f\circ\theta_s\Big] = E^{\cdot}\left[\int_0^\infty Z_s \hat{P}^{X_s}(f)\, dB_s\right] \tag{1}$$

for all positive well-measurable Z and *$\mathscr{F}^*$-measurable f.*

In view of [3], one may and will choose the system $(B,\hat{P})$ with the following properties: $dB_t = dK_t + \sum_{\substack{s\in G\\ X_s\notin F}} \varepsilon_s(dt)$, where K is a continuous additive functional carried by F, with 1-potential $\leqq 1$; $\hat{P}^x=P^x$ if $x\notin F$; $\hat{P}^x\{R=0\}=0$ and $\hat{P}^x(1-e^{-R})\leqq 1$ for all x (which implies that $\hat{P}^x\{R>a\}<\infty$ for all x and all $a>0$); and finally $(X_t)_{t>0}$ is strong Markov with semi-group (P_t) relative to the measure $\hat{P}^x$, for all x.

III. Conditioning on $(A_T, \mathscr{F}_{G_T})$

For each positive random variable S, $\mathscr{F}_S$ denotes the σ-field of the sets $A \in \mathscr{F}$ such that $I_A = Z_S$ on $\{S < \infty\}$ for some well measurable process Z. For example S, $X_S(X_\infty = \delta)$ are $\mathscr{F}_S$-measurable. Here is a technical result.

Proposition 1. 1) *The family* $(\check{\mathscr{F}}_t) = (\mathscr{F}_{G_t})$ *is increasing.*

2) *For each* $(\mathscr{F}_t)$*-stopping time* T, *one has* $\mathscr{F}_{G_T} \subset \check{\mathscr{F}}_T \subset \mathscr{F}_T$.

Proof. 1) Let (Z_t) be a right continuous adapted process. The process (Z_{G_t}) is adapted, since $G_t \leqq t$, and right continuous, since (G_t) is increasing and right continuous. Therefore (Z_{G_t}) is well measurable for each well measurable (Z_t). Let $u \geqq 0$, set $Z_t^u = Z_{G_{t \wedge u}}$; the process $(Z_t^u)_{t \geqq 0}$ is still well measurable and the equality $Z_{G_u} = Z_{G_v}^u$, $u \leqq v$, shows that $\mathscr{F}_{G_u} \subset \mathscr{F}_{G_v}$ if $u \leqq v$.

2) Let T be a stopping time of the increasing family $(\check{\mathscr{F}}_t)$. We have already observed that, if (Z_t) is $(\mathscr{F}_t)$-well measurable, (Z_{G_t}) is $(\mathscr{F}_t)$-well measurable; therefore $\mathscr{F}_{G_t} \subset \mathscr{F}_t$ for all t, and (Z_{G_t}) is $(\check{\mathscr{F}}_t)$-well measurable, which implies that Z_{G_T} is $\check{\mathscr{F}}_T$-measurable (set $Z_\infty = 0$) and that $\mathscr{F}_{G_T} \subset \check{\mathscr{F}}_T$.

Remark. The variable A_T is $\check{\mathscr{F}}_T$-measurable, since (A_t) is right continuous and $(\check{\mathscr{F}}_t)$-adapted, but not necessarily $\mathscr{F}_{G_T}$-measurable.

Theorem 2. *For all positive* $\mathscr{F}^*$*-measurable functions* f *on* Ω *and all* $(\check{\mathscr{F}}_t)$*-stopping times* T *one has*

$$\hat{P}^{A_T, X_{G_T}}(1) > 0 \quad \text{a.s. on } \{G_T < T < \infty\}, \tag{2}$$

$$E^{\cdot}[f \circ \theta_{G_T} | \check{\mathscr{F}}_T] = P^{A_T, X_{G_T}}(f) \quad \text{a.s. on } \{T < \infty\}, \tag{3}$$

where we set

$$\hat{P}^{a,x}(f) = \hat{P}^x(f 1_{\{R > a\}}) \quad \text{if } a > 0,$$

$$P^{a,x}(f) = \frac{\hat{P}^{a,x}(f)}{\hat{P}^{a,x}(1)} \quad \text{if } a > 0 \quad \left(\frac{0}{0} = 0\right),$$

$$P^{0,x}(f) = P^x(f).$$

Remarks. a) If $a > 0$, $\hat{P}^{a,x}(1) = \hat{P}^x\{R > a\} < \infty$, as already observed.

b) $(a, x) \to \hat{P}^{a,x}(f)$ is measurable on $]0, \infty[\times E$ due to the right continuity in a, and $(a, x) \to P^{a,x}(f)$ is measurable on $[0, \infty[\times E$.

c) X_{G_T} is $\mathscr{F}_{G_T}$-measurable. Therefore equality (3) implies the following:

$$E^{\cdot}[f \circ \theta_{G_T} | A_T, \mathscr{F}_{G_T}] = P^{A_T, X_{G_T}}(f) \quad \text{a.s. on } \{T < \infty\}. \tag{3'}$$

d) Let us set, for a positive h on E

$$\hat{Q}_a(x, h) = \hat{P}^x(h \circ X_a I_{\{R > a\}}), \quad a > 0,$$

$$q_a(x, h) = \frac{\hat{Q}_a(x, h)}{\hat{Q}_a(x, 1)} \quad \text{if } a > 0 \quad \left(\frac{0}{0} = 0\right),$$

$$= h(x) \quad \text{if } a = 0,$$

like in [3]. We have $\hat{Q}_a(x,h)=\hat{P}^{a,x}(h\circ X_a)$, $q_a(x,h)=P^{a,x}(h\circ X_a)$ and formula (3) implies

$$E^{\cdot}[h\circ X_T|\check{\mathscr{F}}_T]=q_{A_T}(X_{G_T},h) \quad \text{a.s. on } \{T<\infty\}.$$

Hence Theorem 2 is a direct generalization of Theorem (7.10) of [3].

Proof. Condition (2) was already established in [3], in the form $\hat{Q}_{A_T}(X_{G_T},1)>0$ a.s. on $\{G_T<T<\infty\}$. On the set $\{G_T=T<\infty\}$, equality (3) follows from $\check{\mathscr{F}}_T\subset\mathscr{F}_T$ and from the Markov property at time T. One can reduce the proof of (3) on the set $\{G_T<T<\infty\}$ to the case $T=t$ constant by the argument of [3]. So we have to establish that, for each positive well measurable Z,

$$E^{\cdot}[Z_{G_t}f\circ\theta_{G_t}I_{\{G_t<t\}}]=E^{\cdot}[Z_{G_t}P^{A_t,X_{G_t}}(f)\,I_{\{G_t<t\}}]. \tag{4}$$

Let us firstly give the following extension of formula (1): if we set $G^0=G$ if $R=0$, $G\cup\{0\}$ if $R>0$ and $dB_t^0=I_{\{R>0\}}\,\varepsilon_0(dt)+dB_t$, one has

$$E^{\cdot}[\sum_{s\in G^0}Z_sF(s,X_s,\theta_s)]=E^{\cdot}[\int_{[0,\infty[}Z_s(\omega)\,dB_s^0(\omega)\int F(s,X_s(\omega),\omega')\,\hat{P}^{X_s(\omega)}(d\omega')] \tag{5}$$

for each positive universally measurable function F on $\mathbb{R}_+\times E\times\Omega$.

Now the basic observation is that $s=G_t<t$ iff $s\in G^0$ and $0<t-s<R\circ\theta_s$. Therefore the left side of (4) equals

$$E^{\cdot}[\int_{[0,t[}Z_s\hat{P}^{t-s,X_s}(f)\,dB_s^0]$$

(take $F(s,x,\omega)=f(\omega)\,I_{\{0<t-s<R\}}(\omega)$ in (5)) and the right side of (4) equals

$$E^{\cdot}[\int_{[0,t[}Z_sP^{t-s,X_s}(f)\,\hat{P}^{t-s,X_s}(1)\,dB_s^0]$$

(take $F(s,x,\omega)=P^{t-s,x}(f)\,I_{\{0<t-s<R\}}(\omega)$ in (5)). This yields (4).

Corollary 1. *If $a>0$, let $T^a=\inf\{t\colon A_t>a\}$, $G^a=G_{T_a}$. Then*

$$E^{\cdot}[f\circ\theta_{G^a}|\mathscr{F}_{G^a}]=P^{a,X_{G^a}}(f) \qquad \text{a.s. on } \{G^a<\infty\} \tag{6}$$

for all positive $\mathscr{F}^*$-measurable f.

Proof. T^a is an $(\check{\mathscr{F}}_t)$-stopping time since

$$\{T^a\leqq t\}=\{A_t=a\}\cup(\bigcup_{\substack{r<t\\ r\text{ rational}}}\{A_r>a\})$$

and (A_t) is $(\check{\mathscr{F}}_t)$-adapted. Notice also that $A_{T^a}=a$ on $\{G^a<\infty\}=\{T^a<\infty\}$. Therefore applying (3) to $T=T^a$ yields (6).

Remark. (6) can also be established directly by observing that $s=G^a<\infty$ iff $s\in G^0$, $R\circ\theta_s>a$ and $s\leqq T^a$, and by using equality (5) with $Z_t=I_{\{t\leqq T^a\}}$, like in the proof of (4).

From Theorem 2 one obtains the distribution of the excursion straddling T, conditionally on A_T, X_{G_T} (this was proved by Getoor for $T=t$).

Corollary 2. *Let T be an $(\check{\mathscr{F}}_t)$-stopping time. If* $0<a<r$, $x\in E$, $0<t_1<\ldots<t_n<r$, *one has (symbolically)*

$$P^{\cdot}\{\theta_{G_T}^{-1}\{X_{t_1}\in dy_1,\ldots,X_{t_n}\in dy_n, R>r\}\,|\,A_T=a, X_{G_T}=x\}$$
$$=\hat{Q}_a(x,1)^{-1}\,\hat{Q}_{t_1}(x,dy_1)\,Q_{t_2-t_1}(y_1,dy_2)\ldots Q_{t_n-t_{n-1}}(y_{n-1},dy_n)\,P^{y_n}\{R>r-t_n\}$$
$$=P^{\cdot}\{\theta_{G^a}^{-1}\{X_{t_1}\in dy_1,\ldots,X_{t_n}\in dy_n, R>r\}\,|X_{G^a}=x\}$$

where

$$\hat{Q}_t(x,dy)=\hat{P}^x\{X_t\in dy, R>t\},\qquad Q_t(x,dy)=P^x\{X_t\in dy, R>t\}.$$

Proof. By the Markov property of the measure $\hat{P}^x$, one has

$$P^{a,x}\{X_{t_1}\in dy_1,\ldots,X_{t_n}\in dy_n, R>r\}$$
$$=\hat{P}^{a,x}(1)^{-1}\,\hat{P}^x\{X_{t_1}\in dy_1\, R>t_1\}\,P^{y_1}\{X_{t_2-t_1}\in dy_2, R>t_2-t_1\}$$
$$\ldots P^{y_{n-1}}\{X_{t_n-t_{n-1}}\in dy_n, R>t_n-t_{n-1}\}\,P^{y_n}\{R>r-t_n\}.$$

Hence Corollary 2 follows from Theorem 2 and Corollary 1.

IV. Conditioning on $(A_T, \mathscr{F}_{G_T-})$

For each positive random variable S, $\mathscr{F}_{S-}$ denotes the σ-field of the sets $A\in\mathscr{F}$ such that $I_A=Z_S$ on $\{S<\infty\}$ for some predictable process Z. For example S is $\mathscr{F}_{S-}$-measurable. From now on we shall assume for simplicity that (X_t) is a *Hunt process*. Then X_{S-} is $\mathscr{F}_{S-}$-measurable (we set $X_{\infty-}=\delta$).

In the sequel we shall also assume that *M has (a.s.) no isolated point*, which implies the equivalence $G_t\leqq u \Leftrightarrow t\leqq D_u$ for all $t,u\geqq 0$. It follows that the process (Z_{G_t}) is predictable for all predictable Z. In fact if $Z_t=I_{\{t\leqq U\}}$, where U is a stopping time, $Z_{G_t}=I_{\{t\leqq D_U\}}$ is a predictable process, since D_U is a stopping time. This observation leads to the following result, whose proof is entirely analogous to the proof of Proposition 1.

Proposition 2. 1) *The family* $(\overline{\mathscr{F}}_t)=(\mathscr{F}_{G_t-})$ *is increasing.*

2) *For each $(\overline{\mathscr{F}}_t)$-stopping time T, one has* $\mathscr{F}_{G_T-}\subset\overline{\mathscr{F}}_T\subset\mathscr{F}_T$.

In this paragraph we shall further assume that there exists a "predictable exit system" $(\bar{B},\bar{P})$: $\bar{B}$ is a predictable random measure and $\bar{P}$ is a kernel from $(E,\mathscr{E}^*)$ to $(\Omega,\mathscr{F}^*)$ such that for all positive predictable Z and $\mathscr{F}^*$-measurable f

$$E^{\cdot}\Big[\sum_{s\in G} Z_s f\circ\theta_s\Big]=E^{\cdot}\left[\int_0^\infty Z_s\bar{P}^{X_s}(f)\,d\bar{B}_s\right].$$

Furthermore $\bar{P}^x(R=0)=0$ and $\bar{P}^x(1-e^{-R})\leqq 1$ for all x, and the process $(X_t)_{t>0}$ is strong Markov with respect to (P_t) for each measure $\bar{P}^x$. The existence of such a system is not always satisfied as Getoor and Sharpe showed in [2]. The reader will find sufficient conditions for this existence in the appendix.

Under all previous assumptions, one has the following results (the proofs are analogous to the proofs of paragraph 3 and left to the reader).

Theorem 3. *Let T be an $(\overline{\mathscr{F}}_t)$-stopping time. For all positive $\mathscr{F}^*$-measurable f, $\bar{P}^{A_T, X_{G_T-}}(1)>0$ a.s. on $\{0<G_T<T<\infty\}$ and*

$$E^\bullet[f\circ\theta_{G_T}|\overline{\mathscr{F}}_T]=\tilde{P}^{A_T, X_{G_T-}}(f) \quad \text{a.s. on} \quad \{0<G_T<T<\infty\} \tag{7}$$

where we set

$$\bar{P}^{a,x}(f)=\bar{P}^x(f I_{\{R>a\}}) \quad \text{if } a>0,$$

$$\tilde{P}^{a,x}(f)=\frac{\bar{P}^{a,x}(f)}{\bar{P}^{a,x}(1)}$$

Remarks. A_T is $\overline{\mathscr{F}}_T$-measurable since (A_t) is $(\overline{\mathscr{F}}_t)$-adapted and right continuous. Proposition 2, equality (7) and the fact that X_{G_T-} is $\mathscr{F}_{G_T-}$-measurable imply

$$E^\bullet[f\circ\theta_{G_T}|A_T, \mathscr{F}_{G_T-}]=\tilde{P}^{A_T, X_{G_T-}}(f) \quad \text{a.s. on} \quad \{0<G_T<T<\infty\}. \tag{7'}$$

Corollary 3. *With the notations of Corollary* 1, *one has*

$$E^\bullet[f\circ\theta_{G^a}|\mathscr{F}_{G^a-}]=\tilde{P}^{a, X_{G^a-}}(f) \quad \text{a.s. on} \quad \{0<G^a<\infty\}. \tag{8}$$

Remarks. 1) Corollary 3 follows from Theorem 3 by observing that T^a is an $(\overline{\mathscr{F}}_t)$-stopping time, and that $A_{T^a}=a$ on $\{G^a<\infty\}$. It can also be established directly along the lines of the remark following Corollary 1 (notice that, $Z_t=I_{\{t\leqq T^a\}}$ is predictable). This method has the advantage that *it does not require that M has no isolated point.*

2) In the case where $M=\overline{\{t: X_t=x_0\}}$, $X_{G^a-}=x_0$ a.s. on $\{0<G^a<\infty\}$, since M has no isolated point; therefore the strict past of G^a and the future of G^a are independent, as a consequence of Corollary 3.

Finally Corollary 2 can be rephrased for an $(\overline{\mathscr{F}}_t)$-stopping time by replacing X_{G_T}, X_{G^a}, $\hat{Q}_t$ by X_{G_T-}, X_{G^a-}, $\bar{Q}_t$, where $\bar{Q}_t(x, dy)=\bar{P}^x\{X_t\in dy, R>t\}$.

V. Appendix

In this section, we shall give sufficient conditions for the existence of a "predictable exit system". We assume for simplicity that (X_t) is a Hunt process, and that there exists a universally measurable subset S of F^c such that if $s\in G$, $X_{s-}\in F^c$, $X_s\in F^c$ then

$$X_{s-}\neq X_s \Leftrightarrow X_{s-}\in S.$$

Then one has the following.

Theorem 4. *Under the previous assumptions, there exists a predictable exit system $(\bar{B}, \bar{P})$ such that $\bar{P}^x(R=0)=0$, $\bar{P}^x(1-e^{-R})\leqq 1$ and $(X_t)_{t>0}$ is strong Markov with semi-group (P_t) relative to $\bar{P}^x$, for all x.*

Proof. The set G is the union of the following sets:

$^1G=\{s\in G: X_s\in F\}$,

$^2G=\{s\in G: X_{s-}\in F, X_s\in F^c\}$,

$^3G = \{s \in G \colon X_{s-} \in S, X_s \in F^c\}$,
$^4G = \{s \in G \colon X_{s-} \in S^c \cap F^c, X_s \in F^c\}$.

It is known that $^1G \cap [T] = \emptyset$ a.s. for each stopping time T, whereas $G \setminus {}^1G$ is a countable union of graphs of stopping times.

For each positive, bounded, $\mathscr{F}^*$-measurable function f, define

$$^iu_f = E^{\cdot}\Big[\sum_{s \in {}^iG} e^{-s}(1 - e^{-R_s}) f \circ \theta_s\Big], \qquad i = 1, 2, 3.$$

For each predictable stopping time T, one has $^iG \cap [T] = \emptyset$ a.s., $i = 1, 2, 3$, since $X_{s-} \neq X_s$ for $s \in {}^2G \cup {}^3G$. Hence the functions ^iu_f are regular 1-potentials. By the argument of [3], Theorem (4.1), there exist continuous additive functionals ^iK and kernels ^iN such that

$$^iu_f = E^{\cdot}\left[\int_0^\infty e^{-s}\, {}^iN^{X_s}(f)\, d\,{}^iK_s\right], \qquad i = 1, 2, 3$$

for all f. ^1K and ^2K are carried by F, whereas ^3K is carried by S. Let ^1d, ^2d be densities of ^1K, ^2K with respect to $^1K + {}^2K$ and set

$$\begin{aligned} \bar{P}^x(f) &= \sum_{i=1,2} {}^id(x)\, {}^iN^x(f/1 - e^{-R}) && \text{if } x \in F, \\ &= {}^3N^x(f/1 - e^{-R}) && \text{if } x \in S, \\ &= P^x(f) && \text{if } x \in S^c \cap F^c, \\ d\bar{B}_t &= \sum_{i=1,2,3} d\,{}^iK_t + \sum_{s \in {}^4G} \varepsilon_s(dt). \end{aligned}$$

The random measure $\bar{B}$ is predictable, since $X_{s-} = X_s$ for $s \in {}^4G$. By the same arguments as in [3], one shows that, except for x in a $\bar{B}$-null set N_0, one has $\bar{P}^x\{R = 0\} = 0$, $\bar{P}^x(1 - e^{-R}) \leq 1$ and the strong Markov property of $\bar{P}^x$ with respect to (P_t). If we change the definition of $\bar{P}$ by setting $\bar{P}^x = 0$, if $x \in N_0$, $(\bar{B}, \bar{P})$ is a predictable exit system with all the desired properties.

Remark. By construction $E^{\cdot}\left[\int_0^\infty e^{-s}\, \bar{P}^{X_s}(1 - e^{-R})\, d\bar{B}_s\right] \leq 1$. Therefore the process $\left(\int_0^t \bar{P}^{X_s}(1 - e^{-R})\, d\bar{B}_s\right)$ is a predictable additive functional with 1-potential ≤ 1.

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