Corrections to "An Existence Theorem for the Dirichlet Problem in the Elastodynamics of Incompressible Materials"

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We wish to correct several oversights in the proof of our theorem. The changes described below do not necessitate any modifications in the statement of the theorem. In addition to items 1 through 4, we note that in the sentence preceding (1.12) on page 97 it should read "$\ddot{y}$ must vanish" rather than "y must vanish".

1. In order to ensure the validity of the compatibility condition (C2′) on page 105, we must have $\tilde{u}_{(n)}(x, 0) = y_1(x)$. If $\tilde{u}_{(n)}$ is defined by (2.15) this relation is not necessarily satisfied. An alternative construction of $\tilde{u}_{(n)}$ that does satisfy $\tilde{u}_{(n)}(x, 0) = y_1(x)$ is given below.

2. In (S4′) on page 105, it should be added that the norm of H in $C^1([0, T]; L^2(\Omega)) \cap C([0, T]; H^1(\Omega))$ is bounded by U. In order to ensure that this bound is satisfied during the iteration, we must control the initial value of $\dot{\tilde{u}}_{(n)}$. The construction of $\tilde{u}_{(n)}$ given below achieves the required control.

3. The conditions $\dot{z}(x, 0) = z_1(x)$, $\dot{y}(x, 0) = y_1(x)$ should be added to (5.2) on p. 110. When M is sufficiently large an element of $Z(T, M)$ can be constructed by choosing z with the required properties and then solving (2.17) to find y.

4. In the sixth line from the bottom on page 108, $L^\infty([0, T]; H^{-1}(\Omega))$ should be replaced by $L^1([0, T]; H^{-1}(\Omega))$.

Alternative Construction of $\tilde{u}_{(n)}$

Choose a function χ satisfying

$$\chi \in \bigcap_{k=0}^{2} W^{k,\infty}([0, T]; H^{3-k}(\Omega) \cap H_0^1(\Omega)), \tag{1)$_1$ }$$

$$\chi(x, 0) = y_1(x), \quad \dot{\chi}(x, 0) = y_2(x), \tag{1)$_2$ }$$

and let

$$\alpha_{(n)}(x,t) := \dot{y}_{(n)}(x,t) - \chi(x,t),$$

$$\beta_{(n)}(x,t) := y_1(x) + \int_0^t z_{(n)}(x,\tau) + \lambda(y_{(n)}(x,\tau) - x)\,d\tau - \chi(x,t), \tag{2}$$

$$X := \{u \in L^\infty([0,T]; H^3(\Omega) \cap H_0^1(\Omega)) \cap W^{1,\infty}([0,T]; H^2(\Omega) \cap H_0^1(\Omega)) \mid u(x,0) = 0\},$$

$$Y := \{u \in W^{1,\infty}([0,T]; H^2(\Omega) \cap H_0^1(\Omega)) \cap W^{2,\infty}([0,T]; H_0^1(\Omega)) \mid u(x,0) = \dot{u}(x,0) = 0\},$$

$$Z := \left\{u \in \bigcap_{k=0}^{2} H^k([0,T]; H^{3-k}(\Omega) \cap H_0^1(\Omega)) \mid u(x,0) = \dot{u}(x,0) = 0\right\}. \tag{3}$$

We shall construct a continuous linear mapping $\Pi: X \times Y \to Z$ with the following properties:

(i) $\Pi(u,u) = u$ for every $u \in X \cap Y$.

(ii) The norm of Π has a bound which is independent of T as $T \to 0$.

(iii) Π is also continuous from $\tilde{X} \times \tilde{Y}$ to $\tilde{Z}$, where

$$\tilde{X} = \{u \in C([0,T]; H^2(\Omega) \cap H_0^1(\Omega)) \mid u(x,0) = 0\},$$

$$\tilde{Y} = \{u \in W^{1,\infty}([0,T]; H_0^1(\Omega)) \mid u(x,0) = 0\},$$

$$\tilde{Z} = \left\{u \in \bigcap_{k=0}^{1} H^k([0,T]; H^{2-k}(\Omega) \cap H_0^1(\Omega)) \mid u(x,0) = 0\right\}. \tag{4}$$

We then define $\tilde{u}_{(n)}$ by

$$\tilde{u}_{(n)} = \chi + \Pi(\alpha_{(n)}, \beta_{(n)}). \tag{5}$$

We now describe the construction of the operator Π. Let α be in either X or Y. Then we define a temporally periodic extension $E_1\alpha$ as follows:

$$E_1 a(x,t) = \begin{cases} \alpha(x,t) & \text{if } t \in [0,T], \\ 2\alpha(x,T) - \alpha(x,2T-t) & \text{if } t \in [T,2T], \\ E_1\alpha(x,-t) & \text{if } t \in [-2T,0], \end{cases}$$

$$E_1\alpha(x,t+4T) = E_1\alpha(x,t). \tag{6}$$

We note that the temporal average of $E_1\alpha$ is $\alpha(x,T)$. Let E_2 be an extension operator which maps $H^k(\Omega)$ into $H^k(\mathbb{R}^3)$ for $k = 1, 2, 3$. We choose E_2 to be independent of k.

Let $H_p^k(\mathbb{R}; V)$ denote the space of all $4T$-periodic functions $\mathbb{R} \to V$ with H^k-regularity, and let P be the orthogonal projection from $L_p^2(\mathbb{R}; H^2(\mathbb{R}^3)) \times H_p^1(\mathbb{R}; H^1(\mathbb{R}^3))$ onto the diagonal $L_p^2(\mathbb{R}; H^2(\mathbb{R}^3)) \cap H_p^1(\mathbb{R}; H^1(\mathbb{R}^3))$. The following properties P are easily verified:

(a) P is continuous from $(L_p^2(\mathbb{R}; H^3(\mathbb{R}^3)) \cap H_p^1(\mathbb{R}; H^2(\mathbb{R}^3))) \times (H_p^1(\mathbb{R}; H^2(\mathbb{R}^3)) \cap H_p^2(\mathbb{R}; H^1(\mathbb{R}^3)))$ onto $\bigcap_{k=0}^{2} H_p^k(\mathbb{R}; H^{3-k}(\mathbb{R}^3))$.

(b) If α and β are even functions of time, then so is $P(\alpha, \beta)$.

(c) If α and β are constant (in time), then so is $P(\alpha, \beta)$.

(d) If α and β have zero temporal average, then so does $P(\alpha, \beta)$.

Hence the temporal average of $P(\alpha, \beta)$ depends only on those of α and β. Let γ be the operator of evaluation at $t = 0$. By the trace theorem,

$$\gamma : \bigcap_{k=0}^{2} H_p^k(\mathbb{R}; H(\mathbb{R}^3)) \to H^{\frac{5}{2}}(\mathbb{R}^3) \tag{7$_1$}$$

is continuous. Moreover, the restriction of γ to functions of zero average has a norm that obeys a bound independent of T as $T \to 0$. Let E_3 be a right inverse of γ which maps

$$H^{\frac{5}{2}}(\mathbb{R}^3) \to \{u \in H^3(\mathbb{R}^3 \times [0, T]) \mid \dot{u}(x, 0) = 0\}. \tag{7$_2$}$$

Let R be the operator of restriction to $\Omega \times [0, T]$ and let $Q: H^1(\Omega) \to H_0^1(\Omega)$ be the solution operator $(f \to u)$ for the problem

$$\Delta u = \Delta f, \quad u|_{\partial\Omega} = 0. \tag{8}$$

We define

$$\Pi(\alpha, \beta) := QR\,(\mathrm{Id} - E_3\gamma)\, P(E_2 E_1 \alpha, E_2 E_1 \beta). \tag{9}$$

It is easy to verify that Π has the properties (i)–(iii) above.

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