# *Corrigendum to "Saint-Venant's Problem and Semi-Inverse Solutions in Nonlinear Elasticity"*

Archive for Rational Mechanics and Analysis **102**, 205–229 (1988)

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The mapping $Z: \mathscr{D} \to \mathscr{D}, w \to w - \mathscr{R}G(w)$, given on page 211 is not appropriate for the purposes needed below. It is falsely stated that the derivative $DZ(w)$ can be continued to a bounded operator from $X_s$ into itself, since the trace mapping $\tau: H^s(\Sigma) \to H^{s-1/2}(\partial\Sigma), u \to u|_{\partial\Sigma}$, is only bounded for $s > \frac{1}{2}$ (see [11]), whereas the present context would need $s \in (0, \frac{1}{2})$.

I am grateful to PIUS KIRRMANN for informing me of this serious error.

To close this gap we construct the desired mapping $Z$ by circumvening the use of traces in $H^s(\Sigma)$. Therefore we define a function $\tilde{G}: \mathscr{D} \to [H^{s+1}(\Sigma)]^3$ with $\tilde{G}(w)|_{\partial\Sigma} = G(w)$ for all $w \in \mathscr{D}$, such that its derivative $D\tilde{G}(w)$ can be continued to a bounded operator from $X_s$ into $[H^s(\Sigma)]^3$. Moreover a linear operator $\tilde{\mathscr{R}}$: $[H^{s+1}(\Sigma)]^3 \to \mathscr{D}$ will be constructed which is also bounded from $[H^s(\Sigma)]^3 \to X_s$. Of course this is done such that $\mathscr{B}\tilde{\mathscr{R}}\tilde{G}(w) = G(w)$ for all $w \in \mathscr{D}$, and hence $\mathscr{B}Z(w) = 0$ if and only if $\mathscr{B}w = G(w)$.

Remarkably, the following constructions make the use of non-zero traction boundary conditions in Section 5 superfluous. In particular, Theorem 2.1 can be dropped completely.

To define $\tilde{G}$ we continue the normal vector $n$ from $\partial\Sigma$ into $\Sigma$. Call this vector field $\tilde{n}$ and note that it can be chosen in $\mathscr{C}^2(\bar{\Sigma})$, since $\partial\Sigma$ is of class $\mathscr{C}^3$. Now let

$$\tilde{G}(w) = (\tilde{S}(E(U)) - \lambda(u_{1,1} + u_{2,2} + v_3)\, I - \mu(U + U^t)) \begin{pmatrix} \tilde{n}_1(x) \\ \tilde{n}_2(x) \\ 0 \end{pmatrix},$$

then all the aforementioned properties hold.

For the construction of the linear operator $\tilde{\mathscr{R}}$ we rewrite the boundary conditions $\mathscr{B}w = G(w)$ in the form

$$\mu(u_{1,2} + u_{2,1})\,(n_2^2 - n_1^2) + \mu(u_{1,1} - u_{2,2})\, 2n_1n_2 = \tilde{g}_t := \tilde{G}_1\tilde{n}_2 - \tilde{G}_2\tilde{n}_1, \qquad (1)$$

$$\lambda(u_{1,1} + u_{2,2} + v_3) + 2\mu(u_{1,1}n_1^2 + u_{2,2}n_2^2) = \tilde{g}_n := \tilde{G}_1\tilde{n}_1 + \tilde{G}_2\tilde{n}_2, \qquad (2)$$

$$\mu(u_{3,1} + v_1)\, n_1 + \mu(u_{3,2} + v_2)\, n_2 = \tilde{g}_3 := \tilde{G}_3. \qquad (3)$$

Note that these relations have only to be satisfied on $\partial\Sigma$; hence there is a lot of freedom for choosing $w$ inside of $\Sigma$. As $\mu > 0$, equation (3) is easily solved with $(u_3, v_1, v_2) \in H^{s+2}(\Sigma) \times [H^{s+1}(\Sigma)]^2$ by setting

$$u_3 = 0, \quad v_i = \frac{1}{\mu}\tilde{g}_3\tilde{n}_i \quad \text{for } i = 1, 2.$$

To solve (1) we take the unique solution $\phi \in H^{s+3}(\Sigma)$ of

$$\triangle\phi = \frac{1}{\mu}\tilde{g}_t \quad \text{in } \Sigma, \quad \phi = 0 \quad \text{on } \partial\Sigma,$$

and define $(u_1, u_2)^t = A(x)\,\nabla\phi$. A lengthy but elementary calculation shows that (1) holds whenever $A = A(x)$ satisfies the relations

$$A(x) = \begin{pmatrix} 2n_1n_2 & n_2^2 - n_1^2 \\ n_2^2 - n_1^2 & -2n_1n_2 \end{pmatrix}, \qquad \frac{\partial A_{i1}}{\partial n} n_2 + \frac{\partial A_{i1}}{\partial t} n_1 - \frac{\partial A_{i2}}{\partial n} n_1 + \frac{\partial A_{i2}}{\partial t} = 0,$$
$$\text{for } i = 1, 2,$$

on $\partial\Sigma$ (where $\partial/(\partial n) = n_1\,\partial/(\partial x_1) + n_2\,\partial/(\partial x_2)$ and $\partial/(\partial t) = n_2\,\partial/(\partial x_1) - n_1\,\partial/(\partial x_2)$). However, such functions $A_{ij} \in H^{s+2}(\Sigma)$ can be constructed easily. Thus we have defined a linear operator which maps $\tilde{g}_t \in H^{s+2}(\Sigma)$ into $(u_1, u_2) \in [H(\Sigma)]^2$ such that (1) is valid.

Inserting the function $(u_1, u_2)$ from above into (2) gives a unique $v_3 \in H^{s+1}(\Sigma)$, as $\lambda > 0$. Altogether this shows the existence of the desired operator $w = (u, v) = \tilde{\mathscr{R}}\tilde{g}$ from $[H^{s+1}(\Sigma)]^3$ into $\mathscr{D}$. Obviously, $\tilde{\mathscr{R}} : [H^s(\Sigma)]^3 \to X_s$ is continuous also.

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*(Received June 26, 1989)*