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Our proof of Theorem 1 of [3] p. 107 is not correct since the claim (iii) of Proposition 1 is false. Moreover it is not clear whether the space Cart $(\Omega, \mathbb{R}^N)$ in Definition 1 is closed. Because of this, to minimize functionals in Cart $(\Omega, \mathbb{R}^N)$ becomes unreasonable.

Our idea was to consider the smallest closed set with respect to weak convergence in $\mathscr{C}$ containing the graphs of regular mappings. According to that, the definition of Cart $(\Omega, \mathbb{R}^N)$ has to be changed into the following

Definition 1. Cart $(\Omega, \mathbb{R}^N)$ *is the smallest sequentially closed set in $\mathscr{C}$ (with respect to weak convergence of sequences in $\mathscr{C}$) containing the graphs of C^1 mappings from $\bar{\Omega}$ into $\mathbb{R}^N$.*

Recalling that $T_k \rightharpoonup_{\mathscr{C}} T$ if and only if $\{T_k\}$ has equibounded Cart-norm and $T_k \rightharpoonup T$ in the sense of currents, we can say: Cart $(\Omega, \mathbb{R}^N)$ *is the sequential closure of C^1 graphs with respect to the convergence of currents with equibounded Cart-norms*; in brief, we shall say that Cart $(\Omega, \mathbb{R}^N)$ is the weak sequential closure in $\mathscr{C}$ of C^1 graphs.

Let M be any subset of $\mathscr{C}$ and let ω_1 be the first uncountable ordinal. Classically, compare *e.g.* [1], [4], one can define by transfinite induction the weak sequential closure of M as

$$\text{sw} - \text{cl}\,(M) := \bigcup_{\alpha < \omega_1} M^{(\alpha}$$

where

$$M^{(0)} = M$$

and $M^{(\alpha)}$, for $\alpha \neq 0$, is defined as follows: if α is a limit ordinal, then

$$M^{(\alpha)} := \bigcup_{\beta < \alpha} M^{(\beta)};$$

if $\alpha = \beta + 1$, then

$$M^{(\alpha)} := \{T \in \mathscr{C} \mid \exists \{T_k\} \subset M^{(\beta)} \text{ with } T_k \underset{\mathscr{C}}{\rightharpoonup} T\}.$$

By definition we have

$$\text{Cart}\,(\Omega, \mathbb{R}^N) = \text{sw} - \text{cl (graphs of } C^1 \text{ mappings)}.$$

The proofs of the properties of cartesian currents in [3] can be now reread as the first step of the transfinite inductive proof of the same properties for the elements of our newly defined space Cart $(\Omega, \mathbb{R}^N)$. Since the second step of the transfinite induction goes on essentially in the same way, the old proof provides in fact a complete proof of Theorems 2 and 3, where Cart $(\Omega, \mathbb{R}^N)$ has to be understood in the sense of Definition 1 above. Of course, *a priori* we do not know whether a current T in Cart $(\Omega, \mathbb{R}^N)$ can be approximated weakly in $\mathscr{C}$ by a sequence of smooth graphs.

The previous remarks apply to the other spaces defined in [3]: Dif $(\Omega, \hat{\Omega})$, Dif (Ω), Cart^p $(\Omega, \mathbb{R}^N)$, Dif$^{p,q}(\Omega, \hat{\Omega})$, Dif^p (Ω), Dif^{p,q} (Ω). Their definitions should be modified accordingly by considering the sequential weak closure in the sense of currents with equibounded relative norms of the respective classes of graphs of smooth mappings or diffeomorphisms.

Actually, following our Remark 1, p. 110 of [3] all properties of the elements of Cart $(\Omega, \mathbb{R}^N)$, and moreover, of all other spaces, can be recovered by showing that the same properties are enjoyed by the elements of the analogous spaces with small initial letters. Since this is not always evident from [3], we shall now supply a few precise statements and proofs we think are needed.

We recall that

$$\text{cart}\,(\Omega, \mathbb{R}^N) := \{T \in \mathscr{R}_n\,(U) \mid \|T\|_{\mathscr{C}} < +\infty, \, p_\# T = [\![\Omega]\!], \quad T_{\bar{0}0} \geqq 0, \quad \partial T = 0\}.$$

The following theorem contains essentially Theorems 1 and 2, Section 3 of [3].

Theorem 1. *Let* $T = \tau(\mathcal{M}, \theta, \xi) \in \text{cart}\,(\Omega, \mathbb{R}^N)$.

(i) *For a.e.* $x \in \Omega$, $p^{-1}(x) \cap \mathcal{M}_+$ *contains exactly one element, denoted by* $(x, \tilde{u}(x))$, *and* $\theta = 1$ $\mathscr{H}^n$ *a.e. in* $\mathcal{M}_+$.

(ii) $M_{\beta\bar{\alpha}}(T) := \sigma_\alpha p_\# T_{\alpha\beta}$ *are measures in* Ω; $u^j(T) := p_\#(T_{\bar{0}0} \, \llcorner \, y^j)$ *is absolutely continuous with respect to Lebesgue measure,* $u^j(T) =: u_T^j(x)\,dx$; *moreover,* $u_T \in \text{BV}(\Omega)$ *and*

$$\|u_T\|_{L^1(\Omega)} = \|T\|_{L^1}.$$

(iii) *For all bounded functions* $\phi(x, y) \in C^0(U)$

$$T_{\bar{0}0}(y^j\phi(x, y)) = \int_\Omega \phi(x, u_T(x))\, u_T^j(x)\, dx$$

and $\tilde{u} = u_T$ *a.e. in* Ω.

(iv) *The map* $T \to u_T$ *defines a continuous immersion of* cart $(\Omega, \mathbb{R}^N)$ *into* BV $(\Omega, \mathbb{R}^N)$ *in the sense that*

$$\|u_T\|_{\text{BV}(\Omega, \mathbb{R}^N)} \leqq \|T\|_{\text{Cart}(\Omega, \mathbb{R}^N)} \quad \forall\, T \in \text{cart}\,(\Omega, \mathbb{R}^N).$$

If $T_k \underset{\mathscr{C}}{\rightharpoonup} T$, *then* $u_{T_k} \to u$ *in* $L^1(\Omega, \mathbb{R}^N)$ *and* $Du_{T_k} \rightharpoonup Du_T$ *in the sense of measures.*

(v) *The class* cart $(\Omega, \mathbb{R}^N)$ *is weakly sequentially closed in* $\mathscr{C}$; *thus*

$$\text{Cart} \, (\Omega, \mathbb{R}^N) \subset \text{cart} \, (\Omega, \mathbb{R}^N).$$

Proof. We need only to prove (iii) and (v) since the rest has already been proved in Theorems 2 and 3, Section 3 of [3]. From the area formula we have

$$T_{\bar{0}0}(y^j \phi(x, y)) = \int_\Omega \phi(x, \tilde{u}_T(x)) \, \tilde{u}_T^j(x) \, dx \tag{1}$$

for all $\phi \in C_c^0(U)$. Since $\|T\|_{L^1} < +\infty$, we get $\|\tilde{u}\|_{L^1(\Omega)} < +\infty$; hence (1) holds for all continuous and bounded ϕ. In particular we deduce

$$u^j(T) \, (\phi(x)) =: \int_\Omega \phi(x) \, u_T^j(x) \, dx = \int_\Omega \phi(x) \, \tilde{u}^j(x) \, dx$$

for all $\phi \in C_c^0(\Omega)$, i.e., $\tilde{u} = u_T$. This proves (iii). In order to prove (v) we observe that

$$\|T\|_{L^s} = \left(\int_{\mathscr{M}} |q(z)|^s \, \xi_{\bar{0}0}(z) \, \theta(z) \, d\mathscr{H}^n(z) \right)^{1/s} \quad s \geq 1;$$

thus

$$|T_{\bar{0}0}(\phi(x))| \leq \frac{1}{R} \|T\|_{L^1}$$

for $\phi \in \mathscr{F}_R := \{\phi \in C^0(U) \mid |\phi| \leq 1, \text{spt} \, \phi \subset \Omega \times (\mathbb{R}^n \setminus B_R)\}$. Let $\phi \in C^0(U)$, $|\phi| \leq 1, p \, (\text{spt} \, \phi) \subset\subset \Omega, R > 0$. We split ϕ into $\phi = \phi_1 + \phi_2$ with $\phi_1 \in C_c^0(U)$, $|\phi| \leq 1$ and $\phi_2 \in \mathscr{F}_R$. As

$$\sup_k |T_{k\bar{0}0}(\phi_2)| \leq \frac{1}{R} \sup_k \|T_k\|_{L^1},$$

we conclude that $T_{k\bar{0}0}(\phi) \to T_{\bar{0}0}(\phi)$. This proves (iv), since obviously $T_{\bar{0}0} \geq 0$, $\partial T = 0$, and $T \in \mathscr{R}_n(U)$ by the Federer-Fleming closure theorem. $\square$

As a consequence of Theorem 1, one easily sees that (except for (i)$_1$ and with (ii) replaced by spt $T \subset \bar{\Omega} \times \bar{\hat{\Omega}}$) Theorem 4, p. 114 of [3] holds for $T \in \text{dif} \, (\Omega, \hat{\Omega})$, where

$$\text{dif} \, (\Omega, \hat{\Omega}) := \{T \in \mathscr{R}_n(U) \mid M(T) < +\infty, p_\# T = [\![\Omega]\!], q_\# T$$

$$= [\![\hat{\Omega}]\!], \xi_{\bar{0}0} \geq 0, \xi_{0\bar{0}} \geq 0, \partial T = 0\}.$$

Also, dif $(\Omega, \hat{\Omega})$ is closed with respect to the weak convergence of currents with equibounded masses. As the weak sequential closure of Diff $(\Omega, \hat{\Omega})$ with respect to $\underset{\mathscr{C}}{\rightharpoonup}$ in $\{T \in \mathscr{D}_n(U) \mid M(T) < +\infty\}$, denoted by Dif $(\Omega, \hat{\Omega})$, is contained in dif $(\Omega, \hat{\Omega})$, we also see at once that Theorem 4 holds for $T \in \text{Dif} \, (\Omega, \hat{\Omega})$, too.

The set of cartesian currents in $\Omega \times \mathbb{R}^n$ satisfying

$$\xi_{0\bar{0}} \geqq 0, \tag{2}$$

$$T_{0\bar{0}}(\phi(x, y)) \leqq \int_{\mathbb{R}^n_y} \max_{x \in \Omega} \phi(x, y)\, dy \quad \forall\, \phi \in C^0_c(U),\, \phi \geqq 0 \tag{3}$$

is denoted by dif (Ω). We mention that (2) and (3) essentially appear also in [2].

Proposition 1. *Let* $T = \tau(\mathcal{M}, \theta, \xi) \in \text{dif}\,(\Omega)$ *and*

$$\mathcal{M}^+ := \{z \in \mathcal{M} \mid \xi_{0\bar{0}}(z) \geqq 0\}$$

(i) *For a.e.* $y \in q(\mathcal{M}^+)$ *there exists exactly one* $z \in q^{-1}(y) \cap \mathcal{M}^+$, *denoted by* $(\hat{u}_T(y), y)$; *moreover* $\mathcal{H}^n(q(\mathcal{M}) \setminus q(\mathcal{M}^+)) = 0$, *and* $\theta = 1$ *on* $\mathcal{M}^+$ *and*

$$\mathcal{M}^+ = \{(\hat{u}(y), y) \mid y \in q(\mathcal{M}^+)\}$$

hold $\mathcal{H}^n$ *a.e.*
(ii) dif (Ω) *is closed with respect to the weak* $\mathscr{C}$-*convergence.*

Proof. (ii) is trivial. Let us prove (i). From (3) we easily deduce that

$$T_{0\bar{0}}(\phi(y)) \leqq \int_{\mathbb{R}^n_y} \phi(y)\, dy \quad \forall\, \phi \in C^0_c(\mathbb{R}^n_y),\, \phi \geqq 0,$$

while from the area formula we get for $\phi \geqq 0$

$$T_{0\bar{0}}(\phi(y)) = \int_{\mathbb{R}^n_y} \phi(y) \sum_{z \in q^{-1}(y) \cap \mathcal{M}^+} \theta(z)\, dy \leqq \int_{\mathbb{R}^n_y} \phi(y)\, dy.$$

Thus $\Sigma_{z \in q^{-1}(y) \cap \mathcal{M}^+}\, \theta(z)$ is either 0 or 1 for a.e. $y \in \mathbb{R}^n_y$; as $\theta \geqq 1$ on $\mathcal{M}$, (i) follows. $\square$

We observe that (3) is obviously satisfied for $u \in \text{Diff}\,(\Omega)$, as it amounts to

$$\int_{\Omega} \det Du(x)\, dx \leqq |u(\Omega)|.$$

Consequently, the weak sequential closure in $\mathscr{C}$ of Diff (Ω), denoted Dif (Ω), is contained in dif (Ω).

Let us now discuss the classes $\text{cart}^p(\Omega, \mathbb{R}^N)$ and $\text{Cart}^p(\Omega, \mathbb{R}^N)$, $p > 1$. We set

$$\text{cart}^p(\Omega, \mathbb{R}^N) = \{T \in \text{cart}\,(\Omega, \mathbb{R}^N) \mid \|T\|_{\mathscr{C}^p} < +\infty\}.$$

One easily sees that the $\mathscr{C}^p$-norm in [3], also denoted $\| \, \|_{\text{Cart}^p(\Omega, \mathbb{R}^N)}$, is given by

$$\|T\|_{\text{Cart}^p(\Omega, \mathbb{R}^N)} := \|T\|_{L^p} + \|T\|_{M^p}$$

where for $T = \tau(\mathscr{M}, \theta, \xi)$

$$\|T\|_{L^p} := \left(\int_{\mathscr{M}} |q(z)|^p \, \xi_{\bar{0}0}(z) \, \theta(z) \, d\mathscr{H}^n(z) \right)^{1/p},$$

$$\|T\|_{M^p} := \left(\int_{\mathscr{M}} \xi_{\bar{0}0}^{1-p} \, \theta(z) \, d\mathscr{H}^n(z) \right)^{1/p}.$$

The following theorem provides a version of Theorem 1 in [3], p. 120 for $T \in \text{cart}^p (\Omega, \mathbb{R}^N)$.

Theorem 2. *Let* $T = \tau(\mathscr{M}, \theta, \xi)$ *and* $\{T_k\}$ *belong to* $\text{cart}^p (\Omega, \mathbb{R}^N)$, $p > 1$. *Then we have*:

(i) $\xi_{\bar{0}0} > 0$ $\mathscr{H}^n$-*a.e. in* $\mathscr{M}$, $\mathscr{M} = \mathscr{M}_+$ *and* $\theta = 1$ $\mathscr{H}^n$ *a.e. Also the measures* $M_{\beta\alpha}^-(T) := \sigma_\alpha \, p_\# T_{\alpha\beta}$ *are absolutely continuous with respect to Lebesgue measure, and*

$$T_{\alpha\beta}(\phi(x, y)) = \sigma_\alpha \int_\Omega \phi(x, u_T(x)) \, M_{\beta\alpha}^-(T) \, (x) \, dx \tag{4}$$

for all bounded Borel functions $\phi(x, y)$.

(ii) *Moreover*

$$M_{\beta\alpha}^-(T) \, (x) = M_{\beta\alpha}^-(Du_T(x)) \quad \text{for a.e. } x \in \Omega \tag{5}$$

and

$$\|T\|_{\text{Cart}^p(\Omega, \mathbb{R}^N)} = \|u_T\|_{L^p(\Omega)} + \|M(Du_T)\|_{L^p(\Omega)}. \tag{6}$$

(iii) $T_k \underset{\mathscr{C}^p}{\rightharpoonup} T$ *if and only if* $u_{T_k} \to u_T$ *in* L^p *and* $M_{\beta\alpha}^-(Du_{T_k}) \rightharpoonup M_{\beta\alpha}^-(Du_T)$ *weakly in* L^p.

(iv) $\text{cart}^p (\Omega, \mathbb{R}^N)$ *is closed with respect to the weak convergence in* $\mathscr{C}^p$.

(v) *If* $\sup_k \|T_k\|_{\text{Cart}^p(\Omega, \mathbb{R}^N)} < +\infty$ *and* $u_{T_k} \rightharpoonup w$ *weakly* in* L^1, *then there exists* $T \in \text{cart}^p (\Omega, \mathbb{R}^N)$ *with* $u_T = w$ *and* $T_k \underset{\mathscr{C}^p}{\rightharpoonup} T$.

Proof. From

$$\int_{\mathscr{M}} \xi_{\bar{0}0}^{1-p} \, d\mathscr{H}^n < +\infty$$

we get $\xi_{\bar{0}0} > 0$ a.e. in $\mathscr{M}$; thus $\mathscr{M}_+ = \mathscr{M}$ and $\theta = 1$ $\mathscr{H}^n$-a.e. From the area formula, as in Theorem 1, we deduce that for all $\phi \in C_c^0(U)$

$$T_{\alpha\beta}(\phi(x, y)) = \int_{p_\#(\mathscr{M}_+)} \phi(x, u_T(x)) \frac{\xi_{\alpha\beta}(x, u_T(x))}{\xi_{\bar{0}0}(x, u_T(x))} \, dx. \tag{7}$$

As

$$\frac{\xi_{\alpha\beta}(x, u_T(x))}{\xi_{\bar{0}0}(x, u_T(x))} \in L^1(\Omega),$$

we deduce that (7) holds for all bounded Borel functions $\phi(x, y)$, and therefore

$$\sigma_\alpha M_{\beta\alpha}(T)\,(x) = \frac{\xi_{\alpha\beta}(x,\,u_T(x))}{\xi_{\bar{0}0}(x,\,u_T(x))} \qquad \mathscr{H}^n\text{-a.e. in } \Omega.$$

On the other hand, Theorem 1 yields

$$M_{ji}(T)\,(x) = D_i u_T^j(x),$$

thus (5) follows since T is rectifiable (compare [3], p. 122). Finally,

$$\int_{\mathscr{M}} \xi_{00}^{1-p}\, d\mathscr{H}^n = \int_\Omega \xi_{\bar{0}0}(x,\, u_T(x))^{-p}\, dx = \int_\Omega |M(Du_T)|^p\, dx,$$

and so we have proved (i) and (ii). The claims (iii), (iv), (v) follow from (4) and Theorem 1 (compare also [3], pp. 122–123). □

If we now redefine $\mathrm{Cart}^p\,(\Omega, \mathbb{R}^N)$, $p > 1$, as the weak sequential closure in $\mathscr{C}^p$ of C^1 graphs, we see at once that Theorem 1, p. 120 of [3] is correct. In fact $\mathrm{Cart}^p\,(\Omega, \mathbb{R}^N)$ is then a sequentially closed subset of $\mathrm{cart}^p\,(\Omega, \mathbb{R}^N)$, thus Theorem 2 above holds in $\mathrm{Cart}^p\,(\Omega, \mathbb{R}^N)$, too. Again, the approximation property by smooth graphs cannot be in general deduced; hence Remark 1, p. 123 of [3] is not correct.

Finally let us reconsider the class of weak diffeomorphisms. It is easily seen that Theorem 2 of [3], p. 125 holds for T in

$$\mathrm{dif}^{p,q}\,(\Omega,\,\hat{\Omega}) := \{T \in \mathrm{dif}\,(\Omega,\,\hat{\Omega}) \mid \|T\|_{\mathscr{D}^{p,q}} < +\infty\}$$

provided we replace the spaces in the statement with the same spaces with small initial letters. Obviously $\mathrm{dif}^{p,q}\,(\Omega,\,\hat{\Omega})$ is closed with respect to weak convergence in $\mathscr{D}^{p,q}$. Consequently, Theorem 2 also holds in $\mathrm{Dif}^{p,q}\,(\Omega,\,\hat{\Omega})$ defined as the weak sequential closure of $\mathrm{Diff}\,(\Omega,\,\hat{\Omega})$ in $\mathscr{D}^{p,q}$. We finally observe that for $T = \tau(\mathscr{M}, \theta, \xi)$

$$\|T\|_{\mathscr{D}^{p,q}} = \|T\|_{\mathrm{Cart}^p(\Omega,\mathbb{R}^n)} + \left(\int_{\mathscr{M}} \xi_{0\bar{0}}^{1-q} \theta \, d\mathscr{H}^n \right)^{1/q}$$

Now, for $p, q > 1$, set

$$\mathrm{dif}^p\,(\Omega) := \{T \in \mathrm{dif}\,(\Omega) \mid \|T\|_{\mathrm{Cart}^p(\Omega,\mathbb{R}^n)} < +\infty\},$$

$$\mathrm{dif}^{p,q}\,(\Omega) := \{T \in \mathrm{dif}\,(\Omega) \mid \|T\|_{\mathscr{D}^{p,q}} < +\infty\}.$$

By definition, these spaces are sequentially weakly closed respectively for $\underset{\mathscr{C}^p}{\rightharpoonup}$ and $\underset{\mathscr{D}^{p,q}}{\rightharpoonup}$. According to the above, we redefine $\mathrm{Dif}^p\,(\Omega)$ and $\mathrm{Dif}^{p,q}\,(\Omega)$ as the weak sequential closure of $\mathrm{Diff}\,(\Omega)$ respectively for weak convergence in $\mathrm{dif}\,(\Omega) \cap \mathrm{Cart}^p\,(\Omega, \mathbb{R}^n)$ and in $\mathscr{D}^{p,q}$. Then we clearly have

$$\mathrm{Dif}^p\,(\Omega) \subset \mathrm{dif}^p\,(\Omega),$$

$$\mathrm{Dif}^{p,q}\,(\Omega) \subset \mathrm{dif}^{p,q}\,(\Omega). \tag{8}$$

The two following theorems are a slightly stronger version of Theorems 3 and 4 of [3] Section 4, and, because of (8), they hold respectively in $\text{Dif}^p(\Omega)$ and $\text{Dif}^{p,q}(\Omega)$ too.

Theorem 3. *Let* $T = \tau(\mathcal{M}, \theta, \xi)$ *and* $\{T_k\}$ *belong to* $\text{dif}^p(\Omega)$.
(i) *Set* $u_T(\Omega) := q(\mathcal{M})$. *Then we have*

$$q_{\#}T = [\![u_T(\Omega)]\!]$$

and for all bounded Borel functions $\phi(y)$ *in* $\mathbb{R}^n_y$

$$\int\limits_{u_T(\Omega)} \phi(y)\, dy = \int\limits_{\Omega} \phi(u_T(x))\, \det Du_T(x)\, dx.$$

(ii) $|u_T(\Omega)| \leq |\Omega|^{1/p'} \|T\|_{\text{Cart}^p(\Omega,\mathbb{R}^n)}$.
(iii) *If* $T_k \rightharpoonup T$, *then*

$$\mathscr{H}^n(u_{T_k}(\Omega) \triangle u_T(\Omega)) = 0$$

where $A \triangle B$ *denotes the symmetric difference of* A *and* B.

Proof. Using the area formula and Proposition 1, we have

$$\int\limits_{u_T(\Omega)} \phi(y)\, dy = \int\limits_{\mathcal{M}^+} \phi(y)\, \xi_{0\bar{0}}(x,y)\, d\mathscr{H}^n(x,y)$$

$$= \int\limits_{\mathcal{M}} \phi(y)\, \frac{\xi_{0\bar{0}}(x,y)}{\xi_{\bar{0}0}(x,y)}\, \xi_{\bar{0}0}(x,y)\, d\mathscr{H}^n(x,y) = \int\limits_{\Omega} \phi(u_T(x))\, \det Du_T(x)\, dx.$$

Hence (i) follows.
   (ii) Using the Hölder inequality, we get

$$|u_T(\Omega)| = \int\limits_{\mathcal{M}} \xi_{0\bar{0}}\, d\mathscr{H}^n \leq \left(\int\limits_{\mathcal{M}} \xi_{\bar{0}0}\, d\mathscr{H}^n\right)^{\frac{1}{p'}} \left(\int\limits_{\mathcal{M}} \frac{\xi_{0\bar{0}}^p}{\xi_{\bar{0}0}^{p-1}}\, d\mathscr{H}^n\right)^{\frac{1}{p}} \leq |\Omega|^{\frac{1}{p'}} \|T\|_{\text{Cart}^p(\Omega,\mathbb{R}^n)}.$$

(iii) From Theorem 2 we have $q_{\#}T_k(\phi) \to q_{\#}T(\phi)$ for all continuous and bounded ϕ; as $q_{\#}T = [\![u_T(\Omega)]\!]$ we can conclude

$$\int \phi(y)\, \chi_{u_{T_k}(\Omega)}\, dy \to \int \phi(y)\, \chi_{u_T(\Omega)}\, dy \quad \forall\, \phi \in C^0(\mathbb{R}^n_y),\ \phi \text{ bounded.}$$

Using this for $\phi = 1$, we get $|u_{T_k}(\Omega)| \to |u_T(\Omega)|$. Because $\|\chi_{u_T(\Omega)}\|_{L^2} = |u_T(\Omega)|$ and that $\chi_{u_{T_k}(\Omega)} \rightharpoonup \chi_{u_T(\Omega)}$ in L^2, (iii) follows at once. $\quad\square$

Theorem 4. *Let* $T = \tau(\mathcal{M}, \theta, \xi)$ *and* T_k *belong to* $\text{dif}^{p,q}(\Omega)$, $p, q > 1$.
(i) *We have* $\xi_{0\bar{0}} > 0$ *a.e. in* $\mathcal{M}$. *Also*

$$\mathscr{H}^n(u_T(\Omega)) \geq \mathscr{H}^n(\Omega)^{q'} \|T\|_{\mathscr{D}^{p,q}}^{-1}.$$

(ii) *Let* $\hat{u}_T$ *be the function defined in Proposition 1. For a.e.* $y \in u_T(\Omega)$ *we define the derivatives* $D\hat{u}_T$ *by*

$$D_j\hat{u}_T^i(y) := \frac{\xi_{i\bar{j}}(\hat{u}_T(y), y)}{\xi_{0\bar{0}}(\hat{u}_T(y), y)}.$$

Then we have $\hat{u}_T, \, D\hat{u}_T \in L^q(\hat{\Omega}_T)$.

(iii) *Finally*

$$D\hat{u}_T(u_T(x)) \; Du_T(x) = \mathbb{1}\big|_{\mathbb{R}^n}, \qquad Du_T(\hat{u}_T(y)) \; D\hat{u}_T(y) = \mathbb{1}\big|_{\mathbb{R}^n}$$

$$M_{\alpha\bar{\beta}}(D\hat{u}_T(y)) = \sigma_\alpha \sigma_\beta \, \frac{M_{\beta\bar{\alpha}}(Du_T(\hat{u}_T(y)))}{\det Du_T(\hat{u}_T(y))}$$

for a.e. $y \in u_T(\Omega)$, *and*

$$T_{\alpha\beta}(\phi) = \sigma_\beta \int\limits_{u_T(\Omega)} \phi(\hat{u}_T(y), y) \, M_{\alpha\bar{\beta}}(D\hat{u}_T(y)) \; dy$$

holds for all bounded Borel functions in U.

Proof. From

$$\int\limits_{\mathscr{M}} \xi_{0\bar{0}}^{1-q}\theta \; d\mathscr{H}^n < +\infty$$

we deduce that $\xi_{0\bar{0}} > 0$ a.e. in $\mathscr{M}$. As $\theta = 1$ a.e. in $\mathscr{M}$

$$|\Omega| = \int\limits_{\mathscr{M}} \xi_{\bar{0}0} \; d\mathscr{H}^n \leqq \left(\int\limits_{\mathscr{M}} \xi_{0\bar{0}} \; d\mathscr{H}^n\right)^{1/q'} \left(\int\limits_{\mathscr{M}} \frac{\xi_{\bar{0}0}^q}{\xi_{0\bar{0}}^{q-1}} \; d\mathscr{H}^n\right)^{1/q}$$

$$\leqq |u_T(\Omega)|^{1/q'} \, \|T\|_{\mathscr{D}^{p,q}}.$$

The proof can now be completed as in Theorem 2. $\quad\square$

With the new definitions of the spaces given in this erratum, all statements and proofs of [3], Sections 5, 6, 7 hold true with the same proofs.

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