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Correction to: A Boundary Property of Semimartingale Reflecting Brownian Motions

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The definition in Section 1 of a semimartingale reflecting Brownian motion should include the additional assumption that the Brownian motion $\{X(t)-\theta t, \geq 0\}$ is a martingale relative to the filtration $\{\mathscr{F}_t\}$ under each P_z. This was implicitly assumed in the proof of Theorem 1, in claiming that the integral with respect to dX in Eq. (25) has zero expectation and in applying the Girsanov transformation in Lemma 6. If Y (and hence Z) is adapted to X then the above assumption can be automatically satisfied by choosing the filtration $\{\mathscr{F}_t\}$ to be the natural one generated by X. This is the case for the reflecting Brownian motions constructed by Harrison and Reiman [3] and subsequently analyzed in [4]. In general it is an open problem as to whether such adaptedness holds.

In the proof of Lemma 6, a probability measure P_z^0 as introduced there need not always exist on $(\Omega, \mathscr{F})$. However, assuming X has the additional property described above, it is true that for each m, there is a probability measure P_z^0 on $(\Omega, \mathscr{F}_m)$ (given by the Girsanov transformation) that is equivalent to P_z^θ on $\mathscr{F}_m$ and is such that under P_z^0, $\{X(t), \mathscr{F}_t, t\in[0,m]\}$ is a martingale with mutual variation: $\langle X_i, X_j\rangle_t = \Gamma_{ij}t$, $t\in[0,m]$ (and hence is a $(0,\Gamma)$ Brownian motion on the time interval $[0,m]$). Then the arguments of Lemmas 4 and 5 can be applied to $\{Z(t), t\in[0,m]\}$ under P_z^0 to show that (3) holds P_z^0-a.s. with m in place of ∞ there. But, since P_z^0 is equivalent to P_z^θ on $\mathscr{F}_m$, it follows that this also holds P_z^θ-a.s. Letting $m\to\infty$ yields the desired result that (3) holds P_z^θ-a.s.