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From weakest-link to best-shot: Correction

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My recent paper in *Public Choice*, on alternative social composition functions governing the voluntary provision of public goods (Hirshleifer, 1983), contained a regrettable error in one of the diagrams.

Figure 2 in the paper was intended to picture the extreme case of a 'Weakest-link' social composition function, corresponding to:

$$X = \min_{(i)} x_i \tag{1}$$

Here X is the socially available amount of the public good in a community of I individuals, while x_i represents the contribution of individual i ($i = 1, \ldots, I$). In this Weakest-link situation, for two individuals A and B, the respective preference maps take on drastically different shapes northwest of the 45° line (where $x_A < x_B$) and southeast of it (where $X_A > x_B$). In particular, A will never contemplate operating outside the northwest region, while B will always choose to be in the southeast region.

I should have noticed that A's indifference curves must be strictly *vertical* in his relevant region, while B's must be strictly *horizontal* in his relevant region. (A degree of curvature does exist, in each case, but only in the non-relevant region.) The diagram here, showing only the indifference curves in the relevant regions, is a corrected version of Figure 2 in the original paper. In the corrected diagram there is a best vertical indifference curve for individual A, $\hat{U}_A$, associated with the maximum x_A that A would ever be willing to contribute – regardless of how much greater B's production may be. Consequently, A's Reaction Curve R_A runs from the origin to point F along the 45° line, and thenceforth along the vertical $\hat{U}_A$. For B similarly, his Reaction Curve R_B runs from the origin to point G on the 45° line, and thenceforth along his best indifference curve – the horizontal $\hat{U}_B$.

Consistent with the argument of the original paper, the *equilibrium* will be at the upper limit of the range where the Reaction Curves overlie one another along the 45° line – point F. The range between points F and G is like a 'contract curve' in that one party can gain only at the expense of

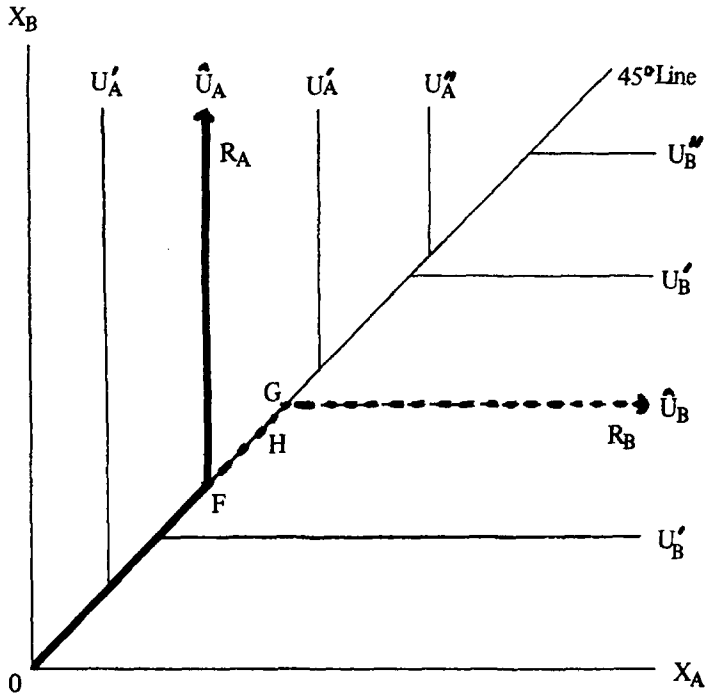

Figure 1. Efficient and equilibrium solutions, Weakest link composition function. (corrected)

the other. When Marginal Costs are equalized between the two parties, some single point like *H* in this range will be the efficient solution (assuming the *MC* functions are rising).

The erroneous diagram in the original paper would have been closer to correct for *intermediate* cases between Summation and Weakest-link, for example:

$$X = w_1 x_1 + w_2 x_2 + \ldots + w_I x_I \qquad (2)$$

Here the w_i are weights attached to the individuals' respective contributions, when these are ranked from the smallest to the largest. In the standard Summation case, all the weights would be unity. In the Weakest-link case, the weights are $w_1 = 1$, $w_2 = \ldots = w_I = 0$. I have discussed this intermediate range of cases, constituting the class of 'descending-weight' social composition functions, in a later paper (Hirshleifer, 1984).

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