ORBIT DETERMINATION FOR A RADAR MAPPING SATELLITE OF VENUS*

E. L. AKIM, Z. P. VLASOVA, V. A. STEPAN'YANTS, and A. G. TUCHIN
Keldysh Institute of Applied Mathematics, USSR Academy of Sciences, Moscow, USSR

In order to obtain high-quality images of Venus and to construct a relief map of its surface by using side-looking radar and radioaltimeter observations from a spacecraft operating in a near-Venus orbit, as well as to provide accurate map control, one should know with high precision the trajectory of the spacecraft relative to the planet. For solving this problem a ground-based and spaceborne measurement data processing system has been developed in the Keldysh Institute of Applied Mathematics (USSR Academy of Sciences). The system has been used successfully for the radar mapping of Venus from the Soviet Venera 15 and 16 spacecraft. Its capabilities are described in this paper. The authors hope that the the methods developed and the experience gained may also be useful in the MAGELLAN Project.

The data processing system is based on a mathematical model of the orbital motion of a Venusian artificial satellite (VAS). In this model the satellite motion in the gravitational field of Venus is affected by perturbations due to the oblateness of the field, the gravity of the Sun and planets, and light pressure on the satellite surface. These perturbations give rise to the satellite's orbit evolution. The evolution due to the Sun and planets is well understood. The oblateness of the Venus gravity field was determined from the Venera-9 and 10 observation data while preparing the present experiment (Akim *et al.*, 1978) and Pioneer Venus Orbiter (Bruce *et al.*, 1987).

Observations of the Venera-15 and 16 spacecraft motion revealed still another cause of orbit evolution: perturbation of the motion of the satellite's c.m. by the attitude control system (due to a difference in thrust between the jets providing rotation about each of two axes). Since the attitude control system operated at each revolution when the images were taken, it caused orbital perturbations at each revolution. The value and direction of the perturbing accelerations vary from revolution to revolution. To take these perturbations into account a mathematical model of a mechanism of their generation has been constructed. The model contains parameters determined by the trajectory measurements. For describing the satellite's motion, osculating elements of its orbit are used. Calculations of perturbations in the orbit elements due to the oblateness of the Venus gravity field, the gravity of the Sun and planets, and light pressure are made by means

* 'Geology and Tectonics of Venus', special issue edited by Alexander T. Basilevsky (USSR Acad. Sci. Moscow), James W. Head (Brown Univ., Providence), Gordon H. Pettengill (M.I.T., Cambridge, Mass.) and R. S. Saunders (J.P.L., Pasadena).

Earth, Moon, and Planets **50/51**: 559–578, 1990.

of numerical integration of differential equations of the satellite's perturbed motion in the Lagrangian form. A numerical-analytical method was used to account for the perturbations caused by the attitude control system operation.

The observational material was supplied mainly by the ground radar measurements of the Venera-15 and 16 Doppler shift. Besides, the measurements of the satellite altitude above the Venus surface, obtained by the spaceborne altimeter-profilograph, were also available. As a result of preliminary processing, the Doppler shift values were interpreted in the form of ballistic values of the satellites' radial velocities with respect to spaced ground measuring stations. Compression of homogeneous trajectory information obtained in each measurement session was performed with its replacement by averaged measurements (AM).

A technique of determining the VAS motion parameters forms the basis for the data processing system. The satellite orbit elements are determined along with unknown parameters of the attitude control system by means of statistical processing of AM of ground trajectory measurements and spaceborne measurements. To enhance the accuracy of determining the satellite trajectory a technique of measurement data processing was developed which enabled us to correlate trajectory measurements over about 1 month. The entire interval of the Venera-15 and 16 flight, during which the radar mapping was carried out, was split into segments of the above given duration. The Venera-15 and 16 trajectory was determined in two stages. At the first stage, by using the ground trajectory measurements from each segment, the orbit elements referring to the beginning of the segment were determined along with additional parameters characterizing the attitude control system operation during this segment. To solve this problem the onboard measurements of the satellite altitude above the planet surface were also used for some segments.

At the second stage the orbital elements and the additional parameters were adjusted for all segments. For this purpose a multiparametric problem was solved to determine L parameters from all the measurement data, where

$$L = \left(6N + \sum_{l=1}^{N} m_i\right);$$

N being the number of segments for both satellites; i, the number of a segment; m_i is the number of additional parameters of the i-th segment. For solving this problem the whole set of ground trajectory measurements (in the form of AM) was used in addition to the onboard measurements of altitudes carried out by different VAS, or by a single VAS on different segments, for the same regions on the planet surface.

Numerical values of the VAS parameters obtained from the first stage were used as good initial values for the iterative second-stage procedure of this large multiparametric problem. Its solution yields a unified correlation of all trajectory measurements performed by Venera-15 and 16 on the mapping interval, and provides higher accuracy in determining the VAS trajectory. The proposed technique using the altitude difference measurements as well as the technique for combined evaluation of the satellites' orbital elements and the parameters of nongravitational perturbations, with the specific features of the spacecraft attitude control system operation taken into account, are original.

By using this data processing system implemented in the form of large computer program complexes the Venera-15 and 16 trajectories were determined with high accuracy for a major part of imaging revolutions by a team of specialists in the Keldysh Institute of Applied Mathematics. The results in the form of orbital elements for each revolution have been used to construct a picture of Venus and its relief, to make the coordinate referencing of radar images. The orbital data were also made available to American specialists.

1. Mathematical Model of Motion of the Venusian Artificial Satellites

1.1 Description of the Venusian artificial satellite motion

The Venera-15 and 16 satellites operate in the gravitational field of Venus. Their motion is perturbed due to oblateness of the field, gravitational effects of the Sun and planets, the light pressure effect upon the satellites surface, and the effect of minor active forces caused by operation of the attitude control systems on the satellites.

The Venera-15 and 16 motion is described in Cartesian coordinates. The origin of the coordinate system coincides with the Venus c.m. and the axes are fixed with respect to stars. The XY plane coincides with the plane of the average planetary equator for epoch 1983, October 1.0 (Moscow time). The Z axis is perpendicular to this plane and directed opposite to the vector of angular momentum of the planet's proper rotation, towards the North of the fixed Laplace plane. The X axis is in Venus's equatorial plane and zero meridian; the Y axis completes the right-handed coordinate system. The zero meridian of Venus is defined by setting the Venusian longitude of the central meridian of the planet as observed from the Earth's center on 20.06.1964 at 00.00 of ET (Ephemeris Time) for 2438566.5 JD (Julian Date) equal to 320°.0. In the geoequatorial coordinate system of epoch 1950.0 the coordinates of the North Pole of Venus are taken as $\alpha_0 = 272°.8$; $\delta_0 = 67°.2$ (System of IAU).

For describing the motion of the satellite, the orbital elements used are as follows: the semimajor axis, a: the Laplace vector components, $\varphi_1 = e \sin\omega$ and $\varphi_2 = e \cos\omega$ (e is the eccentricity and ω is the argument of pericenter — the angular distance to pericenter from the ascending node of the orbit); the inclination, i; the ascending node longitude, Ω; and the time of passage through this node, T_Ω. The angular parameters i, ω and Ω are measured from the XY plane and the X axis using the usual method of celestial mechanics.

1.2. Differential equations of motion

The osculating elements of the satellite orbit at a current time t

$$\bar{q} = \{a, \varphi_1, \varphi_2, i, \Omega, T_\Omega\}, \tag{1.1}$$

are calculated by the formula

$$\bar{q} = \bar{q}_0 + \Delta\bar{q}, \tag{1.2}$$

where

$$\bar{q}_0 = \{a_0, \varphi_{10}, \varphi_{20}, i_0, \Omega, T_{\Omega 0}\}, \tag{1.3}$$

are the osculating orbital elements at the time of the satellite's passage through the first ascending node of the orbit, and are current perturbations. The current perturbations are calculated by integrating numerically the differential equations of the satellite's perturbed motion in the Lagrange form: namely,

$$\begin{aligned}
\frac{d(\Delta a)}{dt} &= -\frac{2a^2}{\mu}\frac{\partial R}{\partial T_\Omega}, \\
\frac{d(\Delta\varphi_1)}{dt} &= \frac{\varphi_2}{c}\cos i\frac{\partial R}{\partial \cos i} - \left(\frac{c}{\mu}\right)^2 \frac{\varphi_1}{(1+\varphi_2)^2}\frac{\partial R}{\partial T_\Omega} + \frac{1}{a}\left(\frac{c}{\mu}\right)\frac{\partial R}{\partial \varphi_2}, \\
\frac{d(\Delta\varphi_2)}{dt} &= -\frac{\varphi_1}{c}\cos i\frac{\partial R}{\partial \cos i} - \left(\frac{c}{\mu}\right)^2 \frac{2}{1+\varphi_2}\frac{\partial R}{\partial T_\Omega} - \frac{1}{a}\left(\frac{c}{\mu}\right)\frac{\partial R}{\partial \varphi_1}, \\
\frac{d(\Delta i)}{dt} &= -\frac{1}{\sin i}\left\{\left[\left(\frac{c}{\mu}\right)^2 \frac{1}{(1+\varphi_2)^2}\frac{\partial R}{\partial T_\Omega} + \frac{\varphi_1}{c}\frac{\partial R}{\partial \varphi_2} - \frac{\varphi_2}{c}\frac{\partial R}{\partial \varphi_1}\right] \times \right. \\
&\qquad \left. \times \cos i + \frac{1}{c}\frac{\partial R}{\partial \Omega}\right\}, \\
\frac{d(\Delta\Omega)}{dt} &= -\frac{1}{c}\frac{\partial R}{\partial \cos i}, \\
\frac{d(\Delta T_\Omega)}{dt} &= \left\{\left(\frac{c}{\mu}\right)^2\left[\frac{\varphi_1}{(1+\varphi_2)^2}\frac{\partial R}{\partial \varphi_1} + \frac{2}{1+\varphi_2}\frac{\partial R}{\partial \varphi_2} - \frac{\cos i}{(1+\varphi_2)^2}\frac{\partial R}{\partial \cos i}\right] + \right. \\
&\qquad \left. + \frac{2a^2}{\mu}\frac{\partial R}{\partial a}\right\},
\end{aligned} \tag{1.4}$$

where μ is the gravity constant of Venus, and c is the angular momentum. The perturbation function R consists of four summands characterizing, respectively, the perturbations in the satellite motion due to oblateness of the Venus gravity field, the Sun gravity effect, the light pressure onto the satellite surface and minor active forces caused by operation of the satellite attitude control system.

1.3. Involvement of perturbations in the satellite motion caused by the gravity effects of the Sun and planets, oblateness of the Venus gravity field and the light pressure onto the satellite surface

Partial derivatives of the perturbation function with respect to the orbital parameters, which are contained in the right-hand sides of Equations (1.4), are calculated by the formulas

$$\frac{\partial R}{\partial q_i} = \bar{F} \cdot \frac{\partial \bar{r}}{\partial q_j}, \quad j = 1, \ldots, 6; \quad i = 1, \ldots, 6, \tag{1.5}$$

where $\bar{F}\{F_x, F_y, F_z\}$ is the vector of the perturbing acceleration in the coordinate system XYZ with the components

$$F_x = \frac{\partial R}{\partial x}, \quad F_y = \frac{\partial R}{\partial y}, \quad F_z = \frac{\partial R}{\partial z} \tag{1.6}$$

and

$$\frac{\partial \bar{r}}{\partial q_j} \left\{ \frac{\partial x}{\partial q_j}, \frac{\partial y}{\partial q_j}, \frac{\partial z}{\partial q_j} \right\} \tag{1.7}$$

are the partial derivatives of the radius vector with respect to the orbital elements (1.1).

The components of the Sun's perturbing acceleration are calculated by the formula

$$\bar{F}_\sigma = \frac{\mu\sigma}{\rho^2} \frac{\bar{r}_\sigma - \bar{r}}{\rho} - \frac{\mu_\sigma}{r_\sigma^2} \frac{\bar{r}_\sigma}{r_\sigma}, \tag{1.8}$$

where μ_σ is the gravity constant of the Sun, $\bar{r}_\sigma\{x_\sigma, y_\sigma, z_\sigma\}$ is the Sun radius vector, $\bar{r}\{x, y, z\}$ is the VAS radius vector in the considered coordinate system, and

$$\rho = \sqrt{(x_\sigma - x)^2 + (y_\sigma - y)^2 + (z_\sigma - z)^2}. \tag{1.9}$$

The gravity effect of planets upon the motion of the satellites is negligibly small on the time intervals under consideration.

Components of the perturbing acceleration caused by the solar radiation pressure are calculated by the formula

$$\bar{F}_x = \kappa \, \mu_\sigma \frac{\bar{r} - \bar{r}_\sigma}{\rho^3}, \tag{1.10}$$

where κ is a coefficient characterizing a value of the light pressure.

The gravitational potential of Venus is presented by a series in spherical functions in the form:

$$U = \frac{\mu}{\tau}\left\{\sum_{n=2}^{N}\sum_{m=0}^{n} P_{nm}(\theta)\left[c_{nm}\cos m\lambda + d_{nm}\sin m\lambda\right]\left(\frac{R_B}{\tau}\right)^n\right\}.$$

Components of the perturbing acceleration due to oblateness of the Venus gravity field are expressed in the form

$$\left(F_G\right)_x = \frac{\partial U}{\partial x}, \qquad (F_G)_y = \frac{\partial U}{\partial y}, \qquad (F_G)_z = \frac{\partial U}{\partial z},$$

where R_B is the average radius of Venus, θ and λ are the latitude and the longitude of the subsatellite point.

The partial derivatives $(\partial\bar{\tau}/\partial q_j)$ are calculated from the formulae

$$\frac{\partial\bar{\tau}}{\partial a} = \frac{1}{a}\left\{\bar{\tau} - \frac{3}{2}\left(t - T_\Omega\right)\bar{v}\right\},$$

$$\frac{\partial\bar{\tau}}{\partial\varphi_1} = -a\left\{\tilde{R}_1\bar{e}_\tau - \left(\frac{\tau}{\rho}\tilde{T}_1 + \tilde{s}^1\right)\bar{e}_N\right\},$$

$$\frac{\partial\bar{\tau}}{\partial\varphi_2} = -a\left\{\tilde{R}_2\bar{e}_\tau + \left(\frac{\tau}{p}\tilde{T}_2 + \tilde{s}_2\right)\bar{e}_N\right\},$$

$$\frac{\partial\bar{\tau}}{\partial i} = \frac{z}{\sin i}\bar{e}_c,$$

$$\frac{\partial\bar{\tau}}{\partial\Omega} = \{-y, x, 0\},$$

$$\frac{\partial\bar{\tau}}{\partial T_\Omega} = -\bar{v};$$

where

$$\tilde{R}_1 = 2\lambda_{11}\cos u + \lambda_{12}\sin u, \quad \bar{e}_\tau = \frac{\bar{\tau}}{\tau}, \quad \bar{e}_N = \bar{e}_c \times \bar{e}_z, \bar{e}_c = \frac{\bar{c}}{c} \tag{1.12}$$

$$\tilde{R}_2 = \lambda_{21}\sin u + \lambda_{22}\cos u, \qquad \lambda_{11} = \frac{\varphi_1}{1+\varphi_2}$$

$$\tilde{s}_1 = 2\lambda_{11}\sin u - \lambda_{12}\cos u, \qquad \lambda_{12} = \frac{1-\varphi_2}{1+\varphi_2}$$

$$\tilde{s}_2 = \lambda_{21}\cos u - \lambda_{22}\sin u, \qquad \lambda_{21} = \frac{\varphi_1\varphi_2}{(1+\varphi_2)^2}$$

$$\tilde{T}_1 = \tilde{s}_1 + (2+\varphi_2)\lambda_{12}, \qquad \lambda_{22} = \frac{1+\varphi_2^2-\varphi_1^2}{(1+\varphi_2)^2}$$

$$\tilde{T}_2 = \tilde{s}_2 - (2 + \varphi_2)\lambda_{21},$$

Here u is an argument of latitude, and $\bar{c}$ is the angular momentum vector of the osculating orbit.

Let us give also the necessary formulae for derivatives of the velocity vector components in orbital elements. They are

$$\begin{aligned}
&\frac{\partial \bar{v}}{\partial a} = \frac{1}{2a}\left\{3(t - T_\Omega)\frac{\mu}{r^3}\bar{r} - \bar{v}\right\},\\
&\frac{\partial \bar{v}}{\partial \varphi_1} = \frac{a}{\rho}\left\{v_N \tilde{R}_1 \bar{e}_N - (v_N \bar{e}_\tau - v_\tau \bar{e}_N)\tilde{T}_1\right\},\\
&\frac{\partial \bar{v}}{\partial \varphi_2} = \frac{a}{\rho}\left\{v_N \tilde{R}_2 \bar{e}_N + (v_N \bar{e}_\tau - v_\tau \bar{e}_N)\tilde{T}_2\right\},\\
&\frac{\partial \bar{v}}{\partial i} = \frac{v_z}{\sin i}\bar{e}_c,\\
&\frac{\partial \bar{v}}{\partial \Omega} = \{-v_y, v_x, 0\},\\
&\frac{\partial \bar{v}}{\partial T_\Omega} = \frac{\mu}{r_3}\bar{r},\\
&v_\tau = \bar{e}_\tau \bar{v}, \qquad v_N = \frac{c}{r}.
\end{aligned} \tag{1.13}$$

Kinematic parameters of the satellite motion $\bar{r}(x, y, z)$ and $\bar{v}(v_x, v_y, v_z)$ are calculated at the current instant of time t using the Keplerian equation with orbital elements $\bar{q}(a, \varphi_1, \varphi_2, i, \Omega, T_\Omega)$ that osculate at this time.

1.4. Involvement of Perturbations in Motion of the Satellite Caused by Operation of Its Attitude Control System

Aside from the gravity forces and the light pressure described by Equations (1.4), the spacecraft motion is affected by the forces caused by operation of the attitude control system. The spacecraft orientation in space is changed by rotating the body coordinate system around the axes OX and OZ. The rotation around each of those axes is performed by two pairs of jets (one pair of forces provides the clockwise rotation of the spacecraft while the other provides the counter-clockwise rotation). In the ideal case the jets of the same pair have identical thrusts and are oppositely directed. The operation of such a pair of jets does not produce additional forces affecting the spacecraft c.m. motion. In real spacecraft, however, the above mentioned conditions are more or less violated. In this case, the arising additional forces generate accelerations of the spacecraft c.m.

Let the real forces arising due to the operation of jets of the i-th pair be designated by $\bar{f}_1^i(t)$ and $-\bar{f}_2^i(t)$. We shall assume that axes of the paired jets are parallel and as the thrust of one jet increases the thrust of the other jet also increases in proportion. In this case the forces $\bar{f}_1^i$ and $\bar{f}_2^i$ can be given in the form

$$\tilde{f}_1^i(t) = \left(1 - \frac{\nu_i}{2}\right)\tilde{f}^i(t),$$

$$\tilde{f}_2^i(t) = \left(1 + \frac{\nu_i}{2}\right)\tilde{f}^i(t),$$

where

$$\nu_i = \frac{f_2^i - f_1^i}{f^i},$$

and

$$\tilde{f}^i(t) = \frac{1}{2}[\tilde{f}_1^i(t) + \tilde{f}_2^i(t)].$$

The perturbing force applied to the spacecraft c.m. is determined by the vector

$$\tilde{f}_2^i(t) - \tilde{f}_1^i(t) = \nu_i \tilde{f}_i(t)$$

To rotate the spacecraft four pairs of jets operate by turns and provide an increase or a decrease in the angular velocity with respect to the axes $\overline{OX}$ and $\overline{OZ}$ of the body coordinate system.

An increment of the velocity vector caused by operation of the i-th pair of jets on the time interval t_j, t_{j+1} can be calculated by the formula

$$\bar{w}_j = \int_{t_j}^{t_{j+1}} \nu_i \frac{\tilde{f}_i(t)}{m}\, dt = \nu_i \int_{t_j}^{t_{j+1}} \frac{\tilde{f}_i(t)}{m}\, dt,$$

where m is the spacecraft mass (we ignore its change).

By taking into account that the value of $f(t)$ in the first approximation is equalled to the design value of the jet thrust, the integrals

$$I_j = \int_{t_j}^{t_{j+1}} \frac{\tilde{f}_i(t)}{m}\, dt,$$

can be calculated explicitly for each switch-on of the attitude control system jets. According to the assumption made above, if the design thrust value $\tilde{f}^i$ increases the values of actual forces $\tilde{f}_1^i$ and $\tilde{f}_2^i$ increase in proportion; therefore the value of $\nu_i = 2(f_2^i - f_1^i)/(f_2^i + f_1^i)$ does not depend on the thrust intensity and characterizes a difference in the thrusts of jets of the i-th pair (it may conditionally be called the norm of the thrust difference of the i-th pair).

2. Determining the Orbital Motion Parameters of Venera-15 and 16 by using the Ground Trajectory Measurement

2.1. Determining the VAS orbit elements by using the Doppler measurements

For determining the Venera-15 and 16 orbits the radar measurements of the Doppler shifts of these spacecraft were interpreted in the form of ballistic values of the radial velocity $\dot{D}$ with respect to the ground measuring stations. In order to obtain values of $\dot{D}$ we calculated the VAS position and the ground measuring station position in the geocentric geoequatorial coordinate system xyz of epoch 1975.0.

The orthogonal coordinates of the measuring station in the system xyz are calculated by using its Greenwich coordinates with involvement of the Earth's proper rotation as well as the Earth's rotation axis procession and nutation in the time interval from epoch 1975.0 to the measurement time.

The measurements of $\dot{D}$ were carried out in the request regime. We give the formulas for calculating values of the radial velocity $\dot{D}$. Let t_2 be the instant of time of emitting a signal by the ground transmitter, $\bar{r}_{tr}$ and $\bar{v}_{tr}$ are the position and the velocity of the transmitter at the time t_2; t_1 is the time instant of the signal reception and transmission by the satellite; $\bar{r}_{rec}$ and $\bar{v}_{rec}$ are the position and velocity of the ground receiver at the time t_0.

For the known t_0, when the signal is received, the times t_1 and t_2 with their respective $\bar{r}$, $\bar{v}$ and $\bar{r}_{tr}$, $\bar{v}_{tr}$ are calculated successively by means of iterative cycles determined by the relations

$$|\bar{r}_{rec}(t_0) - \bar{r}(t_1)| = c|t_0 - t_1|, \tag{2.1}$$

$$|\bar{r}_{tr}(t_2) - \bar{r}(t_1)| = c|t_2 - t_1|, \tag{2.2}$$

where c is the light velocity.

Calculation of values of the function $\psi = \dot{\mathscr{D}}$ under measurement is carried out by the formulae:

$$\begin{aligned}
\dot{\mathscr{D}} &= \tfrac{1}{2}(\dot{\mathscr{D}}_1 + \dot{\mathscr{D}}_2) \\
\dot{\mathscr{D}}_1 &= \frac{1}{\mathscr{D}_1}\{[\bar{r}(t_1) - \bar{r}_{rec}(t_0)][\bar{v}(t_1) - \bar{v}_{rec}(t_0)]\}, \\
\dot{\mathscr{D}}_2 &= \frac{1}{\mathscr{D}_2}\{[\bar{r}(t_1) - \bar{r}_{tr}(t_2)][\bar{v}(t_1) - \bar{v}_{tr}(t_2)]\},
\end{aligned} \tag{2.3}$$

$$\begin{aligned}
\mathscr{D}_1 &= |\bar{r}(t_1) - \bar{r}_{rec}(t_0)|, \\
\mathscr{D}_2 &= |\bar{r}(t_1) - \bar{r}_{tr}(t_2)|.
\end{aligned} \tag{2.4}$$

The method of the Newton's generalized tangents does not require high accuracy of the derivatives. This fact was used to accelerate calculation of isochronous partial derivatives of the functions under measurements. A numerical-analytical technique was developed, in which the satellite trajectory was divided by equal values of the argument of latitude (synphase points) into small-perturbation segm-ents-turns. Within a segment the motion is assumed nonperturbed and the deriva-tives of the orbital element reffered to the beginning of the segment, are calculated by the final formulas of the two-body problem. At the passage from turn to turn the perturbations in the satellite motion due to the Sun gravity effect are taken into account. To involve these perturbations we use the final formulas obtained within "the first approximation" in expanding the solution of differential Equations (1.4) over the time interval of one turn into a series by powers of a small parameter (a ratio of the semimajor axis of the satellite to the distance to the perturbing body–the Sun). The Sun is assumed fluid during the satellite revolution.

The parameters under measurement are designated by ψ and under verification by $\bar{\theta}$. A scheme of calculating the necessary partial derivatives is determined by a succession of the functions

$$\begin{aligned} \psi &= \psi(\bar{x}, t), \\ \bar{x} &= \bar{x}(\bar{q}, t), \\ \bar{q} &= \bar{q}(\bar{\theta}, t), \end{aligned} \tag{2.5}$$

where $\bar{x}(x, y, z, v_x, v_y, v_z)$ is the vector of orthogonal coordinates and velocities of the satellite at the time of measurement, and $\bar{q} = \bar{q}(a, \varphi_1, \varphi_2, i, \Omega, T_\Omega)$ are the osculating elements of the satellite orbit.

Let $\mathcal{T}(\psi/x)$ be the matrix of partial derivatives of the function ψ with respect to the orthogonal coordinates, $\mathcal{T}(x/q)$ be the matrix of partial derivatives of orthogonal coordinates with respect to the orbital elements and $\mathcal{T}(q/\theta)$ is the matrix of partial derivatives of the orbital osculating elements with respect to the parameters under verification. Then the matrix of isochronous partial derivatives of the functions under measurement with respect to the parameters under verification can be given in the form

$$\mathcal{T}\left(\frac{\psi}{\theta}\right) = \mathcal{T}\left(\frac{\psi}{x}\right) \cdot \mathcal{T}\left(\frac{x}{q}\right) \mathcal{T}\left(\frac{q}{\theta}\right). \tag{2.6}$$

Calculation of the matrix $\mathcal{T}(\psi/x)$ is determined by the form of function ψ. For measuring the radial velocity $\dot{D}$ (the Doppler measurements) the vector gradients

$$\bar{\mathcal{L}} = \bar{\mathcal{L}}\{\mathcal{L}_x, \mathcal{L}_y, \mathcal{L}_z\}, \qquad \bar{M} = \bar{M}\{M_x, M_y, M_z\},$$

where

$$\mathcal{L}_x = \frac{\partial \dot{\mathcal{D}}}{\partial x}, \quad \mathcal{L}_y = \frac{\partial \dot{\mathcal{D}}}{\partial y}, \quad \mathcal{L}_z = \frac{\partial \dot{\mathcal{D}}}{\partial z}, \quad M_x = \frac{\partial \dot{\mathcal{D}}}{\partial v_x}, \quad M_y = \frac{\partial \dot{\mathcal{D}}}{\partial v_y},$$

$$M_z = \frac{\partial \dot{\mathcal{D}}}{\partial v_z}$$

have the form

$$\begin{aligned} \mathcal{L} = \frac{1}{2}\Bigg\{&\frac{1}{\mathcal{D}_1}\left[[\bar{v}(t_1) - \bar{v}_{rec}(t_0)] - \frac{\dot{\mathcal{D}}_1}{\mathcal{D}_1}[\bar{r}(t_1) - \bar{r}_{rec}(t_0)]\right] + \\ &+ \frac{1}{\mathcal{D}_2}\left[[\bar{v}(t_1) - \bar{v}_{tr}(t_2)] - \frac{\dot{\mathcal{D}}_2}{\mathcal{D}_2}[\bar{r}(t_1) - \bar{r}_{tr}(t_2)]\right]\Bigg\}, \end{aligned}$$

$$\bar{M} = \frac{1}{2}\left\{\frac{1}{\mathscr{D}_1}[\tau(t_1) - \bar{\tau}_{\tau ec}(t_0)] + \frac{1}{\mathscr{D}_2}[\bar{\tau}(t_1) - \bar{\tau}_{\tau ec}(t_2)]\right\}. \tag{2.7}$$

Within one revolution of the satellite the matrix $\mathscr{T}(x/q)$ is calculated by formulas of nonperturbed motion (1.12) and (1.13).

In the matrix $\mathscr{T}(q/\theta)$ the perturbations in the satellite motion are involved. Let us present $\mathscr{T}(q/\theta)$ at the time of passage through the k-th ascending node in the form

$$\mathscr{T}\left(\frac{q^{(k)}}{\theta}\right) = I\left(\frac{q^{(k)}}{q^{(k-1)}}\right)\mathscr{T}\left(\frac{q^{(k-1)}}{\theta}\right). \tag{2.8}$$

The matrix $I(q^{(k)}/q^{(k-1)})$ involves the perturbations over $(k-1)$ revolutions. It represents a matrix of partial derivatives of orbital elements at a current k-th ascending node with respect to the elements at the $(k-1)$-th ascending node. The initial matrix $\mathscr{T}(q^{(0)}/\theta)$ is a unit matrix. Thus, the matrix $\mathscr{T}(q/\theta)$ changes step-wise at the time of passage through ascending node, remaining constant within one revolution of the satellite.

The passage time is determined by numerical integration of the motion equations (1.4) under the condition $z = 0$ (for $dz/dt > 0$).

To obtain analytic formulas for perturbations in the orbital elements over one revolution within "the first approximation" the solutions of differential Equations (1.4) were expanded into a series by powers of the small parameter.

2.2. Determining the VAS orbit elements along with unknown parameters of the attitude control system operation

The planetocentric orbit elements of VAS

$$\bar{q}\{a, \varphi_1, \varphi_2, i, \Omega, T_\Omega\},$$

are chosen as the motion parameters to be verified. Besides, the parameters ν_1, ν_2, ν_3, and ν_4 that characterize the thrust differences in four pairs of jets in the attitude control system are also included into verification procedure. In addition, we verify the values of additional impulses of velocity, applied to the satellite's c.m. while generating the constant solar and solar-stellar attitude control modes.

In order to decrease the amount of computations the perturbations in kinematic parameters of the satellite motion, $\overline{\Delta x}_i (i = 1, 2, \ldots)$ caused by operation of the attitude control system during one session are referred to the time of orbital pericenter t_ρ and summed over separately for each of the four pairs of jets.

In the linear approximation the corrections $\overline{\Delta x}_i\,(x_\rho)$ to the satellite state vector, which refer to the orbit pericenter, are connected with the corrections to the satellite state vector at the time t_i by the relation

$$\overline{\Delta x}_i(t_\rho) = \frac{\partial \bar{x}(t_\rho)}{\partial \bar{x}(t_i)}\overline{\Delta x}_i(t_i),$$

where $(\partial\bar{x}(t_\rho))/(\partial\bar{x}(t_i))$ is the matrix of derivatives of kinematic parameters at the time t_ρ with respect to the kinematic parameters at the time t_i. The vector $\overline{\Delta x}_i(t_\rho)$ is calculated in the following way. By solving a system of the linear equations

$$\frac{\partial\bar{x}(t_i)}{\partial\bar{q}}\overline{\Delta q}_i = \overline{\Delta x}_i(t_i),$$

the corrections to the orbital elements Δq_i are determined. Then we calculate the product

$$\overline{\Delta x}_i(t_\rho) = \frac{\partial x(t_\rho)}{\partial\bar{q}}\overline{\Delta q}_i.$$

The matrix of the derivatives $(\partial\bar{x}(t_i))/(\partial\bar{q})$ and $(\partial\bar{x}(t_\rho))/(\partial\bar{q})$ of kinematic parameters with respect to the orbital elements is calculated by the formulae of nonperturbed motion.

Let at the times $t_1^y, t_2^y, \ldots, t_M^y$ the satellite obtain instantaneous increments of kinematic parameters as

$$\overline{\Delta x}_1 = v_1\bar{y}_1$$
$$\cdots\cdots$$
$$\overline{\Delta x}_M = v_M\bar{y}_m$$

where $v_1, \ldots, v_M$ are the scalar multipliers; $\bar{y}_1, \ldots, \bar{y}_M$ are the base vectors characterizing the directions along which the increments of kinematic parameters occur.

By using the measurements, $\psi_1, \psi_2, \ldots, \psi_N$ made at the times $t_1^\psi, t_2^\psi, \ldots, t_N^\psi$ we verify the multipliers $v_1, \ldots, v_M$. In this case the restriction

$$v_{i_1} = v_{i_2} = \cdots = v_{i_M} = v_i,$$

may be imposed upon groups of multipliers out of the set $v_1, \ldots, v_M$. Verification of the above parameters together with orbital elements is carried out by the usual least-squares scheme. In Section 2.1 we described the algorithm for calculating discrepancies between the measured and design values of the functions as well as the algorithm for calculating the derivatives of the functions under measurement with respect to orbital elements to be verified. Below we describe the algorithm for calculating the isochronous partial derivatives with respect to the parameters $v_1, v_2, \ldots, v_M$.

Let us designate the kinematic parameters of the satellite, obtained as a result of instantaneous increments, by

$$\bar{x}_1 = \bar{x}_1^- + v_1\bar{y}_1$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\bar{x}_M = \bar{x}_M^- + v_M\bar{y}_M$$

and the satellite parameters before applying the increments by

$$\bar{x}_1^-, \ldots, \bar{x}_M^-.$$

The measured function ψ depends on the kinematic parameters $\bar{x}$ at the time t after applying the k-th increment as

$$\begin{aligned}\psi(\bar{x}) &= \bar{\psi}(\bar{x}(\bar{x}_k(\bar{x}_{k-1}(\ldots \bar{x}_2(\bar{x}_1(\bar{q}))\ldots)))) = \\ &= \psi(\bar{x}(v_k\bar{y}_k + \bar{x}_k^-(v_{k-1}\bar{y}_{k-1} + \bar{x}_{k-1}^-(\ldots v_2\bar{y}_2 \\ &\quad + \bar{x}_2^-(v_1\bar{y}_1 + \bar{x}_1^-(q))\ldots)))).\end{aligned} \tag{2.9}$$

The isochronous partial derivatives of measured functions with respect to the measured multiplier v_1 are calculated by the formula

$$\frac{\partial\psi}{\partial v_i} = \frac{\partial\psi}{\partial\bar{q}}\left[\left(\frac{\partial x_{im}}{\partial\bar{q}}\right)^{-1}\bar{y}_{im} + \cdots + \left(\frac{\partial\bar{x}_{i_1}}{\partial\bar{q}}\right)^{-1}\bar{y}_{i_1}\right]. \tag{2.10}$$

The products of the form

$$\bar{z}_{i_k} = \left(\frac{\partial\bar{x}_{i_k}}{\partial\bar{q}}\right)^{-1}\bar{y}_{i_k},$$

within the square brackets are calculated by solving the system of linear equations

$$\frac{\partial\bar{x}_{i_k}}{\partial\bar{q}}\bar{z}_{i_k} = \bar{y}_{i_k},$$

with respect to the time $\bar{z}_{i_k}$.

The derivatives of the measured functions $\psi_j (j = 1, 2, \ldots, N)$ with respect to the verified parameter v_i that characterized the thrust difference in the i-th pair of jets are calculated by formula (2.10). To calculate the expression within the square brackets, we sum over the terms responsible for the perturbations in the satellite motion caused by operation of the i-th pair of jets.

For determining the parameters of additional velocity impulses applied to the satellite c.m. for generating constant solar and solar-stellar attitude control modes the same algorithm for multiplier verifications is used. The velocity increment vector under verification $\overline{\Delta v}$ is projected onto the radial $\bar{e}_\tau$ transversal $\bar{e}_m$ and binormal $\bar{e}_\mathscr{C}$ directions

$$\overline{\Delta v} = w_\tau\bar{e}_\tau + w_m\bar{e}_m + w_\mathscr{C}\bar{e}_\mathscr{C}.$$

The multipliers w_τ, w_m and $w_\mathscr{C}$ are verified. The vectors

$$\bar{y}_1 = \{0, 0, 0, e_{\tau x}, e_{\tau y}, e_{\tau z}\},$$

$$\bar{y}_2 = \{0, 0, 0, e_{mx}, e_{my}, e_{mz}\},$$

$$\bar{y}_3 = \{0, 0, 0, e_{\mathscr{C}x}, eS_{\mathscr{C}y}, e_{\mathscr{C}z}\},$$

are considered as basic.

2.3. Determination of the VAS orbit elements together with unknown parameters of the attitude control system operation by using a set of the ground and on-board measurements carried out over the whole interval of radar imaging

During the radar imaging the spaceborne radioaltimeter determines distances from the satellite to a subsatellite point on the Venus surface, that reflects a radiosignal. By using these measurements for verifying the orbital motion of the satellite we can considerably increase an accuracy of determining the motion parameters.

In the first approximation it may be assumed that the direction of the signal emitted (by the direction of the signal emitted) by the radio-altimeter coincides with the line connecting the satellite to the Venus c.m. In this case, the distance h from the satellite to the point on the planetary surface reflecting the signal is expressed by the relation

$$h(\bar{\tau}) = \tau - R(\varphi(\bar{\tau}), \lambda(\bar{\tau})),$$

where $\bar{\tau}\{x, y, z\}$ are the VAS coordinates in the fixed-in-Venus system, $\tau = \sqrt{x^2 + y^2 + z^2}$ is the distance of the Venus c.m. to VAS, φ and λ are the latitude and longitude of the subsatellite point, and $R(\varphi, \lambda)$ is the distance of the Venus c.m. to the point on its surface with coordinates φ, λ.

The relief of the Venus surface, described by the function $R(\varphi, \lambda)$ is a priori unknown. Therefore, for calculating the design value of altitude h at the time t one has to be content with the idea of the Venus surface in the form of a sphere with mean radius R_0. At comparison between measured and design values of altitude, along with an instrument error of measurement we come across an error connected with ignorance of the planetary relief such as

$$\Delta_i' = R(\varphi(t_i), \lambda(t_i)) - R_0,$$

whose value may achieve a few km, which greatly surpasses the instrument error Δ_i.

In order to eliminate an effect of the errors due to the relief ignorance, for determining the VAS orbit we use differences between altitude measurements made by the spaceborne radioaltimeter on two different revolutions when the satellite passed above the same point on the planetary surface. This altitude difference obtained between measurements at the time t_1 and t_2 can be written in the form

$$h_1^{ms} - h_2^{ms} = \tau_1^{tr} - R(\varphi_1, \lambda_1) + \Delta_1 - [\tau_2^{tr} - R(\varphi_2, \lambda_R) + \Delta_2],$$

where τ_1^{tr} and τ_2^{tr} are the true values of the distances from VAS to the Venus c.m.

If $\varphi_1 = \varphi_2$, $\lambda_1 = \lambda_2$, $R(\varphi_1, \lambda_1) = R(\varphi_2, \lambda_2)$, the value of the measured altitude difference

$$\Delta h^{ms} = h_1^{ms} - h_2^{ms} = \tau_1^{tr} - \tau_2^{tr} + \Delta_1 - \Delta_2,$$

does not depend on a value of the subsatellite point elevation over the sphere of radius R_0, which approximates Venus. The corresponding design value $\Delta h = \tau_1 - \tau_2$ is also calculated without data on the Venus relief. In this case, it does not matter whether the measurements h_1^{ms} and h_2^{ms} were made from one or two different satellites. It is only essential that both measurements be made above the same point on the Venus surface.

We consider the problem of combined determination of orbital elements and parameters of nongravitational perturbations caused by operation of the attitude control system. For this purpose we use a set of earthbased and onboard trajectory measurements. An interval of trajectory measurements is divided into the segments $M_1, M_2, \ldots, M_n$, each of them having its own set of motion parameters. The number of verified parameters on each segment includes six orbital elements of the satellites and several parameters of nongravitational perturbations on the given segment.

Intervals for trajectory measurements of the radial velocity on each segment are given in Table I. Numbering proceeds in succession regardless of the fact whether a segment refers to Venera-15 or Venera-16.

In order to correlate a whole set of the ground trajectory measurements, made on different parts of the satellite motion, with the dynamics laws the altitude difference measurements are used. These measurements are effective only when a pair of altitudes making the difference are measured above two points on the surface that are sufficiently close to each other. The measurements of the altitudes

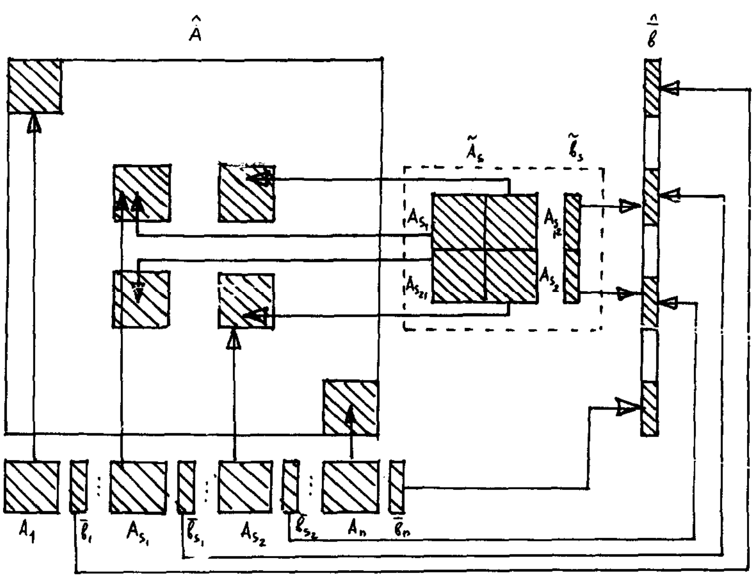

Fig. 1.

TABLE I

N	VAS	Measure interval	Number of $\dot{D}$	Number of PV	$\sigma_0^{(0)}$	σ_0
1	16	31.10.83–27.11.83	185	8	0.41	0.59
2	15	11.11.83–28.11.83	88	11	0.46	0.50
3	16	1.12.83–21.12.83	138	10	0.43	0.52
4	15	18.12.83–11.01.84	122	12	0.53	0.80
5	16	25.12.83–8.01.84	97	10	0.25	0.38
6	16	8.01.84–25.01.84	129	10	0.17	0.44
7	15	18.01.84–8.02.84	105	11	0.58	0.75
8	16	29.01.84–19.02.84	108	11	0.85	1.12
9	15	16.02.84–12.03.84	99	12	0.29	0.50
10	16	23.02.84–4.04.84	144	13	0.56	0.66
11	15	15.03.84–9.04.84	105	10	0.52	0.52
12	16	24.03.84–17.04.84	103	11	0.51	0.56
13	15	14.04.84–15.05.84	130	12	0.36	0.62
14	16	22.04.84–20.05.84	99	13	0.39	0.72
15	15	3.05.84–10.06.84	124	11	1.19	1.30
16	15	25.06.84–11.07.84	66	10	1.01	1.00
17	16	29.06.84–17.07.84	72	11	1.09	1.12
18	15	10.10.83–17.10.83	76	7	0.37	0.36
19	16	14.10.83–22.10.83	76	7	1.03	1.81

forming the pair must belong to two different trajectory segments. In this case they are most valuable from the information point of view.

Two different sessions of altitude measurements performed every day near the orbital pericenter contributed to one file of altitude difference measurements. A list of these files is given in Table II. The measurement data forming the file depend on the satellite orbit parameters on each of the two segments. The orbit parameters are included in the set of parameters under verification, which ensures the correlation of different trajectory segments with dynamics laws.

It is assumed that errors of the above two kinds of measurements are statistically independent and obey a multidimensional normal distribution law. In this case, for determining most probable values of parameters $\bar{Q}$ under verification it is necessary to minimize the functional

$$\hat{\Phi}(\bar{Q}) = \sum_{k=1}^{\hat{N}} P_k \xi_k^2(\bar{Q}),$$

where the vector of determined parameters $\bar{Q}\{\hat{q}_1, \ldots, \hat{q}_n\}$ is composed of verified parameters $\hat{q}_s$ on segments $M_1 \ldots M_n$. In their turn the parameters to be verified $\bar{q}_s\{\bar{q}_s, \bar{q}'_s\}$ represent a pair consisting of orbital elements of the satellite on the s-th segment $\bar{q}_s$ and the parameters of nongravitational perturbations due to the attitude control system jets. In the above expression ξ_i and P_i are, respectively,

TABLE II

NN	Numbers of VAS		Dates		Number of Δh	σ_0	Numbers of intervals	
1	15	16	23.11.83	26.11.83	1589	0.28	2	1
2	15	16	24.11.83	27.11.83	1593	0.23	2	1
3	15	16	28.11.83	1.12.83	1571	0.29	2	3
4	15	16	19.12.83	21.12.83	1583	0.26	4	3
5	15	16	22.12.83	25.12.83	1587	0.26	4	5
6	15	16	28.12.83	30.12.83	1553	0.41	4	5
7	15	16	31.12.83	3.01.84	1576	0.27	4	5
8	15	16	5.01.84	7.01.84	1575	0.34	4	5
9	15	16	10.01.84	12.01.84	1496	0.57	4	6
10	15	16	23.01.84	25.01.84	1571	0.47	7	6
11	15	16	26.01.84	1.02.84	557	0.62	7	8
12	15	16	17.02.84	19.02.84	1556	0.34	9	8
13	15	16	20.02.84	23.02.84	1557	0.28	9	10
14	15	16	26.02.84	28.02.84	1564	0.44	9	10
15	15	16	9.03.84	11.03.84	1572	0.33	9	10
16	15	16	12.03.84	15.03.84	1524	0.27	9	10
17	16	16	24.03.84	24.03.84	1596	0.01	10	12
18	15	16	29.03.84	31.03.84	1579	0.16	11	12
19	15	16	15.04.84	17.04.84	1574	0.21	13	12
20	15	16	21.04.84	23.04.84	1534	0.20	13	14
21	15	16	27.04.84	30.04.84	1201	0.18	13	14
22	15	16	3.05.84	5.05.84	1370	0.17	13	14
23	15	15	4.05.84	4.05.84	1416	0.12	13	15
24	15	15	9.05.84	9.05.84	1552	0.01	13	15
25	15	15	15.05.84	15.05.84	1309	0.15	13	15
26	15	16	15.05.84	18.05.84	1123	0.32	13	14
27	15	16	18.05.84	20.05.84	1567	0.27	15	14
28	15	16	25.06.84	10.07.84	1537	0.34	16	17
29	16	15	4.11.83	1.07.84	1480	0.36	1	16
30	16	15	11.11.83	8.07.84	1395	0.28	1	16
31	16	15	12.11.83	9.07.84	1406	0.25	1	16
32	16	15	13.11.83	10.07.84	1501	0.28	1	16
33	15	16	16.10.83	1.07.84	1401	0.43	18	17
34	16	16	20.10.83	2.07.84	1446	0.32	19	17
35	16	16	16.01.84	29.01.84	226	0.30	6	8
36	16	16	17.01.84	30.01.84	226	0.35	6	8
37	16	16	18.01.84	31.01.84	222	0.33	6	8
38	15	16	25.01.84	16.02.84	170	0.34	7	8
39	16	15	16.04.84	29.04.84	210	0.38	12	13
40	16	15	17.04.84	30.04.84	215	0.28	12	13
41	15	16	4.06.84	29.06.84	334	0.44	15	17
42	15	16	6.06.84	1.07.84	271	0.44	15	17
43	15	16	8.06.84	3.07.84	315	0.47	15	17
44	15	16	9.06.84	5.07.84	315	0.46	15	17
45	15	16	10.06.84	30.06.84	505	0.47	15	17
46	16	15	23.04.84	5.06.84	130	0.10	14	15
47	16	15	12.01.84	7.06.84	131	0.46	6	15
48	15	15	10.01.84	4.06.84	145	0.58	4	15
49	15	15	25.01.84	6.06.84	146	0.20	7	15
50	15	15	19.02.84	5.06.84	161	0.40	9	15
51	16	16	11.11.83	22.01.84	69	0.14	1	6

discrepancies between measured and design values and weight characteristics of all measurements; $\hat{N}$ is a total number of measurements.

The functional representing a sum of weighted squares of all observations is written as the sum $n + m$ of the summands

$$\hat{\Phi} = \sum_{k=1}^{n} \Phi_k + \sum_{s=1}^{m} \Phi_s = \sum_{k=1}^{n} \sum_{i=1}^{N_k} P_{ki}\xi_{ki}^2 + \sum_{s=1}^{m} \sum_{j=1}^{M_s} P_{sj}\xi_{sj}^2, \tag{2.11}$$

so that each of the first n summands Φ_k to contain measurements of radial velocity on the k-th segment, while the rest of the summands must contain m groups of altitude differences of onboard measurements. The index k in (2.11) means that the discrepancy ξ_{ki} and the weight P_{ki} belong to the segment M_k, while index s means that ξ_{si} and P_{si} belong to the s-th group of onboard altitude measurements.

Let the s-th group include altitude differences in measurements made on segments M_{s1} and M_{s2}. Then the corresponding design values $\psi_{sj}(j = 1, 2, \ldots, M_s)$ depend on the motion parameters $\hat{\bar{q}}_{s1}$, $\hat{\bar{q}}_{s2}$ under verification on those segments. The design values $\psi_{ki}(i = 1, 2, \ldots, N_k)$ of radial velocity measurements on the k-th segment depend only on the motion parameters $\hat{\bar{q}}\{\bar{q}_k, \bar{q}_k'\}$ of verification on this segment and do not depend on the motion parameters on other segments.

Thus, the functional depending on the parameters under verification can be written in the form

$$\hat{\Phi}(\bar{Q}) = \sum_{k=1}^{n} \Phi_k(\hat{\bar{q}}_k) + \sum_{s=1}^{m} \Phi_m(\hat{\bar{q}}_{s1}, \hat{\bar{q}}_{s2}),$$

where

$$\Phi_k(\hat{\bar{q}}_k) = \sum_{i=1}^{N_k} P_{ki}\xi_{ki}^2(\hat{\bar{q}}_k),$$

$$\Phi_m(\hat{\bar{q}}_{s1}, \hat{\bar{q}}_{s2}) = \sum_{j=1}^{M_s} P_{sj}\xi_{sj}^2(\hat{\bar{q}}_{s1}, \hat{\bar{q}}_{s2}).$$

Determination of the values of parameters $\tilde{Q}$ that provide minimum of functional $\hat{\Phi}(\theta)$ is reduced to solving the nonlinear equations

$$\frac{\partial\hat{\varphi}}{\partial q_1} = \frac{\partial\hat{\varphi}}{\partial q_2} = \cdots = \frac{\partial\hat{\varphi}}{\partial q_{\hat{n}}},$$

where

$$\hat{n} = 6 \cdot n + \sum_{i=1}^{n} l_i,$$

is a total number of the parameters under verification, l_i is the number of nongravitational parameters on the i-th segment, n is the number of segments.

The linearized system of equations has the form

$$\hat{A}\Delta\bar{Q} = -\hat{\bar{\mathscr{C}}}. \tag{2.12}$$

Elements of matrix $\hat{A}$ and vector $\hat{\bar{\mathscr{C}}}$ are expressed by the formulas

$$\hat{a}_{ij} = \sum_{k=1}^{\hat{N}} \frac{\partial \xi_k}{\partial q_i} P_k \frac{\partial \xi_k}{\partial q_j},$$

$$\hat{\mathscr{C}}_j = \sum_{k=1}^{\hat{N}} \xi_k P_k \frac{\partial \xi_k}{\partial q_j},$$

$$i, j = 1, 2, \ldots, \hat{n}.$$

The matrix $\hat{A}$ and the vector $\hat{\bar{\mathscr{C}}}$ in the system of Equations (2.12) can be written as

$$\hat{A} = A + A', \qquad A = \sum_{k=1}^{n} A_k, \qquad A' = \sum_{s=1}^{m} A'_s$$

$$\hat{\bar{\mathscr{C}}} = \bar{\mathscr{C}} + \bar{\mathscr{C}}', \qquad \bar{\mathscr{C}} = \sum_{k=1}^{n} \bar{\mathscr{C}}_k, \qquad \bar{\mathscr{C}}' = \sum_{s=1}^{m} \bar{\mathscr{C}}'_s,$$

where A_k and $\bar{\mathscr{C}}_k$ are the summands referring to measurements on the k-th segment, A'_s and $\bar{\mathscr{C}}'_s$ are the summands referring to the s-th group of altitude differences.

Values of elements a_{ij} and $\mathscr{C}^k_j$ are different from zero only when the derivatives are calculated with the parameters on the k-th segment. Therefore, the matrix A is a block-diagonal. The k-th block of matrix A as well as the corresponding k-th fragment of vector $\bar{\mathscr{C}}$ can be obtained by processing the trajectory measurements of the k-th segment independently of the rest measurements according to the above described technique.

The elements $(a'_{ij})^s$ of matrix A'_s and $(\mathscr{C}'_{ij})^s$ of vector $\bar{\mathscr{C}}'_s$ contain the derivatives of altitude differences between the s_1-th and s_2-th groups of measurements with respect to the whole set of parameters $\bar{Q}$. The nonzero values of $(a'_{ij})^s$ and $(\mathscr{C}'_j)^s$ are obtained only for parameters $\hat{\bar{q}}_{s_1}\hat{\bar{q}}_{s_2}$ that determine the VAS motion on segments M_{s_1} and M_{s_2}.

Diagrams of matrix $\hat{A}$ and vector $\hat{\bar{\mathscr{C}}}$ are drawn in the figure. Nonzero elements of matrix A'_s and vector $\mathscr{C}'_s$ are distinguished by the dash line. In combination they form the matrix $\tilde{A}_s$ and the vector $\bar{\mathscr{C}}_s$ which can be made up by processing the altitude differences of the s_1-th and s_2 groups independently of measurements on other segments. Calculation of the design values comprising the matrix $\tilde{A}_s$ and the vector $\bar{\mathscr{C}}_s$ is carried out by formulas (2.3) and (2.4). One altitude group contains the measurements obtained in a session of the on-board radioaltimeter operation. The session lasts not more than 3 hours. Within this time the trajectory calculation is carried out by formulas of nonperturbed motion, providing a required accuracy in determining the satellite coordinates. The isochronous partial derivatives of altitude differences with respect to the parameters $\hat{\bar{q}}_{s_1}$ and $\hat{\bar{q}}_{s_2}$ under verification are determined from the relation

$$\frac{\partial \Delta h(t)}{\partial \hat{\bar{q}}_{s_1}} = \frac{\partial \tau_1(\bar{q}_1, t)}{\partial \bar{q}_1} \frac{\partial \bar{q}_1}{\partial \hat{\bar{q}}_{s_1}} - \frac{\partial \tau_2(\bar{q}_2, t)}{\partial \bar{q}_2} \frac{\partial \bar{q}_2}{\partial \bar{q}_{s_2}},$$

where $\bar{q}_i (i = 1, 2)$ are the orbital elements determining the satellite trajectory over the s-th session of the radio-altimeter operation.

Table I lists the characteristics of measure intervals for Venera-15 and 16 participated in combined processing. For each measure interval we have: the number of AM (averaged measurements) $\dot{\mathrm{D}}$; the number of parameters to be verified (PV), involving six orbital elements of VAS, four parameters responsible for different thrusts of the jets and in some cases the values of impulses in the transversal direction, applied at the points where additional perturbations appeared. In the column below $\sigma_0^{(0)}$ we list values of the mean square error of unit weight at independent measurement processing on each interval.

For the combined processing of measurements on all measure intervals, except $\dot{\mathrm{D}}$, the altitude differences between the two satellites (Venera-15 and 16) passing above the same point on the planetary surface were used. Table II shows the numbers of VAS and dates of sessions when the onboard radioaltimeters operated and the altitude differences were calculated. Also given are the number of these differences, and the last two columns give the number of measure intervals to which the dates are referred.

The σ_0 column in Tables I and II contains values of the mean square error of unit weight, obtained for each measure interval and the altitude difference intervals in the combined processing.

References

Akim, E. L., Vlasova, Z. P. and Chuiko, I. V.: 1978, 'Determining the Dynamic Compression of Venus by using the Trajectory Measurements of its First Artificial Satellites "Venera-9" and "Venera-10"'. *Dokl. AN SSSR*, **240**, N 2, p. 556.

Akim, E. L., Tyuflin, Yu. S. et al.: 1986, 'Navigation and Coordinate Referencing of Radar Images Obtained by the Venera-15 and 16 Spacecraft'. *Geodezia*: *Kartographia*, **1**, 38.

Bruce, G. B., Walter, S. K. and Robert, L. Y.: 1987, 'Venus Gravity: A Harmonic Analysis', *J. Geophys. Res.* **92**, 810, p. 10335–10351.