THE SHAPE AND STRUCTURE OF MIMAS

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(Received August 11, 1983)

Abstract. The Voyager images have shown that Mimas and Enceladus have "regular" shapes, with topography of the order of 1% of the diameter. Therefore, we can compare the global shapes of these satellites with the corresponding figures of gravitational equilibrium. In the case of Mimas, this comparison rules out a homogeneous interior, but implies the existence of a denser, presumably rocky core within this small icy satellite.

The Saturnian satellites Mimas and Enceladus, with mean radii of about 200 and 250 km respectively, represent the first couple of solar system bodies in this size range for which detailed images are now available, as obtained during the encounter of the Voyager probes with Saturn's system (Smith *et al.*, 1981, 1982). These images are of great interest because it was not possible to predict in advance whether such objects would have presented a regular, gravity-dominated shape close to an equipotential figure, or they would have resembled rigid, irregular fragments shaped mainly by their collisional history. Since we do not know the solid-state strength of celestial bodies, a similar dilemma occurs for all satellites and asteroids a few hundred kilometers in size; the available evidence does not allow any unequivocal conclusion since, for instance, the large asteroids seem to have nearly-equilibrium shapes (Farinella *et al.*, 1982), whilst Hyperion displays a strongly irregular appearance (Smith *et al.*, 1982).

In the case of Mimas and Enceladus the Voyager images have revealed nearly-spherical, regular shapes, with minor topographic features (craters, peaks, ridges, etc.) not higher than 5–10 km for Mimas and 1–2 km for Enceladus. This topography gives obviously a lower limit to the strength of the material forming the surface layer of the satellites. In the case of Mimas, this limit is of the order of $5\text{–}10 \times 10^6$ dynes cm^{-2}; since the surface is heavily cratered, it is plausible to assume that this gives also a good estimate of the actual value of the strength (because the main factor in limiting the impact-generated topography has been probably the failure of the crust under any too large gravitational load). This conclusion is not valid for Enceladus, whose thermal history has clearly affected the surface features reducing the topographic relief. At any rate, even if thermal mechanisms have reduced the topography in the past, this should cause a *better* fit to the equilibrium

Earth, Moon, and Planets **30** (1984) 39–42. 0167–9295/84/0301–0039$00.60.

figures; thus, no doubt is possible on the fact that the global shape of both the satellites has been moulded by gravitational forces, due to the bodies themselves and (at a minor extent) to Saturn. Thus, as suggested originally by Morrison and Burns (1976) and by Dermott (1979), we should be able to obtain from the theory of gravitational equilibrium figures informations on the physical properties and internal structure of these satellites.

In the first place, we know that for bodies with a homogeneous interior, the tidal deformations due to Saturn's gravity gradient would give rise to triaxial ellipsoids having the longest axis directed toward the planet and the shortest axis normal to the orbit plane. These are the so-called Roche figures, described in detail by Chandrasekhar (1969): for them, a classical theory allows to derive in an unequivocal way the shape parameters (i.e., the differences between the ellipsoid axes) from the knowledge of the density and the spin rate of the body. Soter and Harris (1977) applied to the Martian moon Phobos a well-known linearized version of this theory, yielding for the deviations from sphericity of the satellite's shape the simple equations

$$(a-c) \simeq 3.75\,\Omega^2 R/\pi G\rho, \tag{1}$$

$$(b-c) \simeq (a-c)/4, \tag{2}$$

where $a \geqslant b \geqslant c$ are the ellipsoid's semiaxes, Ω is the rotational (and orbital) angular velocity of the satellite, $R = (abc)^{1/3}$ is its mean radius, ρ is its density and G is the gravitational constant. This linearized treatment is accurate to within few percents as long as $\Omega^2/\pi G\rho < 0.02$. If we assume for the Saturnian satellites the densities derived by Tyler *et al.* (1982) from the Voyager radio-tracking observations, i.e., $1.44 \pm 0.18\,\mathrm{g\,cm^{-3}}$ for Mimas and $1.13 \pm 0.55\,\mathrm{g\,cm^{-3}}$ for Enceladus, $\Omega^2/\pi G\rho$ lies in the range from 0.0175 to 0.0225 for Mimas and from 0.0080 to 0.0231 for Enceladus. Hence, at least for the smaller density values, we have to correct for non-linear effects by applying Chandrasekhar's full theory; in this case, by using the mean radii obtained by Davies and Katayama (1983), we get that $(a-c)$ ranges from 13.8 to 18.0 km for Mimas and from 7.6 to 23.4 km for Enceladus (the large uncertainty is due in this case to the poorly determined density). The corresponding values of $(b-c)$ are too small for a meaningful comparison with the Voyager images, because of the limited resolution (2.2 to 14.0 km for Mimas and 1.9 to 8.2 km for Enceladus) and also of the topographic reliefs.

Before comparing the predictions of the Roche model with the Voyager data, we have to say that if the satellites have a differentiated structure, with a denser silicate core and an icy mantle, the tidal deformations of the equilibrium figures are reduced with respect to the homogeneous case (for the same mean density). The reduction factor H can be computed by applying the theory developed by Dermott (1979). We shall use four different models for the interior of the satellites, with mean densities of 1.26 and $1.62\,\mathrm{g\,cm^{-3}}$ (corresponding to the range of uncertainty of Mimas' density), core densities of 2.2 and $3.7\,\mathrm{g\,cm^{-3}}$ (the "extreme" values for reasonable rocky materials, according to Smith *et al.*, 1981), and a mantle density of $0.93\,\mathrm{g\,cm^{-3}}$ (uncompressed water ice). The resulting values of H (see Table I) show that it can be as small as 0.7, if a dense core of radius more than 1/2 of the total radius is present within the satellite.

TABLE I

Differentiated models for Mimas

ρ_c (g cm^{-3})	ρ_m (g cm^{-3})	$\langle\rho\rangle$ (g cm^{-3})	R_c/R	H	$(a-c)_{\text{Mimas}}$ (km)
3.7	0.93	1.26	0.492	0.749	13.5
2.2	0.93	1.26	0.638	0.694	14.3
3.7	0.93	1.62	0.629	0.689	9.5
2.2	0.93	1.62	0.816	0.809	11.2

ρ_c, ρ_m and $\langle\rho\rangle$ are the core, mantle and mean densities respectively; R_c/R is the ratio between the core's and the satellite's radius; H is the reduction factor of the tidal deformations with respect to homogeneous models (see text); $(a-c)_{\text{Mimas}}$ is the resulting difference between the longest and the shortest semiaxis of Mimas.

Now, the Voyager images allow us to compare the predicted $(a-c)$ values with the actual shapes of the satellites. From a preliminary fit of Mimas's limb profile as imaged by Voyager 1, Farinella *et al.* (1981) obtained an upper limit to the difference $(a-c)$ of about 10 km. Subsequently, Davies and Katayama (1983) performed a systematic fit of a geodetic network of "control points" on the surface of the two satellites, getting $(a-c) = 6 \pm 3$ km for Mimas and 8 ± 5 km for Enceladus. The differences $(b-c)$ in their fits cannot be used as output data, because Equation (2) was assumed as a constraint of the fitting procedure (a reviewer pointed to us that this method is not entirely correct, because the non-linear effects discussed earlier can reduce the ratio $(b-c)/(a-c)$ to about 0.21 when $\Omega^2/\pi G\rho = 0.023$; however, very probably the error is within the stated uncertainty of the results).

The observed value of Mimas's $(a-c)$ leads to the surprising conclusion that the shape of this satellite is clearly (at the 3-σ level) less elongated than predicted by the Roche model. This fact cannot be due to "irregular" topography sustained by the strength of the surface layer, which in case could explain an irregular shape but certainly not a surface too close to sphericity; neither we can solve the problem by the tidal evolution of Mimas' orbit, because its semimajor axis is presently increasing, and thus if the "frozing" of the surface shape occurred long time ago, the tidal distortion would be larger than that corresponding to the present orbital distance, and not smaller as is observed (Soter and Harris, 1977).

The inescapable conclusion appears to be that Mimas has a differentiated core–mantle structure, causing a H value (i.e., a reduction factor of the "Roche" tidal distortion) not much larger than 0.7. In fact, the third model of Table I gives $(a-c) = 9.5$ km, higher than Davies and Katayama's nominal value of 6 km, but consistent with its uncertainty. We note that the shape analysis seems to favour the models of Mimas with a mean density at the high end of the range given by radio-tracking data, and with a dense core including a significant fraction of the total mass. The conclusion of a differentiated structure for Mimas has probably far-reaching implications on the formation process and/or thermal history of this satellite (Consolmagno and Lewis, 1977).

For Enceladus the conclusions have to be much less sharp. Enceladus can be expected to fit the equilibrium shape even better than Mimas, because the lower topography and the extensive resurfacing processes indicate that the strength was much lower in the past, during some phases of a complex thermal history. Unfortunately the mass of Enceladus is not well known, and also the value of $(a - c)$ has been determined with less accuracy than that of Mimas, mostly because the Voyager 2 pictures of Enceladus did not record a complete revolution of the satellite. Therefore we can only state that the available data are consistent with a homogeneous internal structure, even if the presence of a core cannot be ruled out.

Acknowledgements

We thank M. E. Davies for sending us a preprint with the updated results of the shape analysis for Mimas and Enceladus. This work was partially supported by the National Research Council of Italy (C.N.R.).

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