COORDINATES OF THE CENTRE OF THE FIGURE OF LUNAR MARGINAL ZONE

SH. T. HABIBULLIN, L. I. RAHIMOV, and N. G. RISVANOV
Dept. of Astronomy, Kazan State University, U.S.S.R.

(Received August 9, 1983)

Abstract. On the basis of large-scale star-calibrated lunar photographs the rectangular selenoequatorial coordinates of the centre of the figure of lunar marginal zone have been obtained with reference to its mass centre, the position of which has been computed by the ephemerides $j = 2$ and LURE-2. A new definition method of lunar mass centre coordinates by photographic observation in system $j = 2$ and LURE-2 is proposed.

1. Introduction

The study of a number of problems connected with the revolution of the orbit of the Moon – and, in particular, the improvement of the theory of its motion and the setting of exact ephemerides, an analysis of Earth's rotation irregularity and more precise definition of the ephemeris time-scale requires prolonged observations of the lunar positions. Apart from the problems of the techniques of astrometric observations, it is necessary to note that a successful solution of these problems depends on the definition of the accuracy of the position lunar mass centre. The classical methods of lunar observations are based on the measurement of its limb points. They give the coordinates of the centre of approximation circle – i.e., of the lunar disk centre. A lack of coincidence of the latter with the mass centre projection on the pattern plane has been known for a long time. In consequence of the influence of marginal zone relief and mass distribution character in the lunar body, this distinction exceeding $0''.5$ (geocentrically) depends on the lunar orientation with respect to the observer – i.e., on the optical libration.

Usually one takes into account the uneveness of the lunar limb by the relief maps of its marginal zone. One considers that the Watts' maps are most detailed and exact. However, the investigations have shown that the measurements reduction of limb points by the Watts' maps do not give the position of lunar mass centre. Van Flandern (1970), Morrison (1970), Morrison and Appleby (1981), and Mulholland (1971) found the corrections to Watts' maps on the basis of a careful analysis of numerous observations of stellar occultations by the Moon, and also the laser light location. It is considered that these corrections together with Watt's maps allow us to reduce the observations to the lunar mass centre. In what follows shall develop a new method for relating the position of the mass centre with one of figure of lunar marginal zone by star-calibrated lunar photographic observations. One makes a comparison of the results of the definition of the coordinates of lunar mass centre, which were obtained by means of the measurements

Earth, Moon, and Planets **30** (1984) 21–30. 0167–9295/84/0301–0021$01.50.

of improved Watts' maps together with the corrections of Van Flandern, Morrison and Appleby, Mulholland and ours.

2. Relative Positions of Mass Centre and of the Figure of Marginal Zone of the Moon

As the basic coordinates system we shall adopt a rectangular selenoequatorial system of the coordinates ξ, η, and ζ with origin in the ephemeris centre of lunar mass. The ξ-axis is directed to the Mare Crisium, the η-axis along the Moon's axis to its north pole, and the ζ-axis to the direction of the Earth. We shall also introduce a selenocentric rectangular system of coordinates U and V located in a plane normal to the line of sight, such that the U-axis in the direction of increasing right ascension, and the V-axis, in the direction of increasing declination. It is supposed that the observed libration (marginal) zone of the Moon is a belt of width defined by the optical librations. It was denoted by $\mathbf{r}_0(\xi_0, \eta_0, \zeta_0)$ the position vector of the centre of this belt with respect to the lunar mass centre. It can be written as

$$\xi_0 \bar{l} + \eta_0 \bar{m} + \zeta_0 \bar{n} = \bar{r}_0, \tag{1}$$

where ξ_0, η_0, and ζ_0 are the coordinates of the marginal zone centre; $\bar{l}$, $\bar{m}$, and $\bar{n}$ being the direction cosines of the axes of coordinates. The projection of the vector $\mathbf{r}_0$ on the celestial sphere is variable. This change is due to the fact that the optical lunar libration components (l, b, C) are computed with respect to the direction of "lunar mass centre–observer". The problem is to define the coordinates ξ_0, η_0, and ζ_0. For the solution of the problem we used the star-calibrated lunar photographs received on the horizontal telescope at different optical librations (Habibullin and Risvanov, 1983). Every lunar photograph represents the intersection of its marginal zone by the celestial sphere. In accordance with the spherical form of libration zone, the lunar limb is taken as a circle on the astrophotographs. The centre of this circle is that of the marginal zone belt on the celestial sphere at a fixed optical libration. The whole complex of lunar observations (after corresponding reduction) allows to determine the position of marginal zone centre at zero libration. Thus, at first, it is necessary to find the centres of the circles drawn through measured points of lunar limb images on separate photographs, and then, with due regard to the optical libration values, to define the coordinates of the centre of libration zone for the adopted lunar mass centre.

Let x and y be the measured coordinates of the lunar image limb points. Then its standard coordinates in regard to optical plate centre will be

$$\begin{pmatrix} X \\ Y \end{pmatrix}_i = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}_i, \tag{2}$$

where the transformation matrix coefficients are the plate constants, found by reference stars, $i = 1, 2, 3, \ldots, m$ are the lunar limb points. With the aid of the standard coordinates, the polar coordinates of measured limb points θ and ρ are computed in regard to the ephemeris position of the lunar mass centre from the equations

$$\operatorname{tg}\theta_i = \frac{\Delta X_i}{\Delta Y_i}, \qquad \rho = \sqrt{\Delta X_i^2 + \Delta Y_i^2}, \tag{3}$$

where $\Delta X_i = X_i - X_c$, $\Delta Y_i = Y_i - Y_c$, $X_c = (\alpha_c' - \alpha_0') \cos \delta_0'$, $Y_c = \delta_c' - \delta_0'$, α_c' and δ_c' are the ephemeris topocentric coordinates of lunar mass centre; α_0' and δ_0' are the coordinates of the optical plate centre. The equation of the circle in the polar coordinates is of the form

$$\sin\theta_i \Delta X_{\text{obs}} + \cos\theta_i \Delta Y_{\text{obs}} + \Delta R_{\text{obs}}' = \rho_i - R', \tag{4}$$

where ΔX_{obs} and ΔY_{obs} are the unknown corrections to the values X_c and Y_c determining the circle centre position in regard to the lunar mass topocentric centre, and $\Delta R_{\text{obs}}'$ is the correction to the ephemeris topocentric lunar radius R' for an observation moment. The values ΔX_{obs}, ΔY_{obs}, and $\Delta R_{\text{obs}}'$ are computed by the least-squares method from m equations of condition of the form (4). The observed values of the topocentric standard lunar coordinates with respect to the optical plate centre will be

$$X_{\text{obs}} = X_c + \Delta X_{\text{obs}}, \qquad Y_{\text{obs}} = Y_c + \Delta Y_{\text{obs}}. \tag{5}$$

Now by the values X_{obs} and Y_{obs} we can compute the observed topocentric equatorial lunar coordinates α_{obs}' and δ_{obs}'. The latter define the observed topocentrical rectangular lunar coordinates (disk centre) with respect to the ephemeris position of its mass centre

$$U = (\alpha_{\text{obs}}' - \alpha_c') \cos \delta_c', \qquad V = \delta_{\text{obs}}' - \delta_c'. \tag{6}$$

On each photograph, the values of U and V represent the components of the projection of vector $\mathbf{r}_0$ on the celestial sphere.

The connection between the system of coordinates ξ, η, and ζ and U, V can be written (cf Habibullin and Risvanov, 1983) in the form

$$\begin{pmatrix} U \\ V \end{pmatrix}_j = \begin{pmatrix} A_{22} & A_{32} & A_{12} \\ A_{23} & A_{33} & A_{13} \end{pmatrix} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}, \tag{7}$$

where the coefficients A_{pq} can be computed from the values of topocentric optical libration in longitude, latitude and position angle l, b, and C by taking into account the physical libration, and $j = 1, 2, 3, \ldots, n$ are the numbers of the photographs. By solving the $2n$ condition equations (7) by the least-squares method we find the values of ξ_0, η_0, and ζ_0 and their errors.

In effect, at the lunar limb we can observe only one half of the points; and because of several reasons, the positions of the disks centres which were obtained from the measurements of the west and east limbs do not coincide. Therefore, it is necessary to solve Equations (7) separately by lunar observations before and after the full Moon. Their simultaneous solution determines the mean from the two parts of the marginal zone. Let us consider that a such unification defines the position of the centre of the total figure of the lunar marginal zone. In this way the three values of ξ_0, η_0, and ζ_0 are obtained. By this method the 48 star-calibrated lunar photographs have been reduced.

The results of these observations are given in the work of Habibullin and Risvanov (1983). The 300–340 lunar limb points were measured on each photograph. The ephemeris positions by Brown–Eckert's theory for $j = 2$ and LURE-2 are taken as the coordinates of the lunar mass centre. The equinox and equator corrections (Risvanov, 1978) were taken into consideration by a comparison of the observed lunar positions in the FK4-system with the ephemerides $j = 2$ – i.e.,

$$\Delta\alpha_0' = 0^s.050, \qquad \Delta\delta_0' = 0''.017 + 0''.097(T - 19.284), \tag{8}$$

where T is expressed in units of 100 years. The results of the solution are given in Table I. The values ξ_0, η_0, and ζ_0 are expressed in the angular (geocentric) and linear units. The photographic irradiation, the lunar limb unevenness and the errors of the reference stars coordinates can mainly influence the accuracy of the results obtained. Taking into account the sensitometric characteristics of FU-5 plates on which the Moon was photographed, we can assume that, on the linear part of characteristic curve, the form of the photographic image of lunar figure will differ negligibly from its real figure. The eroding of the photographic image edge will enter as the additive component ΔR in the observed value of the lunar radius. If we measure a large number of limb points, the influence of the small marginal zone relief averages out, on the whole, on each plate. The remaining part will appear as accidental errors in the solution of Equation (7), of the order of $\pm (0''.05–0''.10)$. The accidental errors of the positions of reference stars as given in the catalogue are – as was shown in the work of Jazkiv and Gubanov (1980) – close to $\pm 0''.2$. The systematic errors of the reference stars, which are more difficult to estimate, enter the condition Equation (8) as accidental components. By contemporary estimates the ephemerides $j = 2$ accuracy is geocentrically near $0''.5$ for both accidental and systematic errors. The accidental component is averaged by the solution, the systematic one goes through the values ξ_0, η_0, and ζ_0 in full amount. The ephemeris lunar positions determined by the numerical LURE-2 theory can be considered correct taking into account the accuracy of the observations by photographic method. The mean values of the quadratic errors of the coordinates ξ_0 and η_0 definition are near $\pm 0''.15$. This value corresponds to a total influence of the errors inherent in the reference stars and lunar limb relief positions. The mean values of correction $\Delta R'_{\text{obs}}$ is equal to $0''.15$ on all plates. This confirms the conclusion about the influence character of photographic irradiation.

By the laser and space measurements the coordinates of the figure of the Moon relative to its mass centre are $\xi_0 = -1$ km, $\eta_0 = -1$ km, $\zeta_0 = -2$ km (Sagitov, 1979). Our results for the whole disk received on the basis of LURE-2 theory agree with them only quantitatively. The difference is explained by the fact that, in this case, we use not total lunar figure, but only a spherical belt of its libration zone. From Table I it follows that the shift of origin coordinates of the system $j = 2$ with regard to the LURE-2 system is $\Delta\xi = -1.19$ km, $\Delta\eta = 0.36$ km, and $\Delta\zeta = 0.92$ km or in the angular values: $-0''.68$, $0''.21$, and $0''.53$. Those values correspond to the ephemerides $j = 2$ accuracy estimate

TABLE I

Coordinates of the centre of the figure of lunar marginal zone

Limb	Motion theory	Coordinates of centre figures of lunar marginal zone			
East	Brown	0″.14	± 0″.15	0.25	± 0.27 km
		−0 .53	0 .14	−0.92	0.25
		−2 .04	1 .49	−3.54	2.59
	LURE-2	−0 .55	0 .15	−0.95	0.26
		−0 .26	0 .15	−0.46	0.26
		−1 .94	1 .59	−3.38	2.76
West	Brown	0 .49	0 .14	0.85	0.23
		−0 .83	0 .15	−1.44	0.26
		−5 .67	1 .23	−9.85	2.14
	LURE-2	−0 .11	0 .14	−0.19	0.24
		−0 .66	0 .15	−1.15	0.27
		−4 .23	1 .28	−7.35	2.22
Mutual solution	Brown	0 .41	0 .10	0.71	0.18
		−0 .72	0 .10	−1.26	0.18
		−5 .10	0 .92	−8.87	1.61
	LURE-2	−0 .28	0 .10	−0.48	0.18
		−0 .52	0 .10	−0.90	0.18
		−4 .57	0 .92	−7.94	1.61

which was given above. From a compilation of the 264 craters catalogue (Habibullin and Risvanov, 1983) the shift estimate of the origin of coordinates $j = 2$ with respect to LURE-2 gave the values $\Delta\xi = -1.22$ km, $\Delta\eta = 0.46$ km, and $\Delta\zeta = 0.82$ km. A large divergence between the east and west lunar limb indicates asymmetry of the figure of lunar limb zone. However, the difference in the coordinate ξ of $-0''.35$ (Brown) or $-0''.44$ (LURE-2) are smaller than a switch in the longitude from east to west limb, which has been discovered by Markowitz to reach $-1''.2$ (Markowitz *et al.*, 1955). By the data of Van Flandern (1970) and Mulholland (1977) the displacement of the Watts' maps coordinates system centre in regard to the mass centre are given in Table II:

TABLE II

Watts' maps coordinates system

Author	ξ_0	η_0	ζ_0
Van Flandern	0″.25 ± 0″.24	−0″.24 ± 0″.10	4″.51 ± 2″.83
Mulholland	−1 .34 ± 0 .27	0 .03 ± 0 .04	3 .65 ± 1 .29

Van Flandern obtained his results from a large number of observations of stars occultations, while Mulholland used star occultations as well as laser light locations. Mulholland's investigations are based on the numerical UTIE-11 and LURE-2 theories: comparing Tables I and II data one can note that the coordinates system of Watts' maps does not conformed to the results of our investigations of lunar marginal zone.

3. Definition of the Coordinates of Lunar Mass Centre by Photographic Observations

The results of Table I can be used for the reduction of star-calibrated lunar photographic observations to its mass centre in the fixed ephemerides coordinate system. Let the observed coordinates of the lunar centre disk (figure) defined by limb points measurements by means of the attachment to reference stars be α_f and δ_f. Then the coordinates of the observed lunar mass centre can be determined from the expressions (6)

$$\alpha_m = \alpha_f - U \sec \delta_f, \qquad \delta_m = \delta_f - V, \tag{9}$$

where U and V are computed from the Equation (7); the values ξ_0, η_0, and ζ_0 are taken from the Table I; and the coefficients A_{pq} are computed from the optical libration values in longitude, latitude and position angle at the moment of observation. This assumption has been verified from the 16 star-calibrated lunar photographs, obtained with the two horizontal telescopes whose objectives possessed focal distance of 8 m (Habibullin *et al.*, 1974) and 7 m (Bistrov, 1976). From the measurement of 340 points on the limit the topocentric coordinates of lunar disk (figure) centre α'_f and δ'_f were determined on each photograph. With the aid of the topocentric component of the optical libration l, b, and C the matrix elements A_{pq} were computed, and from these the values U and V by Equations (7). In this way two series of the values U and V were obtained: the first one from the coordinates of libration zone centre ξ_0, η_0, and ζ_0 in the ephemerides system $j = 2$, the second one has been obtained by the LURE-2 system. With the aid of Equations (9) two sets of observed lunar mass centre coordinates have been computed: in the system of ephemerides $j = 2$ and LURE-2. For the sake of comparison, these limb points were reduced to the lunar mass centre by Watts' maps, and by Watts' data with corrections of Morrison and Appleby (1981), Van Flandern (1970) and Mulholland (1977). The corrections of Morrison and Appleby are superior to the observations for the ephemerides $j = 2$ and by introducing the correction term $0''.50$ in a coefficient by $\sin Q$ to the lunar mass centre the position of which was determined by the laser altimetry data with SC Apollo 15, 16 and 17 (Sjogren and Wollenhaupt, 1976). On the basis of Morrison's work, it can be supposed that, in his analysis of Watts' maps, Van Flandern used the ephemerides $j = 0$ and the zodiacal stars catalogue of D. Robertson. Mulholland's investigation of the Watts' maps coordinates system is based on the ephemerides UTIE-11 and LURE-2 and the stars positions in the FK4-system.

Thus the seven series of the observed lunar mass centre topocentric coordinates have been obtained. All of them concern the fundamental FK4-catalogue system:

(1) The measurements of the photographs were reduced by the authors' method. The results provided the lunar mass centre in the system of ephemerides $j = 2$.

(2) The measurements were reduced by the Watts' maps. The results refer to the origin of the coordinates of Watts' maps.

(3) The measurements were reduced by the Watts' maps with the corrections of Morrison and Appleby. The results yielded the lunar mass centre in the system of ephemerides $j = 2$.

(4) The measurements were reduced to the Watts' maps with the corrections of Van Flandern as given in Table II. The results furnish the lunar mass centre probably in the system of ephemerides $j = 0$.

(5) The photographic measurements were reduced by the authors' method. The results refer to the lunar mass centre in the system of ephemerides LURE-2.

(6) The measurements were reduced with the aid of Watts' maps. By taking into account the corrections of Morrison and Appleby (the supplementary coefficient 0''.50 was introduced) the observed lunar coordinates have been referred to its mass centre, the position of which Morrison and Appleby harmonised with the laser altimetry results.

(7) The measurements were reduced by Watts' maps. By taking into account Mulholand's corrections given in the Table II the observed lunar coordinates led to a certain point in the lunar body adopted by Mulholland as the Moon's mass centre.

To estimate the accuracy of observations and the different method of leading them to lunar mass centre, we formed the differences $(o–c)$ in the right ascension and declination between the observations and ephemerides coordinates

$$\Delta\alpha \cos\delta = (\alpha'_m - \alpha'_c)\cos\delta'_c, \quad \Delta\delta = \delta'_m - \delta'_c.$$

In Table III are given the mean values of the coordinates differences $\Delta\bar{\alpha}\cos\delta$, $\Delta\bar{\delta}$ and there's mean quadratic errors s_α and s_δ.

TABLE III

Comparison of the different definition methods of lunar mass centre coordinates

Value / Method	Ephemerides	$\Delta\bar{\alpha}\cos\delta$	s_α	$\Delta\bar{\delta}$	s_δ
Authors	$c = j - 2$	−0''.09	± 0''.07	0''.07	± 0''.09
Watts	$c = j - 2$	−0 .25	0 .13	−0 .23	0 .10
Morrison	$c = j - 2$	−0 .16	0 .16	0 .00	0 .10
Van Flandern	$c = j - 2$	0 .11	0 .18	0 .17	0 .16
Authors	$c =$ LURE-2	−0 .04	0 .09	−0 .10	0 .11
Morrison	$c =$ LURE-2	0 .05	0 .12	−0 .01	0 .11
Mulholland	$c =$ LURE-2	−0 .79	0 .16	−0.06	0 .17

For the sake of comparison with the ephemerides $j = 2$ for the equinox system used in the lunar coordinates observation, we introduced the corrections of the equinox and the equator N_2 according to FK4 determined by the expressions (8) $\alpha'_m + \Delta\alpha'_0$ and $\delta'_m + \Delta\delta'_0$. While comparing of observations with LURE-2 ephemerides, we did not take account of the corrections of the equinox and the equator as this theory of Moon's motion leads to the FK4 catalogue system. From Table III it follows that the introduction of corrections by Morrison, Appleby and Van Flandern improves the values

$(o-c)$ obtained by means of the reduction of Watts' maps. The difference between the results obtained by the method of Morrison, Appleby and Van Flandern amount to $0''.27$ and $0''.17$ in the right ascension and declination, respectively. This can be explained by the fact that the Van Flandern investigations, as it was noted above, are based on the ephemerides $j = 0$ and star positions in the zodiacal system. The supplementary coefficient $0''.50$, introduced by Morrison and Appleby in the expression for the correction to the altitudes taken from the Watts' maps, agrees well with the observations of the LURE-2 theory. From an analysis of lunar photographic observations summarized in Table III, it follows that $\Delta\bar{\alpha}\cos\delta = 0$ by the value of the supplementary coefficient $0''.56$. The reduction of the measurements by the Mulholland method leads to $\Delta\bar{\alpha}\cos\delta = -0''.79$. This indicates that the shift in the value of the origin of Watts' coordinates system with respect to the lunar mass centre $X_2 = -1''.30$ (the positive direction of the axis OX_2 coincides with the $O\xi$-axis direction of selenoequatorial coordinates system shown above) is overestimated in the absolute value by approximately on $0''.75$ – i.e., by the value of the equinox correction $\Delta\alpha_0'$. Thus it is necessary to increase the observed values of right ascensions on the value $\Delta\alpha_0'$ to establish agreement between the observed lunar positions obtained by Mulholland's method and the LURE-2 theory. If this is admitted, we obtain $\Delta\bar{\alpha}\cos\delta = -0''.04$ and $\Delta\bar{\delta} = -0''.06$.

The correctness of the definition of the coordinates of lunar mass centre by the proposed method has been verified by the value convergence of the ephemeris (dynamic) time correction. For this we used photographs No. 141, 142 and No. 172, 173 taken on March 31 and April 23, 1980, respectively, with the horizontal telescope of focal distance 7 m. The first two photographs were taken after full Moon, the latter before full Moon. At first the observed topocentric lunar coordinates (of disk centres) α_f' and δ_f' were deduced by the above-described method. Then the differences $\Delta\alpha\cos\delta = (\alpha_f' - \alpha_c')\cos\delta_c'$ and $\Delta\delta = \delta_f' - \delta_c'$ have been formed. By the definition of the corrections of ephemeris time $\Delta T = ET - UT$, it is advisable to use the ephemerides $j = 2$ since, by the determination of ephemeris time-scale, all fundamental investigations are based on the analysis of the positional observations of Moon, Sun, Mercury and Venus with its ephemeris positions computed by classical theories of Brown and Newcomb. Therefore, in the present case, the values α_c' and δ_c' are found by the ephemerides $j = 2$. They are computed as functions of the argument $ET2 = UT2 + (\Delta T2)_0$. As the quality of a preliminary value of the ephemerides time correction, the value $(\Delta T2)_0 = 51^s$ has been adopted. The convergence of the corrections $\Delta\alpha\cos\delta$ and $\Delta\delta$ chracterizes the accuracy of observations, Moon's motion theory and treatment method. By the formulae

$$\delta(\alpha)\cos\delta = \Delta\alpha\cos\delta - U, \qquad \delta(\delta) = \Delta\delta - V \tag{10}$$

these corrections can be used to determine the lunar mass centre. The values U and V are computed by formulae (7) by the values ξ_0, η_0, and ζ_0 in the system $j = 2$. The corrections of ephemeris time $\delta T = \Delta T2 - 51^s$ can be determined by means of the inverse interpolation by the ephemerides separately by the right ascension and declination or by the well-known formula

$$\delta T = \frac{(\alpha_{\text{obs}} - \alpha_c)\cos\delta v_\alpha \cos\delta + (\delta_{\text{obs}} - \delta_c)v_\delta}{(v_\alpha \cos\delta)^2 + v_\delta^2}, \tag{11}$$

where $v_\alpha \cos\delta$ and v_δ are the ephemeris velocities of lunar coordinates changes. By the centre coordinates of lunar disk the corrections of ephemeris time are defined separately by α and δ by inverse interpolation of $\delta T(\alpha)$ and $\delta T(\delta)$. To obtain the coordinates of the lunar mass centre, the corrections δT are computed from Equation (11). The results of this treatment are given in Table IV. In the sixth column the mean values of corresponding values by the four plates are given, while the latter column contains its mean quadratic errors s. The last line contains the observed values of the ephemeris time corrections.

TABLE IV
Ephemeris time corrections

Corrections	141	142	172	173	Mean values	s
$\Delta\alpha \cos\delta$	−0″.67	−0″.60	−0″.26	−0″.30	−0″.46	± 0″.21
$\Delta\delta$	−0 .93	−1 .15	−0 .35	−0 .45	−0 .72	0 .38
U	−0 .74	−0 .74	−0 .43	−0 .43		
V	−0 .88	−0 .88	−0 .31	−0 .31		
$\delta(\alpha) \cos\delta$	0 .07	0 .14	0 .17	0 .14	0 .13	0 .04
$\delta(\delta)$	−0 .05	−0 .27	−0 .04	−0 .14	−0 .12	0 .07
$v_\alpha \cos\delta$	0 .468	0 .468	0 .501	0 .501		
v_δ	−0 .162	−0 .162	−0 .120	−0 .120		
$\delta T(\alpha)$	$-1^{\text{s}}.43$	$-1^{\text{s}}.28$	$-0^{\text{s}}.53$	$-0^{\text{s}}.60$	$-0^{\text{s}}.96$	$0^{\text{s}}.46$
$\delta T(\delta)$	5 .74	7 .09	2 .96	3 .79	4 .90	1 .87
δT	0 .17	0 .45	0 .35	0 .33	0 .32	0 .12
$\Delta T2$	51 .17	51 .45	51 .35	51 .33	51 .32	0 .12

From Table IV it follows that the scatter of $\delta(\alpha)\cos\delta$, $\delta(\delta)$ is considerably smaller than that of $\Delta\alpha\cos\delta$ or $\Delta\delta$. The errors of the latters are five times as large. An introduction of the corrections U and V practically removes a discontinuity observed in the values $\Delta\alpha\cos\delta$ and $\Delta\delta$ by a transition through the full Moon. We should also note that the algebraic signs of the corrections $\Delta\alpha\cos\delta$ do not correspond to those of $v_\alpha \cos\delta$. Therefore, the ephemeris time from the coordinates values of lunar disk centre are determined, not by Equation (11), but by means of the inverse interpolation. Thus on the basis of the results compiled in Table IV the conclusion can be reached that the utilization of the proposed method raises considerably the accuracy of the definition of the correction for the ephemeris time by the photographic star-calibrated lunar observations.

4. Conclusions

The results of the definition of the coordinates of the centre of lunar libration zone with respect to the ephemeris positions of its mass centre by the proposed method can be used to establish the coordinates of lunar mass centre by photographic observations. The high accuracy of the LURE-2 theory allows us to expect that the observed values of the coordinates of the lunar mass centre will be near to the true position of its mass centre. The coordinates of marginal zone centre can be improved by use of a more accurate theory of Moon's motion for the comparison. By this the definition accuracy of lunar mass centre coordinates from the observations is raised. The proposed method of reduction of the measurements of limb points on star-calibrated lunar photographs to the mass centre in the system of ephemerides $j = 2$ increases the accuracy of definition of the ephemeris (dynamic) time correction – both in the accidental and systematic sense.

The star-calibrated lunar photographs of large scale, and the results of photoelectric registrations of the star occultation by the Moon, can be combined with the laser probing data. It is expedient to make the observations synchronously or quasi-synchronously. The combined solution allows to decrease the uncertainty of mathematicaly treatment of the observations, and to obtain more accurate results.

Acknowledgements

The authors thank V. K. Abalakin and M. A. Fursenko for the placing at our disposal a number of results used in this work.

References

Abalakin, V. K.: 1978, Trudi Instituta Teoreticheskoy Astronomii, *AN SSSR*, vip. XVII, 82.
Bistrov, N. F.: 1976, Isvestija Glavnoj Astronomicheskoy Observatorii, *AN SSSR* **194**, 158.
Habibullin, S. T. and Risvanov, N. G.: *Earth, Moon, and Planets* **30**, 1 (this issue).
Habibullin, S. T., Risvanov, N. G., and Bistrov, N. F.: 1974, *The Moon* **11**, 125.
Jazkiv, J. S. and Gubanov, V. S.: 1980, *Sb. Statey Geodinamika i Astrometrija*, Kiev, izd. Naukova Dumka, p. 110.
Markowitz, W., Hall, R., and Edelson: 1955, *Astron. J.* **60**, 171.
Morrison, L. V.: 1970, *Mon. Not. R.A.S.* **149**, 81.
Morrison, L. V. and Appleby, G. M.: 1981, *Mon. Not. R.A.S.* **196**, 1013.
Mulholland, J. D.: 1977, *Astron. J.* **82**, 306.
Risvanov, N. G.: 1978, *Pisma v Astronomicheskiy Zhurnal* **4**, 520.
Sagitov, M. U.: 1979, *Lunnaja Gravimetrija.* Nauka, Moskva.
Sjogren, W. L. and Wollenhaupt, W. R.: 1976, *The Moon* **15**, 143.
Van Flandern, T. C.: 1970, *Astron. J.* **75**, 744.