INFRARED BRIGHTNESS OF A COMET BELT BEYOND NEPTUNE

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Abstract. We consider the infrared brightness of a flattened comet belt beyond the orbit of Neptune using a disk-like model with a power-law density distribution of comets. We compare this spectrum with the emission from a model zodiacal dust cloud in the ecliptic and with published IRAS data and present some consequences of dust in the comet belt.

1. Introduction

Oort (1950) modeled the source of long period comets as a spherical shell located in a region beyond 10 000 AU. If this cloud is a natural by-product of the formation of the solar system, then we wish to examine whether a relic of the protoplanetary disk exists in the region between 50 and 10 000 AU (Kuiper, 1951; Cameron, 1962). This 'inner Oort cloud' might contain up to 100 Earth masses of comets (Hills, 1981; Bailey, 1983c). The inner edge of the inner Oort cloud, called here the 'comet belt', has been suggested as a source for short period comets (Fernandez, 1980; Bailey, 1986). Its perturbational effects on Neptune have been considered by Whipple (1964). Hamid *et al.* (1968) estimated that the orbits of intermediate period comets limit the mass of the comet belt to a few Earth masses. Bailey (1983b) noted the effect of a dense inner cloud on the orbit of Neptune. Using spacecraft tracking data, Anderson and Standish (1986) place an upper limit of five earth masses on a comet belt with an inner boundary near Neptune. IRAS observations of dust clouds around main sequence stars are consistent with cometary disks as sources of IR excesses (Weissman, 1984). An extensive review of the inner Oort cloud is found in Weissman (1985).

2. Infrared Brightness

Bailey (1983b) and Bailey *et al.* (1984) calculated the infrared and dynamical constraints on a spherically-symmetric inner Oort cloud with a power-law density distribution. They concluded that up to 300 Earth masses of comets might reside in such a cloud. The inner part of a comet cloud beyond 40 AU will probably be quite flattened (Fernandez, 1984). There is the possibility that over the age of the solar

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system the inner cometary disk could have been stirred by random field stars (Hills, 1983), but unless the passage were recent, inelastic collisions would have caused the distribution to relax back to the near flat configuration from which it originated (Brahic, 1977). Following Bailey (1983) the structure of the comet belt between 40 and 10 000 AU is modeled with a distribution function

$$\nu(r, z, m) = kr^{-a} e^{-z^2/z_0^2} m^{-b}, \tag{1}$$

where r, z and ϕ are cylindrical coordinates and m is mass. Here, k is a normalizing constant and z_0 is the scale height determined by Cuzzi *et al.* (1979)

$$z_0^2 = \frac{2}{3}\frac{V_e^2}{\Omega^2} = C_0 r^3, \tag{2}$$

where Ω is the angular frequency at r. For an axisymmetric distribution of comets, $\nu(r, z, m)$ is the number of comets per unit volume in the range $(r, r + \mathrm{d}r)$, $(z, z + \mathrm{d}z)$ and masses m in the range $(m, m + \mathrm{d}m)$. We let the vertical thickness of the disk be determined, as in the primordial solar planetesimal disk, by a scale height dependent on the dispersion velocity of the disk (Safronov, 1972). Apart from factors of order unity, this random velocity is the escape velocity V_e, of the largest body in the population, given by

$$V_e^2 = \frac{2M_u G}{R_u}, \tag{3}$$

where M_u is its mass and R_u its radius.

The comets are considered as spherical black bodies of uniform composition radiating at uniform temperature

$$T = 280/\sqrt{R}\,(\mathrm{K}), \tag{4}$$

and the distance, R, is in AU.

If the cometary disk is limited by an inner boundary r_0 and an outer boundary r_u, then the mean brightness in the plane of the ecliptic is given by

$$I_\nu = \int_{r_0}^{r_u} \mathrm{d}r \int_{m_L}^{m_u} \mathrm{d}m\, \pi \left[\frac{3m}{4\pi d}\right]^{2/3} B_\nu[T(r)]\nu(r, 0, m), \tag{5}$$

where d is the mass density of a single comet and $B_\nu[T(r)]$ is the Planck function. The factor $\pi[3m/4\pi d]^{2/3}$ is the average projected area per comet. The Earth-Sun distance is considered to be small compared to r.

The normalization constant k in the distribution function is determined by the total mass of the cloud. If m_L is the mass of the smallest body, then the total mass is given by

$$m_T \simeq 4\pi \int_{m_L}^{m_u} \int_{r_L}^{r_u} \int_0^\infty \nu(r, z, m) mr\, \mathrm{d}r\, \mathrm{d}z\, \mathrm{d}m. \tag{6}$$

The radial index will be constrained by the form of the surface density of the cometary disk. Polyachenko and Fridman (1972) and Torbett *et al.* (1982) have used a surface density that varies inversely with the 1.5, 2, and 3 power, respectively. We define the surface density as

$$\sigma(r) = 2 \int_{m_L}^{m_u} m \, \mathrm{d}m \int_0^{\infty} \mathrm{d}z \, \nu(r, z, m). \tag{7}$$

For values of $a = 1.5$, 2, and 3, the mass of the inner belt will be less than 5 Earth masses in accordance with the results of Anderson and Standish (1986). A calculation of the mass contained between 40 and 50 AU shows that for $a = 1.5$, 2 or 3 this constraint is satisfied.

The brightness can be written as

$$I_\nu = \frac{1}{4C_0} \frac{M_T}{m_L} \frac{(3m_L/4\pi d)^{2/3}}{r_0^{5/2}} \frac{(2-b)}{(5/2-b)} \left[\frac{u_m^{5/3-b} - 1}{\mu_m^{2-b} - 1} \right] A(x_u) \tag{8}$$

with $\mu = m_u/m_L$ and $x_u = r_u/r_L$. For a surface density varying inversely with the square of the distance,

$$A(x_u) = \frac{1}{\ln(x_u)} \int_1^{x_u} x^{-7/2} B_\nu[T(x)] \, \mathrm{d}x; \tag{9}$$

and for a surface density varying inversely with the cube of the distance,

$$A(x_u) = \frac{x_u^2}{x_u^2 - 1} \int_1^{x_u} x^{-11/2} B_\nu[T(x)] \, \mathrm{d}x. \tag{10}$$

3. Zodiacal Emission

For the spatial distributions given above, the spectrum will peak at a temperature of 40 to 50 deg Kelvin for an inner boundary at 40 AU. We compare the spectrum at 100 microns with the tail of the spectrum of the zodiacal dust in the ecliptic. The standard 'fan' model of the small matter complex in the solar system described in Giese and Kinatede (1985) and Leinert *et al.* (1980) will be used to compute the spectrum or IR brightness. The dust distribution has the form

$$\nu(r, \beta, m) = K r^{-1.3} e^{-2.1 \sin \beta} m^{-b}, \tag{11}$$

where β is the ecliptic latitude. The mass distribution is taken to be in approximate collisional equilibrium and the spatial distribution is empirically derived by Grun *et al.* (1985).

The normalization constant K will be determined by the requirement that the spatial mass density match that measured at 1 AU, 9.6×10^{-23} gm cm^{-3} (Grun *et al.* 1985). With the mass limits $m_L = 10^{-8}$ gm and $m_u = 1$ gm (Grun *et al.*, 1985) and the spatial limits from 1 to 5 AU, the constant K is 2.3×10^{-6}. The brightness of the zodiacal dust is then

$$I_\nu \simeq \frac{K}{r_0^3} \left[\frac{3}{4\pi d} \right]^{2/3} e^{-2.1 \sin \beta} m_L^{5/3-b} \left[\frac{\mu_u^{5/3-b}}{5/3-b} \right] \int_1^{x_u} Q_{\mathrm{abs}} B_\nu(x) x^{-1.3} \, \mathrm{d}x. \tag{12}$$

We set Q_{abs}, the *IR* absorption coefficient, to unity, treating the particles as black bodies.

4. Comet Belt Parameters

Shoemaker and Wolfe (1984) show that evolution of planetesimals in the outer solar system results in a population that is peaked towards the inner edge of the cloud. Thus we choose the steeper power laws of 2 and 3 for the parameter, *a*. It is difficult to estimate the upper mass in the population of comets. Theoretical considerations (Goldreich and Ward, 1973) suggest that the upper mass should be about 10^{21} grams. We choose a conservative lower limit of one kilogram. The ensemble of particles considered is in collisional equilibrium; hence, the value of *b* is 11/6 (Dohnanyi, 1969). The inner edge of the comet belt is placed at 40 AU. An outer limit of 1000 AU will provide a safe margin from the disturbance by passing field stars and the galactic tide (Bailey, 1986) which tend to give the comet cloud a more thermalized spherical distribution.

With these parameters specified we numerically integrate the spectrum for the comet belt and for the zodiacal dust. The spectra are plotted in Figure 1. The two cloud models displayed correspond to surface densities of r^{-2} and r^{-3}, respectively. We present the published data from the IRAS (Hauser *et al.*, 1984) for comparison.

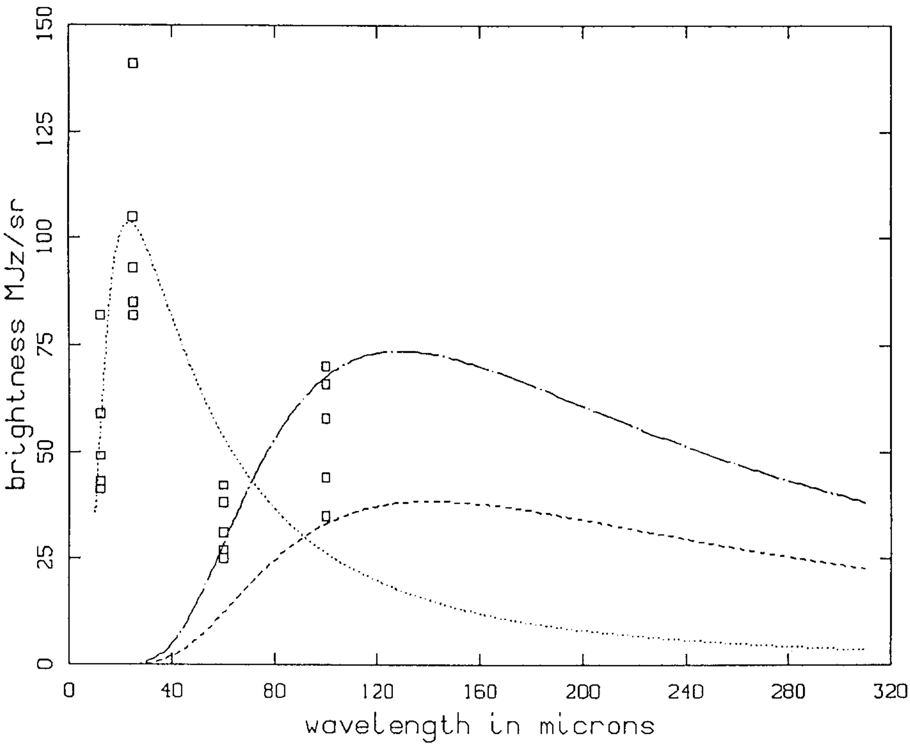

Fig. 1. The infrared brightness, in the ecliptic, of a comet belt with inner edge at 40 AU. The dashed curve is for a comet disk with a surface density that varies inversely with the square of the distance and the chain dotted is for one that varies inversely with cube of the distance. The dotted curve is the infrared brightness of the zodiacal dust cloud and the open squares are measured IRAS data as given in Hauser *et al.* (1984). Model parameters are described in the text.

5. Collisional Dust

Consider a simple case for the inner edge of the comet belt and suppose that there are five or less Earth masses distributed in the region between 40 and 50 AU with half height of 1 AU. If each comet has a diameter of approximately 10 km and a mass of about 10^{18} grams in the bounded volume of 4150 AU, the number density, n, is $\sim 10^6$ comets per cubic AU. If the comets have the dispersion velocity, v, of 0.1 km sec^{-1} and a collisional cross section, σ, then the collision rate is given by

$$R = n\sigma v, \tag{13}$$

implying almost 10 collisions per year between equal-sized objects. A mass distribution will give more collisions. At velocities of 0.1 km sec^{-1}, the collision energy per unit volume is $\sim 10^6$ ergs cm^{-3}, which exceeds the impact strength of model comet ice structures (Weidenschilling *et al.*, 1984).

What of the debris of these collisions? The infrared brightness of a cloud of dust in the ecliptic plane is estimated by

$$I_\nu \approx \int_{r_L}^{r_u} \int_{s_L}^{s_u} s^2 Q_{\text{abs}}[\pi B_\nu(T_{\text{ave}})] f(s) g(r) \, \mathrm{d}r \, \mathrm{d}s, \tag{14}$$

where s is the particle radius, Q_{abs} is the infrared absorption coefficient, $B_\nu[T_{\text{ave}}]$ is the Planck function, and $f(s)$ and $g(r)$ are the size and radial distributions, respectively.

If the dust dynamics is governed solely by collisions and radiation forces, the spatial distribution can be shown to be given (Trulsen and Wikan, 1980) by

$$g(r, z) = A_1 r^{-a} e^{-z^2/r^2}, \tag{15}$$

where the radial index would be 1 for Poynting-Robertson force alone but greater than 1 if collisions are included. We set the radial index to 3/2. The size distribution should be typical of a collisional system (Dohnanyi, 1969). Thus we consider

$$f(s) = A_2 S^{-3}. \tag{16}$$

We normalize these distributions by taking

$$M_{\text{Dust}} = \frac{4}{3}\pi d \, A_2 \int_{S_L}^{S_m} S^3 f(s) \, \mathrm{d}S, \tag{17}$$

$$I = 4\pi A_1 \int_{r_L}^{r_u} \int_0^{\infty} r^{-3/2} e^{-z^2/r^2} r \, \mathrm{d}r \, \mathrm{d}z, \tag{18}$$

where M_{Dust} is the total mass of dust, and A_1 and A_2 are constants.

Setting Q_{abs} to unity, we estimate the temperature, T, from (4) to be 42 K After normalization, the calculated brightness at 100 microns is given by

$$I_\nu(100\ \mu\text{m}) \approx \frac{9(\pi B_\nu) M_{\text{Dust}}}{16\pi^{3/2} \, dS_m \, r_u^{3/2} r_L^{1/2}} \ln(S_L/S_m). \tag{19}$$

The largest particle size will be taken as that surviving Poynting-Robertson drag over 10^9 yr, or about 1 cm. Radiation pressure will effect particles with a small complex index of refraction such as the dust debris of dirty ice. Particles smaller than 0.1 micron will be blown away (Burns *et al.*, 1979). For a belt of dirty ice dust located between 40 and 50 AU, the brightness will be

$$I(100\ \mu\text{m}) \approx 10^{-23} M_{\text{Dust}}(\text{MJz}/sr). \tag{20}$$

The average brightness at 100 microns is $I = 57\text{MJz}/sr$ (Hauser *et al.*, 1984), which implies that $M_{\text{Dust}} \lesssim 10^{23}$ grams. For a uniform distribution, this implies a spatial density of $\sim 10^{-20}$ gm cm^{-2}, about 1000 times that at 1 AU (Grun *et al.*, 1984). The zodiacal cloud has a collisional lifetime of about 10^5 yr. Since this time is inversely proportional to the number density of the collisional dust, this annular ring would have a lifetime of about 100 yr. Ten comet collisions per year would supply only 10^{19} grams; thus dust debris which meets the IR constraints would be seriously depleted in a short time. A more detailed calculation of catastrophic collisions (Dohnanyi, 1969) produces as much as 10^{15} gm sec^{-1}, which shortens the dust lifetime.

6. Conclusions

Comparison of the spectra in Figure 1 suggests that it may be possible to detect a comet belt by examining the diffuse IR emission in the ecliptic plane, and that the tail of the zodiacal brightness curve would not swamp the measured data. However, because of the number of free parameters in the problem, it is difficult at this time to draw any definitive conclusions. Nature could conspire to hide the proposed comet belt by placing the inner boundary far enough out of the solar system that its brightness becomes quite small. If most of the mass of the cloud lies too far away it will become too difficult to distinguish between radiation from a cometary cloud and the cosmic microwave background. If the cloud mass is small enough, the smooth galactic component will dominate.

The inner edge of a comet belt having a power law distribution may be indirectly detectable. If the number of collisions per year is not so excessive that catastrophic grinding shortens the dust lifetime, Poynting-Robertson drag may supply a component of the small particle population in the outer solar system. Thus dusty ice from the comet belt may be a source of the ice meteoroids hypothesized by Zook (1980) from data by Humes *et al.* (1974).

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References

Anderson, J. and Standish, E.: *The Galaxy and the Solar System*, University of Arizona (in press).

Bailey, M. E.: 1983a, *Nature* **302**, 399.

Bailey, M. E.: 1983b, *Monthly Notices Roy. Astron. Soc.* **205**, 47.

Bailey, M. E.: 1983c, *Monthly Notices Roy. Astron. Soc.* **204**, 603.

Bailey, M. E., McBreen, B., and Ray, T. P.: 1984, *Monthly Notices Roy. Astron. Soc.* **209**, 881.

Bailey, M. E.: 1986, *Nature* **27**, 350.

Brahic, A.: 1977, *Astron. Astrophys.* **54**, 895.

Burns, J. A., Philippe Lamy, and Steven Soter: 1979, *Icarus* **40**, 1.

Cameron, A. G. W.: 1962, *Icarus* **1**, 13.

Cameron, A. G. W.: 1978, S. F. McDermott (ed.), in *The Origin of the Solar System*, John Wiley and Sons, New York, p. 49.

Cuzzi, J. N., Durisen, R. H., Burns, J. A., and Hamill, P.: 1979, *Icarus* **38**, 54.

Dohnanyi, J. S.: 1969, *J. Geophys. Res.* **74**, 2531.

Fernandez, J. A.: 1980, *Monthly Notices Roy. Astron. Soc.* **192**, 481.

Fernandez, J. A.: 1984, in A. Carusi and G. Valsecchi (eds.), *Dynamics of Comets: Their Origin and Evolution*, D. Reidel Publ. Co., Dordrecht, Holland, p. 45.

Giese, R. H. and Kinatede, G.: 1986, in R. G. Marsden (ed.), *The Sun and Heliosphere in Three Dimensions*, D. Reidel Publ. Co., Dordrecht, Holland, p. 441.

Goldreich, P. and Ward, W. P.: 1973, *Astrophys. J.* **183**, 1051.

Grun, E., Zook, H. A., Fechtig, H., and Giese, R. H.: 1985, *Icarus* **62**, 244.

Hauser, M. G., Gillett, F. C., Low, F. J. Gautier, T. N., Beichman, C. A., Neugebauer, G., Aumann, H. H., Baud, B., Boggess, N., Emerson, J. P., Houck, J. R., Soifer, B. T., and Walker, R. G.: 1984, *Ap. J. Lett.* **278**, L15.

Hills, J. G.: 1981, *Astron. J.* **86**, 1730.

Humes, D. H., Alvarez, J. M., O'Neal, R. L., and Kinard, W. H.: 1974, *J. Geophys. Res.* **79**, 3677.

Kissel, J., Brownlee, D. E., Buchler, K., Clark, B. C., Fechtig, H., Grun, E., Hornung, K., Igenbergs, E. B., Jessberger, E. K., Kruger, F. R., Kuczera, H., McDonnell, J. A. M., Morfill, G. M., Rahe, J., Schwehm, G. H., Sekanina, Z., Utterback, N. G., Volk, H. J., and Zook, H. A.: 1986, *Nature* **321**, 336.

Kuiper, G. P.: 1951, in J. A. Hynek (ed.), *Astrophysics*, McGraw Hill Book Co., New York, p. 357.

Leinert, C., M. Hanner, I. Richter, and E. Pitz: 1980, *Astron. and Astrophys.* **82**, 328.

Oort, J. H.: 1950, *Bull. Astron. Inst. Neth.* **11**, 91.

Polyachenko, V. L. and Fridman, A. M.: 1972, *Sov. Astrn.* **16**, 23.

Safronov, V. S.: 1972, *Evolution of the Protoplanetary Cloud and Formation of the Earth and Planets*, NASA TT F–677.

Shoemaker, E. M. and Wolfe, R. R.: 1984, *Lunar and Planet. Sci. Conf. XV* 780.

Torbett, M., Greenberg, R., and Smoluchowski, R.: 1982, *Icarus* **49**, 313.

Trulsen, J. and Wikan, A.: 1967, in *Zodiacal Light and the Interplanetary Medium*, NASA-SP 150, 409.

Whipple, F. L.: 1964, *Proc. Nat. Acad. Sci.* **51**, 711.

Weidenschilling, S. J.: 1984, in R. Greenberg and A. Brahic (eds.), *Planetary Rings*, University of Arizona Press, p. 367.

Weissman, P. R.: 1984, *Science* **224**, 987.

Weissman, P. R.: 1985, *Space Science Reviews* **41**, 299.

Whipple, F. I.: 1964, *Proc. Nat. Acad. Sci.* **51**, 711.

Zook, H.: 1980, 'Evidence for Ice Meteoroids Beyond 2 AU', in I. Halliday and B. McIntosh (eds.), *Solid Particles in the Solar System*, D. Reidel Publ. Co., Dordrecht, Holland, p. 375.