# RADIATION PRESSURE CORRECTION TO METEOR ORBITS

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**Abstract.** We present a method to calculate the radiation pressure force to gravity ratio on meteoroids from their atmospheric flight. Radiation pressure corrections to meteor orbits are negligible for fireballs; of the order of or less than the measurement errors ($\approx$ 1 %) for photographic meteors; of the order of and in some cases substantially larger than the measurement errors ($\approx$ 10 %) for radar meteors.

**Key words:** Radiation pressure, meteors

## 1. Introduction

Numerous studies have been devoted to the reduction of heliocentric meteoroid orbits from multistation photographic and radar data. These typically correct the observed velocity vector for diurnal aberration and zenith attraction due to the Earth's gravity (Whipple and Jacchia, 1957), but they neglect the effect of radiation pressure during translation of the state vector to heliocentric orbital elements. The radiation correction to orbits of massive meteoroids is negligible, but radiation pressure can shift the orbits deduced from faint meteors significantly.

Radiation pressure from sunlight partially cancels solar gravity. The resulting field equations on a stationary object have the same form as the unperturbed gravity field around a factor 1-$\beta$ less massive star where $\beta$ is the ratio of the radiation pressure force to gravity. It follows that because the potential energy of a meteoroid at 1 AU increases with $\beta$, the heliocentric orbit corresponding to a given state vector also increases in size. We must know $\beta$ to estimate the true orbital energy.

Radiation pressure also distorts and amplifies existing distortions in the central force field. As radiation pressure increases, meteoroids bond less strongly to the Sun and the effect of planetary and other perturbations are amplified. This can be seen formally in the Gaussian perturbation equations where the perturbing force is expressed as a ratio to the central force. The distortion of the central force field has a negligible effect on the instantaneous orbit but affects the orbit's evolution over time. Orbital integrations to estimate a meteoroid's orbit at other epochs must account for this perturbation and for the Poynting-Robertson drag resulting from light absorption and scattering (Gustafson, 1994).

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The Poynting-Robertson drag causes meteoroids to spiral toward the Sun at rates proportional to $\beta$. The spiraling motion results as the orbital energy and angular momentum gradually transfer to the scattered or reemitted light. The determination of $\beta$ can therefore also be used to estimate the dispersion rate of meteoroids in a stream and give a measure of the age of meteor streams (Froeschlé *et al.*, 1993). Such analysis and orbital integrations of meteoroid trajectories to past epochs (Gustafson, 1989b; 1990) have been plagued by the lack of direct knowledge of the value of $\beta$, which inspired this study. We show how $\beta$ can be calculated from a meteoroid's atmospheric trajectory and how the immediately preceding heliocentric orbits of fireballs, photographic meteors, and radar meteors are affected by radiation pressure.

## 2. Calculation of $\beta$ from meteor data

The ratio $\beta$ is proportional to a meteoroid's effective cross-sectional area to mass ratio. The efficiency factor for radiation pressure, $Q_{pr}$, accounts for particle morphology and exposure geometry. For a perfect absorber we have $Q_{pr} \equiv 1$ independently of the shape and orientation. This is also a good approximation for any dark chondritic meteoroid producing photographic meteors and most detectable radio meteors. These meteoroids exceed a few times $10\mu$m across (Bronshten, 1983) and are opaque to sunlight so that the large particle approximation and geometric optics apply (Gustafson, 1994). The optically large meteoroids have $Q_{pr}$ values that are strictly independent of the particle shape when averages are made over random orientations Gustafson (1989a) as long as they are convex in shape. We can therefore choose any convex shape when we evaluate the efficiency factor $Q_{pr}$. This is easily done using geometric optics for a sphere of albedo $w$ (van de Hulst, 1957) where $Q_{pr} = 1 - wg$. The gemoetrical factor $g$ is inside the interval from unity (for totally forward scattering particles) to $-1$ (for total backscattering). We note that $Q_{pr}$ is close to unity at the small albedos of a few percent that are typical for cometary and asteroidal chondritic material and we adopt $Q_{pr} = 1$. We can therefore evaluate $\beta$ from the equation

$$\beta = C\, Q_{pr}\, A_0/m_0, \tag{1}$$

where the proportionality factor $C = 7.6 \times 10^{-5}\,\mathrm{g\,cm^{-2}}$ (Gustafson, 1994) and the subscripts denote that the cross-sectional area $A_0$ and the mass $m_0$ are values prior to atmospheric entry. We evaluate $A_0/m_0$ directly from meteor data.

The atmospheric flight of a meteoroid is a complex process. Severe problems emerge as we allow the meteoroid to be nonspherical, non-homogenous and fragmenting. Problems also arise in the modelling of transition between flow regimes. We expect to find a more reliable value of $A_0/m_0$ by confining our analysis to the uppermost part of the trajectory so that we can neglect fragmentation (unless the meteor has an abrupt beginning or has already started to fragment) and we can assume that the physical properties of the meteoroid remain constant. Further down along the trajectory this is not true.

To estimate $A_0/m_0$ we use classical single body meteor theory (e.g. Bronshten, 1983) where we treat only the cross-sectional area $A$ and the mass $m$ of the meteoroid as free parameters. It is then possible to integrate the system of classical equations to obtain two complementary estimates of $A_0/m_0$;

$$A_0/m_0 = \left(C_D \int_{h_1}^{h_2} \rho\, dh\right)^{-1} \cos z\, [\mathrm{Ei}(\kappa\, \mathrm{v}_1^2) - \mathrm{Ei}(\kappa\, \mathrm{v}_2^2)]\, \exp(-\kappa\, \mathrm{v}_0^2) \qquad (2)$$

and

$$A_0/m_0 = -2\,\dot{v}\,/(C_D\,\rho\, v^2)\, \exp[\kappa(v^2 - v_0^2)], \qquad (3)$$

where $C_D$ is the drag coefficient, $\rho$ the air density at height $h$, $\cos z$ the zenith distance, and $v_0$ the speed of the meteor at the onset of intensive evaporation. $\mathrm{Ei(x)} = \int_{-\infty}^{\mathrm{x}} \mathrm{e^t/t\, dt}$ is the Exponential integral and $\kappa = (1-\mu)\Lambda/(2C_DQ)$, where $\mu$ is the so called shape variation parameter, $\Lambda$ the heat-transfer coefficient (fraction of energy that goes into meteoroid ablation) and $Q$ the heat of ablation (the amount of energy needed to ablate a unit mass of meteoroid material). With introduction of the ablation coefficient $\sigma = \Lambda/(2C_DQ)$ and with the assumption of a self similar meteoroid, it is possible to write $\kappa = \sigma/6$.

We notice that Eq. (2) requires speeds, $v_1$ and $v_2$, at two heights, $h_1$ and $h_2$, in the atmosphere, and that Eq. (3) requires the speed, $v$, and the deceleration, $\dot{v}$, in one point. Equation (2) reduces to Eq. (3) if we let $h_2 \rightarrow h_1$ and substitute $dh/dt = -v \cos z$. It should also be noted that Eq. (2) is actually the integral of Eq. (3).

In Figure 1 we plot $\beta$, computed using the estimated $A_0/m_0$ (average of Eqs. (2) and (3)) in Eq. (1), using atmospheric flight data for twenty Harvard Meteor Program Geminid meteoroids (circles) and nine Perseids (crosses) from Jacchia *et al.* (1967). The $\beta$-values are plotted as a function of the photographic mass ($m_{\infty 2}$ adopted from Jacchia *et al.*, 1967). Based on their Knudsen numbers (c.f. Bronshten, 1983), all meteoroids were in the transition regime between the free-molecular flow regime and slip-flow in this part of their trajectory. From interpretation of photographic observations (Ceplecha, private communication) we use $\sigma = 0.012\,\mathrm{s^2\,km^{-2}}$ for Geminids and$\sigma = 0.042\,\mathrm{s^2\,km^{-2}}$ for Perseids with assumption of self similar meteoroids. We also adopt $C_D = 2$ (free-molecular flow) to estimate $A_0/m_0$ from Eq. (2) and Eq. (3). We notice that the Geminids seem to be

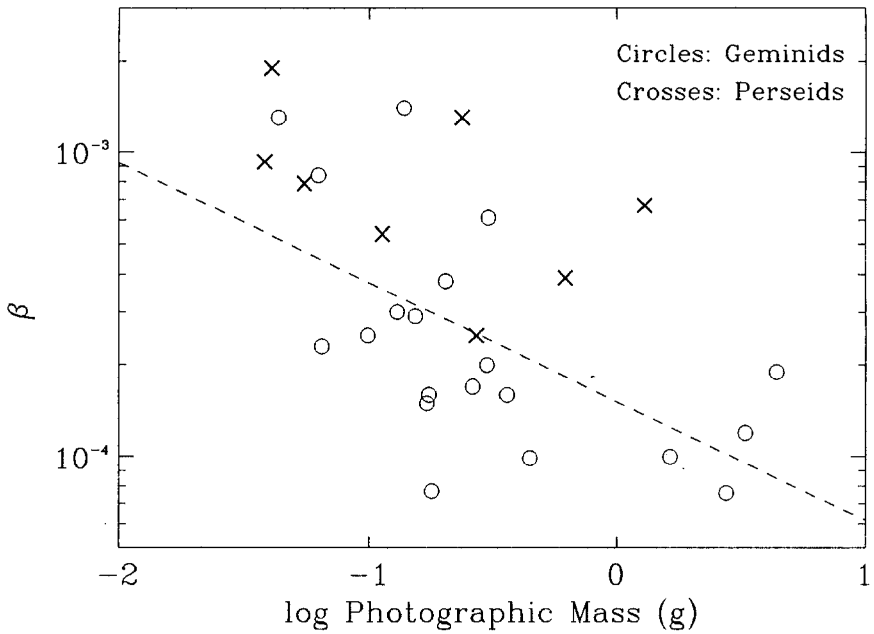


Fig. 1. Radiation pressure force to gravity ratio, $\beta$, as a function of photographic mass for twenty Geminid meteoroids (circles) and nine Perseids (crosses) photographed by Jacchia *et al.* (1967). The dashed line corresponds to the theoretical value, Eq. (1), for a sphere of bulk density $1\,\mathrm{g\,cm^{-3}}$.

denser than the Perseid meteoroids. This is consistent with previous investigations from which the Geminids are thought to be made of relatively tough, dense material. To provide a reference, the dashed line was generated using Eq. (1) and the surface area to mass ratio of a sphere of density $1\,\mathrm{g\,cm^{-3}}$. We notice that many points are located above this line indicating that the meteoroids are less dense or that they are aspherical. Another explanation might be that the photographic mass is overestimated leading to a left shift of all points in Figure 1. In any case, the value of $\beta$ is not affected by these uncertainties and $\beta$ is the only quantity used to generate new heliocentric orbits in the next section.

## 3. Radiation pressure correction to heliocentric orbits

Radiation pressure on a meteoroid vanishes as the meteoroid enters the Earth's shadow. Therefore, as we calculate a heliocentric orbit from meteor data, and integrate from the top of the Earth's atmosphere into interplanetary space, we need to account for radiation pressure ($\beta > 0$) as soon as the meteoroid leaves the Earth's shadow. The inclination and the longitude of the ascending node are not affected as radiation pressure only reduces the central force field. The effect on the argument of perihelion is automatically calculated in a full trajectory integration along with corrections to the semimajor axis $a$ and the eccentricity $e$. Complete results will be presented in a separate article.

In this section, we use analytic formulae for the change in $a$ and $e$ from the *Vis Viva* integral and estimate the magnitude of change in $a$ and $e$ by comparing the cases $\beta = 0$ (denoted $a_0$ and $e_0$) and $\beta > 0$ (denoted $a_\beta$ and $e_\beta$). Because the shift in the argument of perihelion is quite small it is not discussed here. To the first order in $\beta$ we can then write

$$a_\beta - a_0 = a_0 (2a_0/r - 1)\,\beta \tag{4}$$

and

$$e_\beta - e_0 = (a_0/r - 1)(1 - e_0^2)\,\beta/e_0, \tag{5}$$

where $r$ is the heliocentric distance. Realizing that $a_0 > r/2$, we see that the semimajor axis always increases with $\beta$. The eccentricity of a meteoroid's orbit decreases with $\beta$ when the semimajor axis is in the interval $r/2 < a_0 < r$. Otherwise, the eccentricity increases and the orbit is hyperbolic whenever $\beta > e_0/[(1+e_0)(a_0/r - 1)]$.

The correction for radiation pressure thus depends on the semimajor axis and $\beta$ which in turn depends most strongly on the meteoroid's size. Let us consider the Geminids, $a = 1.36\,\mathrm{AU}$ and $e = 0.896$, and the Perseids, $a = 28\,\mathrm{AU}$ and $e = 0.965$ (Cook, 1973). We assume for illustrative purposes, that the meteoroids can be represented by spheres of bulk density $1\,\mathrm{g\,cm^{-3}}$. We then compare the effect on meteoroids of masses $10^5$ g (fireballs), $10^0$ g (photographic meteors), and $10^{-5}$ g (radar meteors) using Eqs. (1), (4) and (5). We notice from Eqs. (4) and (5) that generally, the effect will be larger for the Perseids than for the Geminids since their semimajor axes are larger. We also notice that the normalized correction (%) in semimajor axis is approximately one order of magnitude larger than the correction in eccentricity.

We find that the correction for fireballs is less than $\approx 0.01\,\%$. This is negligible in comparison with the uncertainty in semimajor axis and eccentricity at the one standard deviation level as given by Ceplecha *et al.* (1983) for European Network fireballs. The semimajor axis increases by $\approx 0.5\,\%$ for a Perseid and $\approx 0.02\,\%$ for a Geminid photographic meteor. These corrections equal (Perseids) or are a magnitude less (Geminids) than the standard deviations given by Jacchia and Whipple (1961); we can not neglect the radiation pressure correction to photographic meteors with large semimajor axes. The correction to radar meteors is slightly less than 1 % for Geminids and 24 % for Perseids. The best radar observations made today allow us to determine $a$ to about 10 % (Baggaley *et al.* 1992). We notice that the radiation pressure correction to radar meteors is actually substantially larger than the measurement errors. In addition, the correction is systematic.

In conclusion, we can not always neglect radiation pressure when computing a meteoroid's heliocentric orbit prior to atmospheric entry. Furthermore, it is important to determine the value of $\beta$ even when it is numerically small so that we may investigate the long-term dynamical evolution of meteor streams, their life-span as a stream, and retrace the orbits to their origin following Gustafson (1989b).

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