LONG-TERM BEHAVIOR OF THE MOTION OF PLUTO OVER 5.5 BILLION YEARS

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Abstract. The motion of Pluto is said to be chaotic in the sense that the maximum Lyapunov exponent is positive: the Lyapunov time (the inverse of the Lyapunov exponent) is about 20 million years. So far the longest integration up to now, over 845 million years (42 Lyapunov times), does not show any indication of a gross instability in the motion of Pluto. We carried out the numerical integration of Pluto over the age of the solar system (5.5 billion years $\approx$ 280 Lyapunov times). This integration also did not give any indication of chaotic evolution of Pluto. The divergences of Keplerian elements of a nearby trajectory at first grow linearly with the time and then start to increase exponentially. The exponential divergences stop at about 420 million years. The divergences in the semi-major axis and the mean anomaly (equivalently the longitude and the distance) saturate. The divergences of the other four elements, the eccentricity, the inclination, the argument of perihelion, and the longitude of node still grow slowly after the stop of the exponential increase and finally saturate.

1. Introduction

Sussman and Wisdom (1988) carried out the numerical integration of Pluto for 845 million years (Myr hereafter) with the Digital Orrery, a special purpose computer. This integration took about three months. In this computation Pluto was a massless particle disturbed by other giant outer planets, Jupiter, Saturn, Uranus, and Neptune. Their integration indicates that the long-term motion of Pluto is chaotic and the largest Lyapunov exponent is about $10^{-7.3}$ year^{-1} (the Lyapunov time is about 20 Myr). Laskar (1990) numerically integrated for 100 Myr the secular Hamiltonian system of the

Earth, Moon, and Planets **72**: 165-173, 1996.

whole solar system (excluding Pluto) that is obtained after elimination of short periodic terms and is second-order with respect the disturbing masses. His integration showed that the solar system is chaotic with the Lyapunov time of only 4 Myr. Sussman and Wisdom (1992) carried out an integration of the whole solar system including Pluto with the Toolkit, another special purpose parallel computer, which took about 1000 hours. Their computation confirmed Laskar's results in direct integration instead of averaged equations. The evolution of Pluto in this 100 Myr integration is similar to that of Pluto found in 845 Myr integration of the outer planets system (Sussman and Wisdom 1988). Although the motion of Pluto is chaotic in the sense that the largest Lyapunov exponent is positive, the integration does not give any indication of a gross instability in the motion of Pluto. According to Wisdom (1992), "Since the system is apparently chaotic, we cannot rule out the possibility of gross instability. Recall some chaotic asteroid trajectories have been seen evolve chaotically for 100 Lyapunov times at low eccentricity and then suddenly jump to large eccentricity. It will be very interesting to see a number of integrations of the whole solar system for the age of the solar system and longer." We carried out the integration of the outer planets system over 5.5 billion years which is about 280 Lyapunov times. In this computation Pluto is treated as a massless particle in order to compare our results with those of Sussman and Wisdom (1988). The integrator is a 12th-order linear symmetric multistep method (Quinlan and Tremaine 1990). The error estimate of our integration is given in section 2 and preliminary results are then presented.

2. Method of Numerical Integration and its Accuracy

Pluto is integrated as a massless particle that is disturbed by four giants planets (Jupiter, Saturn, Uranus, and Neptune). The masses of the inner planets are added to the Sun. The planetary masses and their initial conditions are taken from DE245, which is the most recent planetary ephemerides developed at JPL.

As an integration formula, we adopt a linear symmetric multistep integrator (LSMI), which is one of linear multistep integrators whose coefficient in the formula are symmetric (Quinlan and Tremaine 1990). One of the great merits of LSMI is that the truncation (discretization) errors do not produce secular errors in the energy and the angular momentum, in other words no secular errors in the semi-major axis, eccentricity, and inclination.

The numerical computation was carried out on an FMR70-HX3 with an accelerator board. FMR70-HX3 is a personal computer whose cpu is Intel's i386 with clock speed of 25MHz and which does the job of input and output of data between FMR and the accelerator board. The accelerator board uses Intel's i860 cpu with clock speed of 40MHz and does only numerical integrations of equations of motion.

In order to reduce round-off errors, we evaluated the 12 th-order LSMI (Quinlan and Tremaine 1990) in the following form:

$$x_{n+k} = -\alpha_{k-1} \otimes x_{n+k-1} \ominus \cdots \ominus \alpha_0 \otimes x_n \oplus h^2 \sum_{j=0}^{k-1} \beta_j f_{n+j}, \qquad (1)$$

where $\alpha_i = \alpha_{k-i}, \beta_i = \beta_{k-i}, i = 0, \cdots, k = 12$, and $\alpha_{12} = 1$.

The $\otimes, \oplus, \ominus$ symbols mean multiplications, additions, and subtractions in quadruple precision. The quadruple arithmetic operations are carried out by software which is written in assembler. Other operations (the evaluation of the force and the last part of the right-hand side of (1)) are done with double precision.

Because of the symmetry in the coefficients of LSMI, the result by LSMI has a time-reversal nature. Therefore the time reversal test (integrating forward and then backward) does not give any information on the accumulation of the truncation errors. For checking the accuracy of orbits obtained by the 12 th-order LSMI, we made reference orbits of one Myr integrated by the extrapolation method developed by Gragg(1965), which are computed in quadruple precision. The accuracy of the reference orbits themselves are examined by the time reversal test. Assuming that the longitude error increases proportional to square of time, the longitude error of Jupiter is about 0.016 arcsecond after 5.5×10^9 years, which is precise enough to test the accuracy of orbits computed in lower precision. The difference in the longitude of Jupiter after one Myr between the orbits obtained with 12-th LSMI and the reference orbits is $1.^\circ 6 \times 10^{-5}$. Since the longitude error due to the truncation grows linearly for the LSMI and the longitude error due to round-off errors increases with the power of 3/2 of the time, the round-off errors become dominant soon. Extrapolation by the power of 3/2 of the longitude error of Jupiter over 5.5 billion years is about $6^\circ.5$. Similarly the longitude error of Pluto is about $0.^\circ 3$ after 5.5 billion years.

The timespan of one run of our integration was 4×10^{10} days $\approx$ 110 Myr, and one run took 53 hours using the FMR with the accelerator board. We carried out 50 runs of this computation whose total time was 110 days. We made output of the positions and velocities of the 5 outer planets and a nearby orbit of Pluto in double precision for every 2 million days $\approx$ 5500 years, whose total amount is 296 mega bytes. These data are available on request.

3. Results

Figure 1 shows four Keplerian elements of Pluto (the eccentricity, the argument of perihelion, the inclination, and the longitude of node referred to the longitude of Neptune's node) for the first 100 Myr of 5.5 billion years integration, and Figure 2 exhibits those over the last 100 Myr. There is no indication of global instability of Pluto's motion.

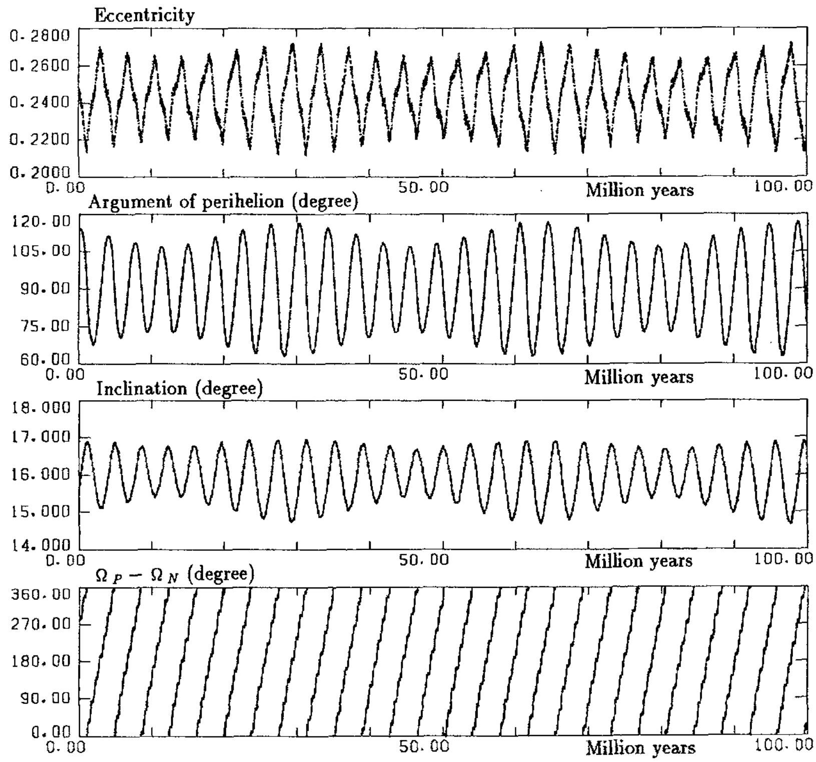

Figure 1. Keplerian elements of Pluto (the first 100 million years).

In the motion of Pluto three resonances are found in the past works.
1) Pluto is in the mean motion resonance with Neptune. The critical argument $\theta_1 = 3\lambda_P - 2\lambda_N - \varpi_P$ of the 3:2 mean motion resonance librates around 180 degrees with the amplitude 81.2 degrees and the libration period is 2.0×10^4 years. The dominant periodic component in the variation of the semi-major axis is the libration of the critical argument θ_1. The amplitude in the semi-major axis is 0.15AU
2) The argument of perihelion $\theta_2 = \varpi_P - \Omega_P$ librates around 90 degrees and its period is 3.8 Myr. The dominant periodic variations of the eccentricity

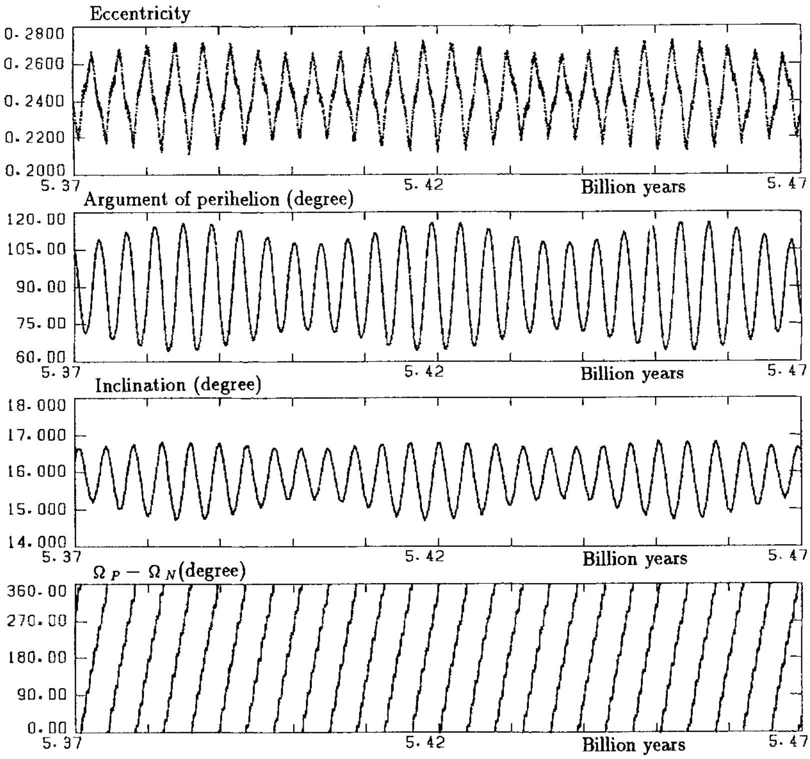

Figure 2. Keplerian elements of Pluto (the last 100 million years).

and the inclination are synchronized with the libration of the argument of perihelion, which is expected from the secular perturbation theory (Kozai 1962). The variations of θ_2, the eccentricity, the argument of perihelion, and the inclination are modulated with 34 Myr periodicity.

3) Moreover the longitude of Pluto's node referred to the longitude of Neptune's node, $\theta_3 = \Omega_P - \Omega_N$, circulates and the period of this circulation is equal to the period of the libration of θ_2. When θ_3 becomes zero, the inclination of Pluto referred to the invariable plane takes a maximum, the eccentricity reaches a minimum, and the argument of perihelion is 90 degrees. When θ_3 becomes 180 degrees, the inclination takes a minimum, the eccentricity reaches a maximum, and the argument of perihelion is again 90 degrees.

This new type of resonance was conjectured by Williams and Benson (1971) and was confirmed by Milani et al. (1989) who called this kind of resonance the 1:1 super resonance. This type of resonance is also called a secondary resonance. All these three resonances are well kept over 5.5 billion years.

Williams and Benson (1971) discussed the behavior of the argument $\theta_4 = \varpi_P - \varpi_N + 3(\Omega_P - \Omega_N)$. In the Longstop 100-Myr integration (Nobili et al. 1989) the argument θ_4 seems to circulate with a period of about 246 Myr and in the Digital Orrery (Sussman and Wisdom 1988) θ_4 seems to librate, from the consideration that $\dot{\theta}_4$ is consistent with zero. Milani et al. (1989) suggested the possibility that the argument θ_4 alternately librates and circulates and this may be the origin of the positive Lyapunov exponent. Figure 3 shows the variation of θ_4 over 5.5 billion years. θ_4 clearly and stationary librates around 180 degrees with 570 Myr period. There is no indication of interchange of libration and circulation.

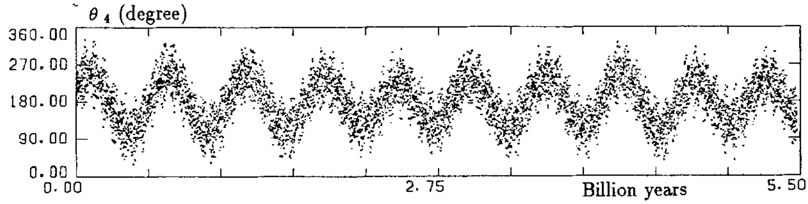

Figure 3. Argument $\theta_4 = \varpi_P - \varpi_N + 3(\Omega_P - \Omega_N)$ over 5.5 billion years.

Figure 4 shows the deviations of the Keplerian elements between Pluto and its nearby orbit whose initial conditions are slightly different (the relative distance in the phase space is 10^{-12}). The deviations first increase proportionately with time and then from about 150 Myr start to increase exponentially. The time scale of the exponential increase is about 20 Myr, is in good agreement with the Lyapunov time (the inverse of Lyapunov exponent) of 20 Myr (Wisdom et al. 1998 and Nakai et al. 1992). The deviations of the semi-major axis and the mean anomaly saturate after 420 Myr and after do not increase. This saturation is related to the mean motion resonance. In fact the deviation of the critical argument θ_1 of the two orbits saturates at 162.4 degrees which is twice of the amplitude of the libration of the critical argument, and the deviations of the mean longitudes and the mutual distance of two orbits saturate at 70 degrees and 44 AU, respectively.

The exponential increase of the deviations of the eccentricity, inclination, argument of perihelion, and longitude of node stops after 420 Myr and then increases slowly and seems to finally saturate (see Figure 4). The final saturation after 5.5 billion years in these four elements is related to other two resonance lockings, the libration of the argument of perihelion and the secondary resonance between θ_2 and θ_3. Figure 5 shows the inclination of Pluto referred to the invariable plane versus the argument θ_3. Due to the secondary resonance, the inclination oscillates as a stationary

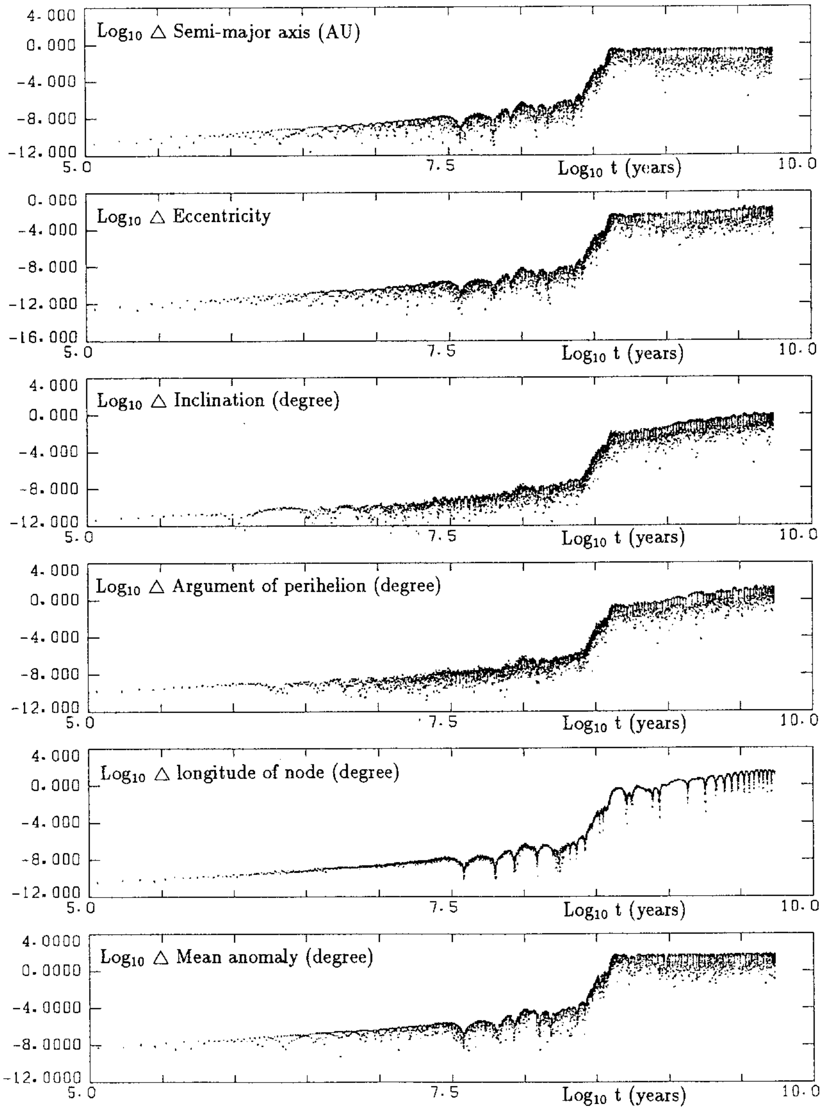

Figure 4. Deviations of the Keplerian elements between Pluto and its nearby orbit.

wave, which goes up and down with the period of 34 Myr. The width of this stationary wave is about 1.2 degree, which is equal to the saturated deviation of the inclination of two orbits. Similar discussions can be applied to the eccentricity and the argument of perihelion.

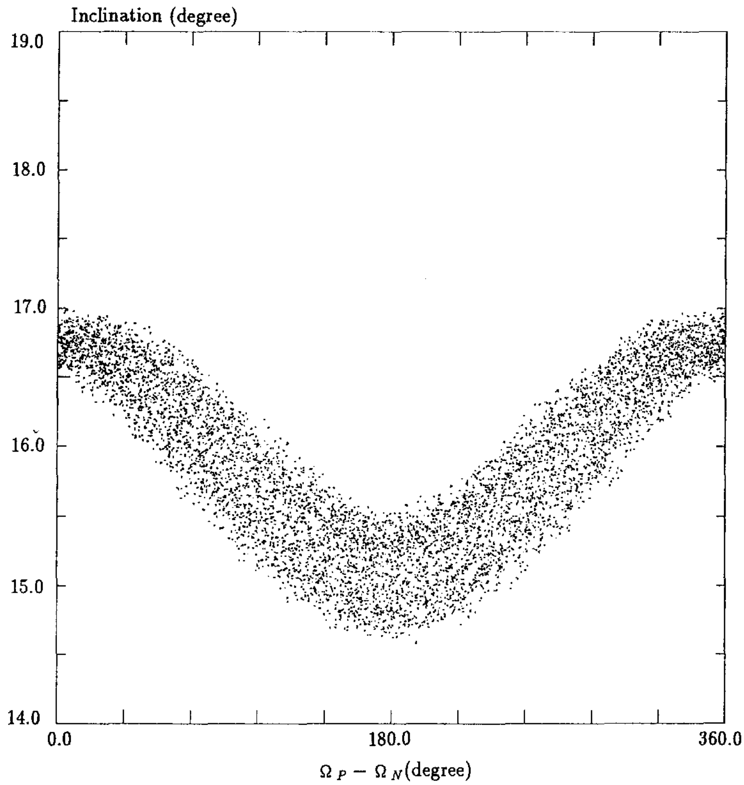

Figure 5. The inclination of Pluto referred to the invariable plane versus the argument $\theta_3 = \Omega_P - \Omega_n$.

4. Concluding Remarks

One peculiar fact on Pluto's motion for us is that the duration of the linear deviation of nearby orbits with time is too long, 150 Myr $\approx$ 600,000 revolutions of Pluto. The duration of the linear growth and the exponential divergence do depend on the initial distance (Nakai et al. 1992).

As mentioned at the beginning of Section 3, Pluto is locked in three resonances. Moreover the argument θ_4 librates around 180 degrees with a 570 Myr period. The behavior of θ_4 depends on the initial conditions. We carried out several numerical integrations with different initial values of θ_1 (the critical argument of the mean motion resonance) keeping other elements unchanged (Nakai and Kinoshita 1994). When the initial value of θ_1 is small, θ_4 circulates with prograde direction. As the initial value of θ_1 increases, the behavior of θ_4 changes from prograde circulation to libration around 180 degrees, and then to retrograde circulation, and other orbital elements do not show any irregular variations. From these experiments, the behavior of θ_4 does not seem to relate to the global stability of Pluto's motion. When the libration amplitude of θ_1 reaches about 90 degrees, the secondary resonance is destroyed, but orbital elements do not show any irregularities. However when the amplitude of θ_1 becomes larger than about 110 degrees, the second resonance (the libration of the argument of perihelion) is destroyed and all orbital elements show irregular changes. Even though from a very limited number of experiments we cannot derive a general conclusion of Pluto's motion, we do think the secondary resonance does not have an important role in the stabilization of Pluto's motion.

In this paper we integrated Pluto's motion towards the future. From the time reversibility of equations of motion, we do think 5.5 billion years integration of Pluto towards the past do not show any essential difference from the results of this paper. In this sense, in order to investigate how Pluto evolves to the present state of three resonance lockings, we have to take account of a non-conservative mechanism in the early stage of the solar system.

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