DYNAMICAL EVOLUTION OF NEAS: CLOSE ENCOUNTERS, SECULAR PERTURBATIONS AND RESONANCES

PATRICK MICHEL, CHRISTIANE FROESCHLÉ and PAOLO FARINELLA*
Observatoire de la Côte d'Azur, Dept. Cassini, B.P. 229, 06304 Nice Cedex 4, France.

Abstract. We discuss the main mechanisms affecting the dynamical evolution of Near-Earth Asteroids (NEAs) by analyzing the results of three numerical integrations over 1 Myr of the NEA (4179) Toutatis. In the first integration the only perturbing planet is the Earth. So the evolution is dominated by close encounters and looks like a random walk in semimajor axis and a correlated random walk in eccentricity, keeping almost constant the perihelion distance and the Tisserand invariant. In the second integration Jupiter and Saturn are present instead of the Earth, and the 3/1 (mean motion) and ν_6 (secular) resonances substantially change the eccentricity but not the semimajor axis. The third, most realistic, integration including all the three planets together shows a complex interplay of effects, with close encounters switching the orbit between different resonant states and no approximate conservation of the Tisserand invariant. This shows that simplified 3-body or 4-body models cannot be used to predict the typical evolution patterns and time scales of NEAs, and in particular that resonances provide some "fast-track" dynamical *routes* from low-eccentricity to very eccentric, planet-crossing orbits.

Key words: Near-Earth asteroids, Resonances, Close encounters

1. Introduction

Near-Earth asteroids (NEAs) are widely believed to be continuously injected into Earth-approaching orbits through a few different resonant channels, which collect fragments randomly ejected from main-belt asteroids as a consequence of energetic interasteroidal collisions (see e.g. Wetherill, 1985, 1987; Farinella *et al.*, 1993, 1994). Subsequently, these fragments undergo a fairly complex orbital evolution process, driven by (mean motion and secular) resonances, by non-resonant secular perturbations, and by a sequence of close encounters with the inner planets. This process has been and is being studied by numerical techniques (Milani *et al.*, 1989; Farinella *et al.*, 1994; Froeschlé *et al.*, 1995; Valsecchi *et al.*, 1995), with the integrations showing a puzzling variety of phenomena and behaviours. However, a basic qualitative understanding of the main mechanisms at work would also be important, both to interpret and classify the integration outputs, and to devise some simplified, statistical model of the dynamical evolution process to be applied not to single objects, but to entire populations.

* On leave from the Department of Mathematics, University of Pisa, Via Buonarroti 2, 56127 Pisa, Italy, thanks to the "G. Colombo" fellowship of the European Space Agency.

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So far, such statistical models have been developed (Wetherill 1985, 1987; Melosh and Tonks, 1993; Bottke et al., 1994, private communication) under the following simplifying assumptions: (i) a Monte Carlo algorithm together with Öpik's (1976) analytical theory is used to assess the occurrence of planetary encounters and predict their outcomes; (ii) secular perturbations are usually taken into account only by assuming uniformly precessing apses and nodes; and (iii) resonant effects are treated as occurring at fixed values of the semimajor axis (2.5 AU for the 3/1 mean motion resonance with Jupiter, 2.05 AU for the ν_6 secular resonance, etc.) and causing essentially rapid jumps in eccentricity and/or inclination. Thus, whenever resonances are not at work, the orbital evolution is modelled basically as an encounter–driven random walk in the semimajor axis–eccentricity–inclination (a–e–I) space; if encounters with only one planet are possible (or if a planet plays a dominant role owing to its larger mass, e.g. the Earth vs. Mars), this random walk is constrained to occur near a T = constant surface, where T is the Tisserand invariant relative to the dominant planet (assumed to have a quasi–circular orbit). For a detailed discussion of the corresponding evolutionary patterns, we refer to the recent review by Greenberg and Nolan (1993).

In this paper we are going to argue that these models, albeit providing useful qualitative insights, do not account for all the main features of the orbital evolution of NEAs. The main reason is that secular perturbations play a role at least as important as that of encounters in guiding the orbital evolution in the a–e–I space, and that resonances need to be modelled in a more complex way to obtain realistic predictions of their effects. An example of this is the recent finding that resonant effects can pump up the eccentricity to almost unity on a time scale $< 10^6$ yr, thus causing a significant fraction of NEAs to end up hitting the Sun (Farinella *et al.*, 1994; Froeschlé *et al.*, 1995).

We shall discuss some of these problems by using as a typical example of NEA evolution the numerically integrated orbit of asteroid (4179) Toutatis. Our integrations have spanned a time scale of 1 Myr (such as required to detect and interpret most secular effects), and used several different dynamical models to point out the main mechanisms at work. As discussed by Whipple and Shelus (1993) and by Benest *et al.* (1994), who integrated the same orbit over shorter time spans (up to $\approx 10^5$ yr), the orbit of Toutatis is extremely chaotic, with a Lyapounov characteristic time of the order of 100 yr due to frequent encounters with the inner planets, and with the 3/1 Jovian resonance affecting the orbit on a longer time scale. Here we shall extend the afore-mentioned studies with a particular emphasis on the aspects important not just for Toutatis itself, but rather for the long–term evolution of the entire NEA population. In Sec. 2 we shall describe our integrations and discuss the most interesting results, whereas in Sec. 3 we shall elaborate on the significance and implications of these results.

2. Numerical Experiments on Toutatis

2.1. Integration method and initial conditions

Given that close approaches with planets cause fast variations of the orbital elements of NEAs, it is necessary to use an accurate method to integrate the equations of motion. However, whereas a small time step is required when the integrated object is close to a planet (or the Sun) and moves very fast relative to it, a much longer time step can be used when the NEA is far from the planets. Therefore, an integration method using a variable stepsize turns out to be the optimal choice. We adopted the Bulirsch–Stoer extrapolation method ((Stoer and Bulirsch, 1980)) which, through a controlled variable stepsize, allows to handle close approaches with planets much more accurately than with a fixed stepsize. After a number of tests, we chose a restrictive value (10^{-12} instead of the usual 10^{-8}) of the convergence parameter ϵ, which determines the difference between two successive estimates in the iteration.

It is important to emphasize that, even if the integration method is very accurate, for such strongly chaotic orbits the results cannot be deterministic over a time span much longer than the Lyapounov time, and the integrations can provide only qualitative information on the long–term orbital evolution. The reasons are the following. First, the assumed dynamical model always neglects some real small perturbations, whose effects get amplified in a stochastic fashion. Second, owing to the exponential increase of small numerical errors, two different integration methods applied to the same dynamical model and the same initial conditions will give two different orbital evolutions (with similar qualitative behaviors). Likewise, the same integration method used on two different computers will yield quantitatively different results, due to different round–off errors. Thus, while for the sake of simplicity we will talk about the real NEA Toutatis, the orbital evolutions computed with even the most accurate method and the most realistic dynamical model are not necessarily predictive of the behavior of the real asteroid.

Initial conditions for the planets and Toutatis, for the epoch 1993 January 13.0 (JD = 244900.5), were kindly provided to us by G. Hahn (1994, personal communication). The orbital angles are referred to the J2000 equator and equinox. Table I contains the initial conditions of Toutatis and Table II gives the masses of the planets taken from the JPL ephemeris DE200. Note that we combined the Earth and the Moon into a single body located at the

Earth–Moon barycenter, and added the mass of Mercury to that of the Sun. Toutatis is assumed to be massless.

TABLE I
Osculating Orbital Elements of Toutatis, Referred to the J2000 Ecliptic and Equinox

Epoch 1993 January 13.0	(JD= 244900.5)
$M = 15^\circ.05349$	$a = 2.5054645AU$
$e = 0.6398550$	$i = 0^\circ.46674$
$\omega = 276^\circ.28112$	$\Omega = 126^\circ.48206$

2.2. The three dynamical models

In order to discriminate and assess the significance of each of the mechanisms which affect the orbit of Toutatis, we used three different dynamical models:

(1) In the first model (Sun–Earth–asteroid), the perturbations are mainly due to close approaches with the Earth, located between the Sun and Toutatis, almost in the same plane.

(2) In the second model (Sun–Jupiter–Saturn–asteroid), the orbit of Toutatis is mainly affected by resonance mechanisms with the outer planets, as no close approaches occur.

(3) The third model (Sun–Earth–Jupiter–Saturn–asteroid) takes into accoun all the main perturbation forces that affect the orbit of the real Toutatis, and is therefore the most realistic (and complex) one.

TABLE II
Masses of Planets

Planets	Masses ($M_\odot = 1$)
Sun+Mercury	1.000000166013679527193
Earth+Moon	0.0000030404327387 1084
Jupiter	0.000954786104043042
Saturn	0.000285877644368210

For all the three models, we numerically integrated the Newtonian equations of motion of the planets along with Toutatis over a time span of 1 Myr.

2.3. Results

2.3.1. *The Sun–Earth–asteroid model*

The evolution of Toutatis in this model is characterized by a random walk of the semimajor axis a and of the eccentricity e of its orbit due to close approaches with the Earth (see Fig. 1). Actually, the small inclination of Toutatis increases significantly the frequency of these encounters, as they can occur also relatively far from the mutual nodes. As a consequence, the random walk is fairly effective, with the semimajor axis changing by about 1 AU over the integration time span. Thus for this type of orbits, Earth encounters are sufficient to cause a comparatively fast evolution in the orbital element space and in particular (as we shall see later on) to jump frequently between different resonant states.

In Fig. 1 a strong correlation is also apparent between the semimajor axis and the eccentricity of the asteroid, with the perihelion distance staying almost constant. This fact can be explained in a simple way by using Gauss' perturbation formulae (see e.g. Bertotti and Farinella, 1990, Ch. 11). If we treat close approaches as causing fast, quasi–impulsive changes of the osculating Keplerian elements, these changes can be expressed as a function of the impulsional velocity increment δV. We have:

$$\delta a = \frac{2}{n\sqrt{1-e^2}}[\delta V_1 + e(\delta V_1 \cos f + \delta V_2 \sin f)] \tag{1}$$

$$\delta e = \frac{\sqrt{1-e^2}}{na}[\delta V_2 \sin f + \delta V_1(\cos f + \frac{e+\cos f}{1+e\cos f}] \tag{2}$$

where f is the asteroid's true anomaly at the time of the encounter and n is its mean motion. δV_1 and δV_2 are the transverse and radial components of the impulsional velocity increment. Assuming that the geometry of Toutatis' orbit is such that the approaches with the Earth always occur near perihelion (a good approximation, as the perihelion distance keeps close to 1 AU), one can approximate those equations by taking $f \approx 0$. Then, from Eq. (1), we have:

$$\delta V_1 \approx \frac{n\delta a\sqrt{1-e^2}}{2(1+e)}, \tag{3}$$

and substituting this expression in Eq. (2), we obtain:

$$\delta e \approx \frac{\delta a}{a}(1-e). \tag{4}$$

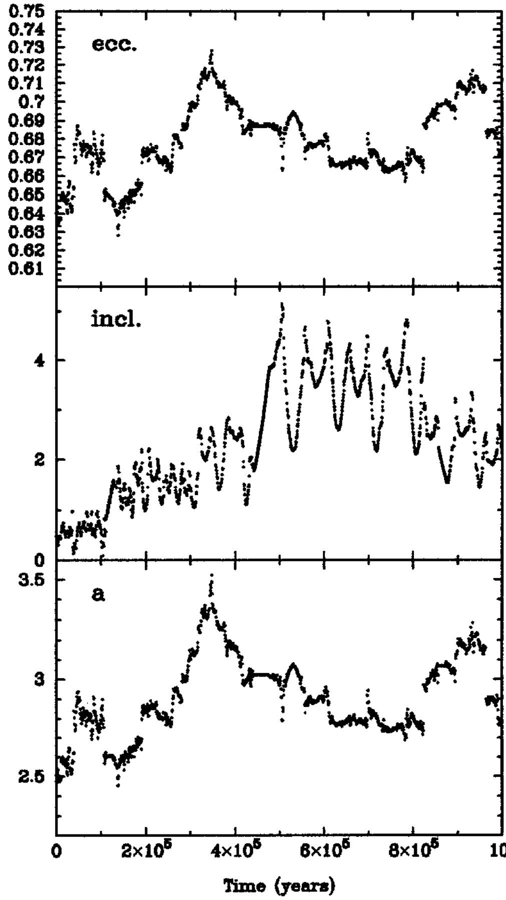

Fig. 1. Evolution of the semimajor axis (in AU), the inclination (degrees) and the eccentricity of Toutatis' orbit over 1 Myr in the Sun–Earth–asteroid model.

Thus δe is proportional to δa and the evolutions of e and a are well correlated. Note that Eq. (4) implies that the perihelion distance q of the orbit of Toutatis is almost constant. This is consistent with the (approximate) conservation of the Tisserand invariant of this problem. Indeed, if we neglect the Earth's orbital eccentricity, the model provides just a restricted three-body problem, for which the Jacobi constant is conserved and the Tisserand parameter

$$T \ = \ 1/a + 2[a(1-e^2)]^{1/2}\cos I \tag{5}$$

(a being expressed in units of the perturbing planet's semimajor axis, namely, in our case, AU) is not modified by the encounters. Thus the orbit must remain close to a surface T = constant in the orbital element space. Actually, Fig. 2 shows the contour lines for different values of the Tisserand invariant

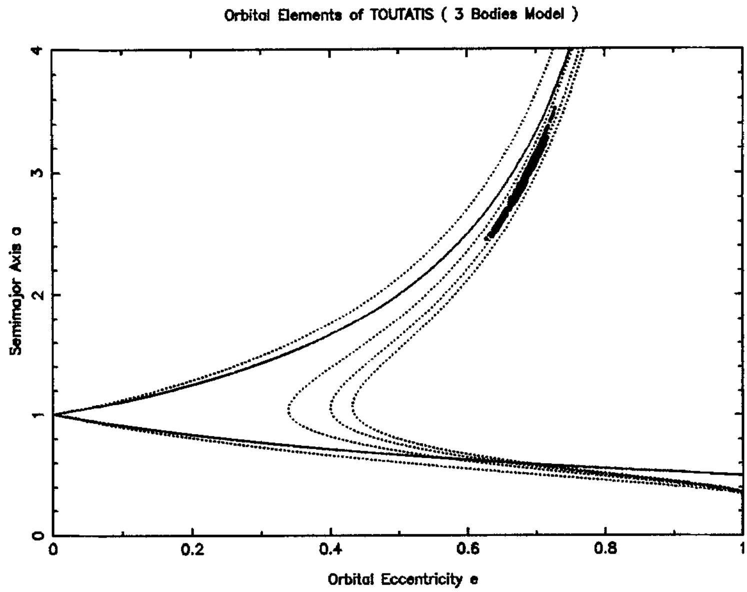

Fig. 2. The black dots show the evolution in the semimajor axis a (in AU) vs. eccentricity e plane for the numerically integrated orbit of Toutatis over 1 Myr, with the Sun–Earth–asteroid model. The two solid lines correspond to the perihelion q and aphelion Q distances equal to 1 UA. The dotted lines are the contours of the Tisserand constant T at zero inclination and correspond (from left to right) to $T = 3$, 2.88, 2.83 and 2.80.

for $I = 0$, and shows that the integrated orbit of Toutatis always stays close to the contour line $T = 2.83$, corresponding to its initial osculating elements. Note also that as $q = a(1 - e)$, another expression for T is:

$$T = \frac{1-e}{q} + 2[q(1+e)]^{1/2} \cos I . \tag{6}$$

Then, if $q = 1$ and e is small, we can expand T into a series of e:

$$T = 1 - e + 2\left[1 + \frac{1}{2}e + \mathrm{O}(e^2)\right] \cos I , \tag{7}$$

which, for $I = 0$, becomes:

$$T = 3 + \mathrm{O}(e^2) . \tag{8}$$

Therefore, the orbit of a body with small inclination and for which $T \approx 3$ will evolve near the line $q = 1$, thus causing frequent Earth approaches.

2.3.2. *The Sun–Jupiter–Saturn–asteroid model*

The evolution of the orbital elements of Toutatis (see Fig. 3) shows that the asteroid is locked in the 3/1 mean motion resonance with Jupiter, near

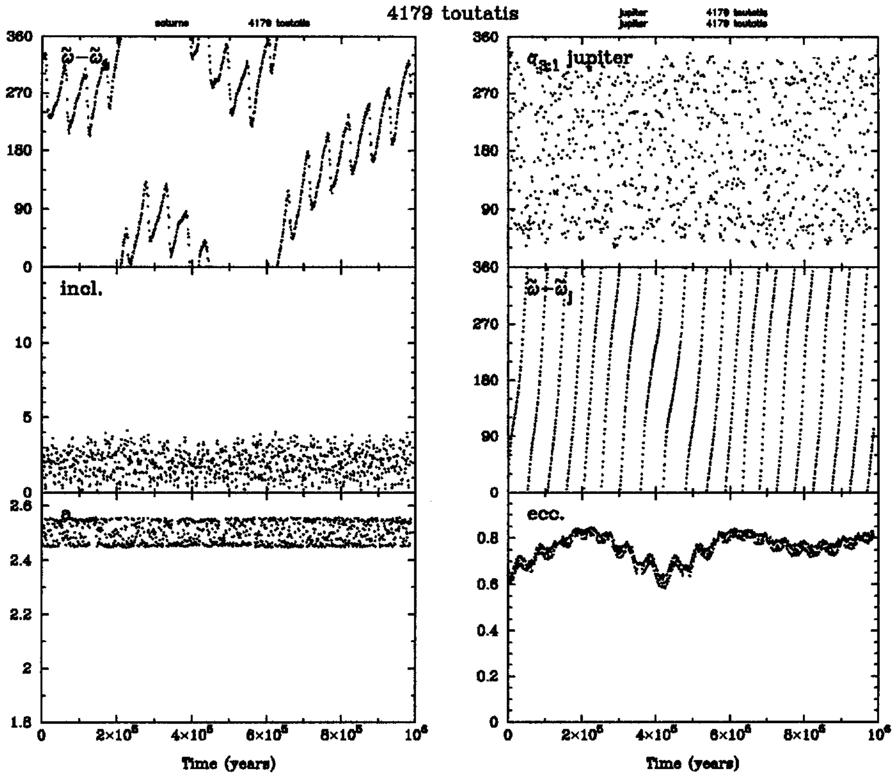

Fig. 3. Evolution of the orbital elements of Toutatis over 1 Myr in the Sun–Jupiter–Saturn–asteroid model. Besides semimajor axis, eccentricity and inclination, the figure shows also three critical arguments: $\sigma_{3:1}$ of the 3/1 mean motion resonance with Jupiter [see Eq. (9)]; $\varpi - \varpi_j$ and $\varpi - \varpi_s$ of the ν_5 and ν_6 secular resonances (ϖ_j and ϖ_s being the perihelion longitudes of Jupiter and Saturn, respectively).

$a = 2.5$ AU, during the whole integration span. This is confirmed by the fact that the critical argument $\sigma_{3:1}$, defined by:

$$\sigma_{i:k} = k\lambda - i\lambda_j + (i-k)\varpi\,, \tag{9}$$

with $i = 3$, $k = 1$, always librates (with a large amplitude) around 180°. Here λ is the mean longitude and ϖ is the longitude of perihelion of Toutatis, while λ_j is the mean longitude of Jupiter. It is worth noting that Toutatis' orbit is also inside the secular resonance ν_6 with Saturn (perturbed by Jupiter) during the first 6×10^5 yr of the integration, as shown by the libration of the secular critical argument $\varpi - \varpi_s$ around 0°. When the orbit is locked in the ν_6 resonance, wide oscillations of the eccentricity occur. This is consistent with the findings of Morbidelli and Moons (1993) and Moons and Morbidelli (1995) on the behavior of secular resonances inside mean motion resonances.

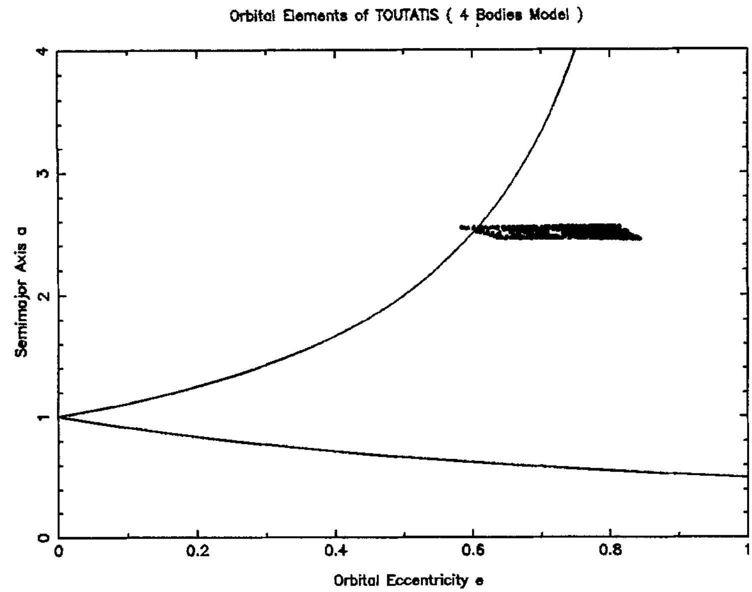

Fig. 4. A plot of the semi-major axis a vs. eccentricity e for the numerically integrated orbit of Toutatis over 1 Myr with the Sun–Jupiter–Saturn–asteroid model. The lines $q = 1$ AU and $Q = 1$ AU are also shown.

As the Earth is not present in this model, however, there are no close approaches and no random walk of the semimajor axis during the integration time. This confirms that the stochasticity of the evolution is mostly due to encounters with the inner planets (Whipple and Shelus, 1993; Benest *et al.*, 1994). In the a vs. e plane (see Fig. 4), the motion occurs along a narrow horizontal strip, which crosses the $q = 1$ AU line.

2.3.3. *The Sun–Earth–Jupiter–Saturn–asteroid model*

This model is the most realistic one. As many dynamical mechanisms are at work at the same time, Toutatis has a complex behavior (see Fig. 5), switching between several mean motion and secular resonances. During the first 9×10^4 yr, the orbit is locked in the 3/1 resonance with Jupiter, with the semimajor axis staying around 2.5 AU. Then, due to a close approach with the Earth, it is ejected from this resonance, with the semi-major axis jumping to lower values. But soon it enters in ν_6 secular resonance (as shown by the librations of the corresponding critical argument). Subsequently, for a fairly long time (between about 4×10^5 and 7.5×10^5 yr), the semimajor axis stays around 2.06 AU, corresponding to the 4/1 mean motion resonance with Jupiter. Actually, during this interval the corresponding critical argument $\sigma_{4:1}$ alternates between libration around 0° and circulation (see Fig. 6). In the libration intervals, the orbit also gets locked in the ν_5 secular resonance

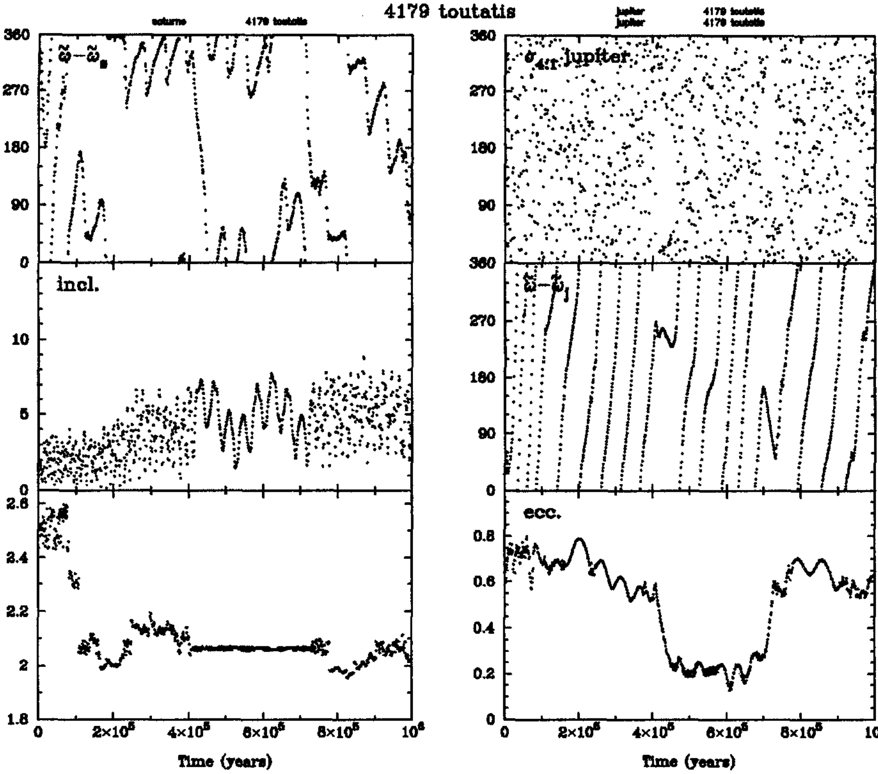

Fig. 5. The same as Fig. 3 but for the Sun–Earth–Jupiter–Saturn–asteroid model and with the critical argument $\sigma_{4:1}$ instead of $\sigma_{3:1}$.

and the eccentricity undergoes significant changes (Moons and Morbidelli, 1995, and Yoshikawa, 1989).

Fig. 7 shows that when the outer planets are taken into account in the integration, the orbit of Toutatis does not random walk any more along the contour lines of the Tisserand invariant relative to the Earth. The corresponding path is quite complex, with vertical (semimajor axis) changes caused by encounters and large horizontal strips due to both mean motion and secular resonance effects, with superimposed oscillations due to non-resonant secular perturbations. The inclination also grows up to almost 10°. Therefore, even if the orbit's stochasticity is still dominated by close approaches with the Earth, any dynamical model considering only the effects of the encounters (i.e., conserving approximately the Tisserand invariant) cannot reproduce the trends and time scales of the real evolution. Therefore, using such models to predict the evolution of the NEA population is likely to yield misleading results.

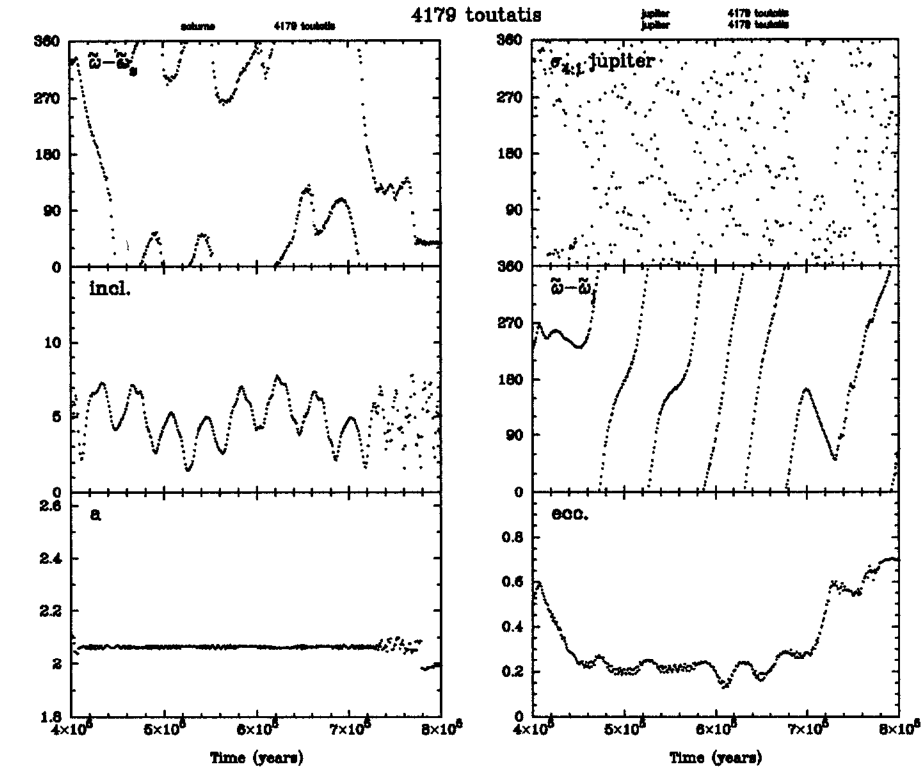

Fig. 6. The same as Fig. 5 but with the horizontal scale enlarged to better show the time interval between 4×10^5 and 8×10^5 yr.

3. Conclusions

The main conclusions of the work described above can be summarized as follows:

(1) When only the Earth is included in the dynamical model, the evolution of Toutatis is dominated by close encounters with it. The orbit is strongly chaotic and the changes of a and e have the typical features of a random walk (albeit with a strong correlation between the two elements). However, the perihelion distance and the Tisserand parameter stay almost constant, as predicted by simple analytical arguments based on the restricted three–body problem.

(2) On the other hand, the presence of the outer planets in the dynamical model causes Toutatis' orbit to get locked into resonances, in particular the 3/1 mean motion resonance with Jupiter and the ν_6 secular resonance with Saturn's longitude of perihelion (perturbed by Jupiter). In order to get the latter resonance, both outer planets have to be present in the model. Resonances cause significant variations of e but keep a almost constant.

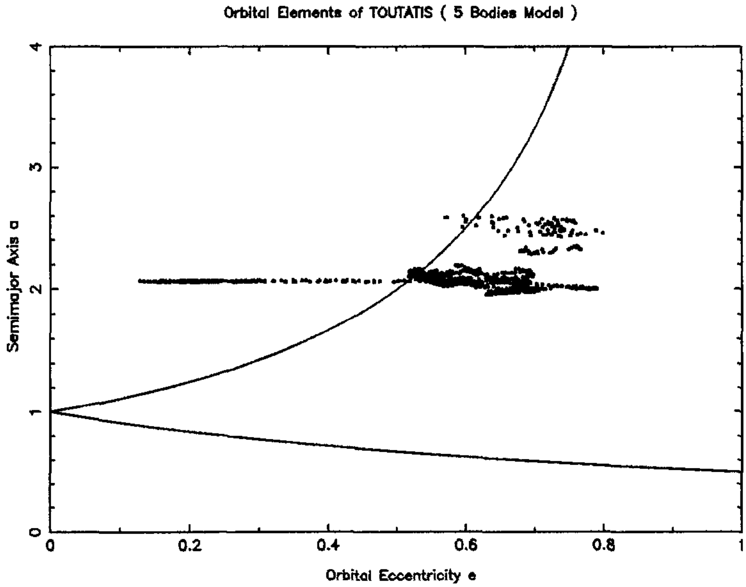

Fig. 7. The same as Fig. 4 but for the Sun–Earth–Jupiter–Saturn–asteroid model.

(3) The 5–body model including the Earth, Jupiter and Saturn together results in a very complex interplay of dynamical effects. In summary, Earth encounters switch Toutatis' orbit between several different resonant states, including the 3/1 and 4/1 mean motion and the ν_5 and ν_6 secular resonances. The eccentricity changes are dominated by resonance effects, whereas in a mainly encounter–related variations are apparent. None of the previous two models has even a qualitative resemblance to the evolution pattern resulting in this case. In particular, the Tisserand invariant relative to the Earth is not conserved at all, as the evolution of the eccentricity is mainly controlled by the outer planets. Indeed, strong and fast eccentricity changes appear to provide a "fast–track" dynamical *route* between low– and high–eccentricity asteroid orbits.

We stress that even the third, most realistic dynamical model does not provide a quantitatively predictive description of the behavior of the real asteroid, for at least two reasons: (i) the strong stochasticity of the orbit, mainly related to Earth encounters; (ii) and the fact that the model is still an approximate one, with a number of missing perturbation effects. For instance, another numerical integration of Toutatis with a model including all the planets from Venus to Neptune (Farinella *et al.*, 1994; Valsecchi *et al.*, 1995) shows a variety of dynamical mechanisms at work as in our third model, but the evolution of the orbital elements is markedly different. Actually, in this integration, Toutatis gets ejected from the solar system on a comet–like hyperbolic orbit some 6×10^5 yr in the future, after an encounter with

Jupiter! Of course, this just shows that for such strongly chaotic, fast–track evolving orbits, very different final fates are possible, including a hyperbolic ejection, a collision with the Sun or with a planet.

Another important point to be stressed is that not all NEAs are currently evolving along the fast–track resonant routes of the type we have discussed for Toutatis. The evidence from the recent numerical work quoted above is that probably only a minor fraction (possibly some 20%) of the existing NEAs at any given time is on fast–track orbits. Other orbits (in particular those classified in the Geographos class by Milani *et al.* (1989)) actually evolve in a slower, random–walk fashion, with Earth encounters playing the dominant role, in a way qualitatively similar to that shown in Fig. 1. This is also the case for many Amor objects, including the largest NEAs, (1033) Ganymed and (433) Eros: in these cases, the evolution is still slower, as only Mars encounters are possible and the Martian mass is an order of magnitude smaller than that of the Earth. However, from the point of view of the "demography" of NEAs, it is likely that the slow–track objects represent more the exception than the rule, and that they are over–represented in the existing population just because of their much slower evolution and longer lifetime. Also, since all NEAs probably start their independent life (after collisional ejection from main–belt parent asteroids) inside Toutatis–like resonant channels, slow–track nonresonant bodies probably are the outcome of "lucky" encounters with Mars or the Earth, removing them from the resonances and putting them into long–lived "parking zones". Thus, the conclusion appears likely that there is no *typical* dynamical evolution or even lifetime for all NEAs, but that the transfer of bodies from the main asteroid belt to the Earth–crossing zone is a complex process, with a variety of time scales, dynamical mechanisms, and exchanges between different classes of objects. This scenario is further discussed by Froeschlé *et al.* (1995) and Menichella *et al.* (1995).

Much more numerical and modelling work appears needed to understand the remaining open problems. For instance: how many different "dynamical classes" are required to correctly classify the orbital evolution patterns over time spans of 10^7 to 10^8 yr? How frequently do exchanges occur among different classes, and more in general between fast–track and slow–track orbits? What are the locations of secular resonances in the little–known region with $a < 2$ AU, where the secular perturbations of the terrestrial planets need to be taken into account? And how effective are the mean motion resonances with the inner planets to protect NEAs from encounters, as in the case of the Toro–class objects of Milani *et al.* (1989)? We plan to deal with these problems in the future to obtain a plausible, self–consistent evolutionary scenario for this important and intriguing population of near–Earth interplanetary bodies.

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