ORBITS OF SHORT PERIOD COMETS CAPTURED FROM THE OORT CLOUD

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Abstract. Oort cloud comets occasionally obtain orbits which take them through the planetary region. The perturbations by the planets are likely to change the orbit of the comet. We model this process by using a Monte Carlo method and cross sections for orbital changes, i.e. changes in energy, inclination and perihelion distance, in a single planet–comet encounter. The influence of all major planets is considered. We study the distributions of orbital parameters of observable comets, i.e. those which have perihelion distance smaller than a given value. We find that enough comets are captured from the Oort cloud in order to explain the present populations of short period comets. The median value of $\cos i$ for the Jupiter family is 0.985 while it is 0.27 for the Halley types. The results may explain the orbital features of short period comets, assuming that the active lifetime of a comet is not much greater than 400 orbital revolutions.

1. Introduction

There is a steady flux of "new" Oort cloud (Oort 1950) comets through the inner Solar System. By integrating the orbits of a large number of such new comets with jovian perturbations upon repeated perihelion passages, Everhart (1972) concluded that short-period comets evolve from new comets by changes in orbit. However, it has been subsequently claimed that the number of short period comets which results from captures from the Oort cloud is not large enough (Joss 1973, Fernández and Ip 1983) or that the distribution of orbital parameters of the observed short period comets does not match with the expected capture population (Duncan *et al.* 1988; Wetherill 1991; Fernández and Gallardo 1993). The situation is still rather unclear (Stagg and Bailey 1989; Valtonen *et al.* 1992).

In this paper we present an improved calculation of the orbital evolution of Oort cloud comets through the Solar System and show that under certain (not implausible) conditions the statistics of short-period comets are well reproduced if they evolve by dynamical means from the Oort cloud.

2. The method of calculation

The most direct way to attack the problem of orbital evolution of Oort cloud comets is to integrate the orbits of large numbers of comets (a million

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or so) over the relevant time periods, up to the age of the Solar System. Quinn *et al.* (1990) opted for this approach but found that the required time of computation is overwhelming unless the problem is somehow simplified. The simplifications considered were: (1) to have only one planet, Jupiter, to influence the orbits of comets, or (2) to have very massive outer planets in place of Saturn, Uranus and Neptune. Both simplifications change the original problem significantly.

In view of the difficulty with excessive computer time in direct integrations, Wetherill (1991) carried out Monte Carlo calculations using the Öpik (1951) scheme for calculating comet–planet encounters. However, this scheme may not be accurate enough for all the encounters of importance in the current problem (Carusi *et al.* 1990). Instead one should use data from real comet–planet encounters, based on accurate orbit integrations. Following pioneering work by Rickman and Vaghi (1976), Everhart (1977) and Froeschlé and Rickman (1980), Stagg and Bailey (1989) and Fernández and Gallardo (1993) started a push toward this direction by integrating the orbits of comets through the Solar System and using the distribution of the changes of orbital parameters as input to a Monte Carlo scheme. The problem in this previous work is the limited statistical data, which becomes quite obvious upon comparison with the results of Zheng (1994).

The approach used in the present work is outlined by Valtonen *et al.* (1992). By using several millions of accurate orbit integrations, the probability distributions for changes in the cometary orbital elements a (semi-major axis), i (inclination) and q (perihelion distance) are evaluated for orbits of given initial a_0, i_0 and q_0, assuming the remaining orbital elements to be randomly distributed. The planetary orbits are assumed circular. The probability distributions are reported in Zheng (1994), and they are used in a Monte Carlo scheme similar to the approach of Fernández and Gallardo (1994).

We assume that comets arrive from the Oort cloud with a uniform distribution of $\cos i_0$ and with a constant semi-major axis $a_0 = 20\,000$ AU. Several different distributions of q_0 ($\lesssim 30$ AU) were studied. Here we report results based on the Galactic tide model of Matese and Whitman (1989) where the number of new comets increases with q_0 qualitatively like in the stellar perturbation models of Weissman (1985) and Fernández (1982), though differing in the details:

$$N(q_0) = \begin{cases} 1 + 0.014\, q_0^{1.82} & \text{when } q_0 < 13 \text{ AU} \\ 5 & \text{when } 13 \text{ AU} < q_0 < 30 \text{ AU} \end{cases} \tag{1}$$

(in units of the near-parabolic flux at Earth's orbit). For each comet with given initial values of q_0, a_0 and i_0, the probability of scattering by each major planet is calculated and a particular planet is selected according to this weighting. The orbital change caused by the selected planet is then chosen from the relevant, tabulated probability distributions and new orbital elements a, i and q are computed. The comet is then assumed to undergo N revolutions with the new elements until the next scattering, at which time a perturbing planet is again selected and a new orbital change is computed in the same way. This number of revolutions is determined by the inverse of the scattering probabilities for all the planets. This process is continued by selecting new scatterings and new orbits consecutively until the comet escapes from the Solar System or is considered to have become unobservable due to physical decay. As scatterings we only consider strong perturbations, corresponding to $|\Delta 1/a| > 5 \times 10^{-4}\,\mathrm{AU}^{-1}$ for Jupiter, and decreasing proportional to M_p/a_p where M_p is the mass of the planet and a_p its orbital radius, to $|\Delta 1/a| > 5 \times 10^{-6}\,\mathrm{AU}^{-1}$ for Neptune.

We consider a comet to live through N_{max} close encounters with the Sun, where a close encounter is defined by $q < q_{lim}$. We use the values of $N_{max} = 400$, 1000 and 4000 and $q_{lim} = 1.5$ and 2.6 AU. The calculations are continued with a steady influx rate of new comets up to the time limit of T_{max}. We have considered T_{max} ranging from $3 \cdot 10^6$ yr to $300 \cdot 10^6$ yr. Here we report only the case of $T_{max} = 300 \cdot 10^6$ yr, $q_{lim} = 2.6$ AU and $N_{max} = 400$ (see e.g. Delsemme 1973, 1979; Kresák 1981). The full details will be published elsewhere (Zheng *et al.*, in preparation).

3. Captured comets

The model reported here considers the evolution of 10^6 comets, injected at a steady rate of one comet every 300 years. The comets initially have nearly parabolic orbits with semi-major axes $a_0 = 2 \cdot 10^4\,\mathrm{AU}$, but those considered in the model are the ones that initially suffer scatterings into more tightly bound orbits, leaving them with new semi-major axes less than a_1, where $a_1 = 2000\,\mathrm{AU}$ for Jupiter encounters and $a_1 = 18\,000\,\mathrm{AU}$ for Neptune encounters. Such comets, i.e. those that are perturbed to bound orbits in this sense, constitute approximately 15% of the original near-parabolic flux (see below), so the corresponding near-parabolic flux in the model is one comet every 45 yr with perihelion distance less than 30 AU.

During the simulation any comet which achieves a perihelion distance $< q_{lim}$ is counted as a 'visible' comet; data are reported here for the particular case $q_{lim} = 2.6\,\mathrm{AU}$. If the orbital period P when the comet is visible is less than 20 yr, the comet is called a Jupiter-family (JF) comet, and each time it passes perihelion in the simulation with such a period and perihelion distance one unit is added to the total number of revolutions as a JF comet in the simulation.

Similarly, if the orbital period P is in the range $20 < P < 200\,\mathrm{yr}$ when the comet is visible, it is called a Halley type (HT) comet, while if $200 < P < 90\,000\,\mathrm{yr}$ (corresponding to $a = 2000$ AU), it is called a long-period (LP) comet. The total number of revolutions of the HT-phenomenon and LP-phenomenon in the simulation is calculated in the same way as for the JF comets. Any comet that undergoes more than N_{max} revolutions as a visible comet is removed from the simulation, being deemed to have been destroyed by physical decay. The results here refer to the particular case $N_{max} = 400$.

We calculate the number of short-period comets both in the Jupiter family $N(JF)$ and the Halley types $N(HT)$ as well as the number of long-period comets $N(LP)$. With the parameters used here we find $N(JF) = 3044$, $N(HT) = 68$ and $N(LP) = 190$. The median values of $\cos i$ for the Jupiter family and for the Halley types are 0.985 and 0.27, respectively. These are close to the observed values. The high value for the Jupiter family is obtained only when $N_{max} = 400$, and the agreement with observations is completely lost when $N_{max} = 4000$.

The numbers of comets $N(JF)$, $N(HT)$ and $N(LP)$ refer to the total number recorded over the calculation period T_{max}. At any point in time only a fraction $N_{max} \cdot \langle P_x \rangle / T_{max}$ of them is observed, so comparison with observed comet numbers can be made after $N(JF)$, $N(HT)$ and $N(LP)$ are multiplied by this fraction. We use $\langle P_{JF} \rangle = 8$ yr for the Jupiter family, $\langle P_{HT} \rangle = 80$ yr for the Halley types and $\langle P_{LP} \rangle = 800$ yr for the long-period comets.

The true influx rate of new comets is obtained as follows. We take the new Earth-crossing comet flux of Bailey and Stagg (1988) of 4.6 comets/AU/yr, which includes a correction for comets missed at the low end of the luminosity function (Everhart 1967; Hughes 1988) to absolute magnitude $H_{10} = 10.8$, and multiply by the fraction of new comets which suffer strong encounters (new $a < a_1$), i.e. by 0.15 (Everhart 1968; Marsden and Williams 1994). In addition, we divide by 1.5 since the present day comet flux may exceed the average over the past few million years by this factor (Matese *et al.* 1995). The resulting long term average flux of new comets which become captured into orbits of $a < a_1$ is therefore about 0.5 comets/AU/yr at 1 AU. But since the comet flux in our model increases with q_0, there is an additional factor of 3.5 which has to be taken into account, as we consider the average Oort cloud flux over the 30 AU interval. Thus the numbers $N(JF)$, $N(HT)$ and $N(LP)$ should be multiplied by $50\ /\mathrm{yr} \cdot T_{max}/10^6$ in order that they correspond to the true Oort cloud flux. Since only the fraction $N_{max}\langle P_x \rangle / T_{max}$ is observed at any time, the expected number of comets N_x of type x is

$$N_x = 5 \cdot 10^{-5} \cdot N(x) \cdot N_{max} \cdot \langle P_x \rangle \qquad (2)$$

With our parameters Eq. (1) gives us the expected numbers $N_{JF} = 487$, $N_{HT} = 109$ and $N_{LP} = 3040$.

We should note at this point that our model neglects orbital evolution via distant encounters with planets. We estimate that the inclusion of scattering by small energy steps would increase the HT and LP families by a factor 2–3, while the Jupiter family is less affected by this process. This has to be verified by future work.

Fernández *et al.* (1992; see also Fig. 2 in Fernández and Gallardo 1994) estimate that the Jupiter family has about 550 members out to $q = 2.5$ AU at the magnitude limit of 10.8. Hughes (1988) estimates the numbers of comets in different dynamical groups on the basis of the cumulative number of discovered comets brighter than H_{10}. Using his Fig. 1, we find 42 Jupiter family comets at $H_{10} = 10.8$ where the discovery rate still appears to be complete. At magnitude $H_{10} = 7.0$ where a break (indicating discovery incompleteness) appears to occur for Halley type comets, these are about 3 times more numerous than the Jupiter family comets. Thus the corrected number of Halley type comets should be ~ 126 at $H_{10} = 10.8$, if the luminosity functions are the same. About one quarter of the long period comets listed by Marsden and Williams (1994) corresponds to our LP-category. At magnitude $H_{10} = 5.8$ they are 50 times more numerous than Jupiter family comets, or there should be ~ 2100 of them at $H_{10} = 10.8$.

The known population of Halley type comets has $q \lesssim 1.5$ AU, and it is not known how to correct the numbers to correspond to the limit of $q = 2.6$ AU. A simple linear extrapolation between the two limits predicts 218 Halley type comets out to $q = 2.6$ AU. Note that some incompleteness of the discovered Halley type comets is due to arise from the orbital periods being comparable to the past interval of efficient comet discovery. For the LP comets we should expect a fourfold increase of the number observed during the last two centuries in order to fit with $\langle P_{LP} \rangle = 800$ yr. Considering the uncertainties (e.g. the proper value of N_{max} in Eq. (1), the Oort Cloud flux at large q, etc.) there is hence no major discrepancy between the (extrapolated) observed comet numbers and the numbers calculated from theory.

Thus the agreement between the Oort cloud capture products and the observed comet families is quite satisfactory, assuming that N_{max} is not very far from 400. This agreement is particularly striking in the inclination distributions which are likely to be better determined observationally than the total population numbers. Within the uncertainties of observational data we may thus state that the short-period comets may derive entirely from the Oort cloud.

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