ORIGIN AND EARLY EVOLUTION OF THE PLANETARY SYSTEM

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Abstract. It is shown by linear stability analysis that a preplanetary (presatellite) disk of dust and gas with Keplerian velocity field can become unstable due to the collective self-gravity of the disk. The radial distribution of rings, which may result from this instability, is derived. These rings later on can be the formation sites for planets around the Sun and for satellites around the planets. The derived orbits are shown to be in good agreement with that of the planets and the satellites (of Jupiter, Saturn, and Uranus). Predictions and conclusions seem to be possible for the existence of three yet unknown Uranian satellites, the origin of the early Moon and the possible radial extension of the planetary system.

1. Introduction

The great majority of very different and numerous planetogonic models for origin and early evolution of the planetary system and the ('evolved') satellite systems of Jupiter, Saturn, and Uranus increasingly shows a consensus in assuming a thin 'preplanetary disk' as a predecessor of these systems (Stiller *et al.*, 1980). This seems to be based mainly on the evidence that, with few exceptions only, orbital inclinations of planets and satellites are small.

The differences in planetogonic models result therefore from the differing assumptions about origin and further evolution (structurization) of preplanetary disks.

This paper will start with the existence of a non-structurated disk of dust and gas, which was focussed to a plane due to the action of gravitational centrifugal and frictional forces (Cameron and Pine, 1973; Cameron, 1978; Safronov, 1969) and by magnetic fields (Alfvén and Arrhenius 1976; Möhlmann, 1984). Furthermore, only those disks around a central mass M_c shall be discussed, which are stable against a direct gravitational collapse of local regions, producing there giant protoplanets. This evolutionary way seems to be possible for massive disks with $m_{\text{disk}} \geqslant M_c$ only. (Cassen and Moosmann, 1981; Weidenschilling, 1983) Consequently, preplanetary disks with $m_{\text{disk}} \ll M_c$ shall be discussed in this paper.

It will be shown by stability analysis within the frame of linear perturbation theory, that for a sufficient low thermal pressure and due to collective self-gravity the disk becomes unstable with a quasiperiodic radial distribution of the density disturbance. It is assumed then, that in the nonlinear regime rings are formed due to the collective self-gravitation at the maxima of the linear disturbances in the density. With further decreasing pressure (caused by cooling and condensation) these rings, may become

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unstable too by an 'azimuthal instability' (Möhlmann, 1984a; Boss, 1982). Consequently, planetary bodies may form from the rings approximately at their orbital radii. The resulting radial structure of the rings and derived planets fits quite well the observations in the planetary system and the satellite systems of Jupiter, Saturn, and Uranus (see Fig. 2).

2. Basic Equations

A hydrodynamic description of a preplanetary disk has to start from Navier–Stokes equations

$$\frac{\partial}{\partial t} v + (v \,\mathrm{grad}) v = -\,\mathrm{grad}(V + \phi) - \frac{1}{\rho}\,\mathrm{grad}\, p + \nu \Delta v, \tag{1}$$

where $V = -\gamma M_c / r$ is the gravitational potential of the central mass M_c and ϕ represents the gravitational potential of the disk of density ρ, connected with ϕ by Poisson's equation

$$\Delta \phi = 4\pi\gamma\rho. \tag{2}$$

As equation of state $p = \rho c^2$ shall be used in this context with the approximation $c^2 = \mathrm{const}$.

An estimation of the relative importance of thermal pressure can be found by comparing the gravitational force of the central body and the radial pressure gradient over planetary scales, giving

$$c^2 < \frac{\gamma M_c}{r} \tag{3}$$

as a condition for the existence of a disk, dominated by the gravitation of the central body. Using $c^2 = kT/m$, with Boltzmann's constant $k = 1.38 \times 10^{-23}$ *Ws/grd* and $m = m_p$ as the proton mass for hydrogen gas, we find (with $T \leqslant 10^3$ K) that $c \leqslant 3 \times 10^3 \,\mathrm{m\,s^{-1}}$. This restricts the radial scales of the preplanetary disk to $r < 10^{13}$ m, while the corresponding scales for the presatellite disks of Jupiter, Saturn, and Uranus are $r_j < 10^{11}$ m, $r_s < 10^{11}$ m, and $r_u < 10^{10}$ m respectively. It shall be noted here, that the above-given estimation cannot be applied to azimuthal scales, which are not influenced directly by the gravitation of M_c and which are expected to be axisymmetric as long as pressure is effective.

The relative importance of thermal pressure and collective self-gravitation of the disk can be found by comparison of grad ϕ and $\rho^{-1}\,\mathrm{grad}\; p = c^2 \rho^{-1}\; \mathrm{grad}\, \rho = c^2 (4\pi\gamma\rho)^{-1}$ grad $\Delta\phi$. Consequently, pressure is important for

$$c^2 \geqslant \omega_c^2 L^2 \tag{4}$$

where $\omega_c^2 = 4\pi\gamma\rho$ and $\Delta \sim L^{-2}$ as an order of magnitude estimation. With $\rho \geqslant 10^{-9} \,\mathrm{kg\,m^{-3}}$, what is implied by the actual planetary masses, and with the above-given numerical values, it follows that (4) was satisfied in early preplanetary phases, indicating the essential action of pressure, counteracting any early structurization in the density-distribution.

Structuring processes can become effective if the above-described situation reverses, and collective self-gravitation overcomes the disintegrating action of pressure. This can have been realized, if most of the gas has been condensed out to small particles and droplets. To investigate the characteristics of these later structuring processes, and if pressure is not mentioned explicitly, it shall be assumed throughout this paper, that the preplanetary (presatellite) disk consists mainly of condensed particles and grains, interacting gravitationally.

If we neglect kinematic viscosity, which might have been essential in earlier phases, Equation (1) reduces to

$$\frac{\partial}{\partial t} v + (v \operatorname{grad}) v = -\operatorname{grad}(V + \phi), \tag{5}$$

which together with (2) and the equation of continuity

$$\frac{\partial}{\partial t} \rho = -\operatorname{div} \rho v \tag{6}$$

completes a complete set of the equations.

To investigate the stability of a special 'ground-state' with v_0, ρ_0, ϕ_0 of a preplanetary disk under the influence of the central body and its own gravitation a linearization according to $v = v_0 + \mathbf{u}$, $\rho = \rho_0 + \delta\rho$, $\phi = \phi_0 + \Psi$ with $|\mathbf{u}| \ll |v_0|$ and $\delta\rho \ll \rho_0$ gives

$$\frac{\partial}{\partial t} v + (v_0 \operatorname{grad})\mathbf{u} + (\mathbf{u} \operatorname{grad}) v_0 = -\operatorname{grad} \Psi. \tag{7}$$

If we assume now a static and axisymmetric 'ground state' which shall be described in cylindrical coordinates by $v_0 = \mathbf{e}_\varphi v_0(r)$, it follows from (5)

$$v_0(r) = \left\{ \gamma \frac{M_c}{r} + r \frac{\partial}{\partial r} \phi_0 \right\}^{1/2}. \tag{8}$$

As can be shown simply by expansion with Legendre polynomials, the gravitational potential of a disk with radius 'a' and constant mass-density σ_0 for $r < a$ in spherical coordinates is given by

$$\phi_0 = -2\pi\gamma\sigma_0 a \left\{ \left(1 - \frac{r}{a}\right) |P_1| + \frac{1}{2}\left(\frac{r}{a}\right)^2 P_2 - \frac{1}{8}\left(\frac{r}{a}\right)^4 P_4 + \right.$$
$$\left. + \frac{1}{16}\left(\frac{r}{a}\right)^6 P_6 - \frac{5}{128}\left(\frac{r}{a}\right)^8 P_8 \pm \dots \right\}. \tag{9}$$

With a surface density σ_0 of the order of $(10^1\text{–}10^4)\,\mathrm{kg\,m^{-2}}$ (see Figure 1) of the preplanetary disk, there follows with $M_c = 1.99 \times 10^{30}\,\mathrm{kg}$ that $\gamma M_c/r \gg r(\partial/\partial r)\phi_0$. Therefore, the axisymmetric disk potential is not important under these circumstances for the description of the undesturbed 'ground state' and the approximation

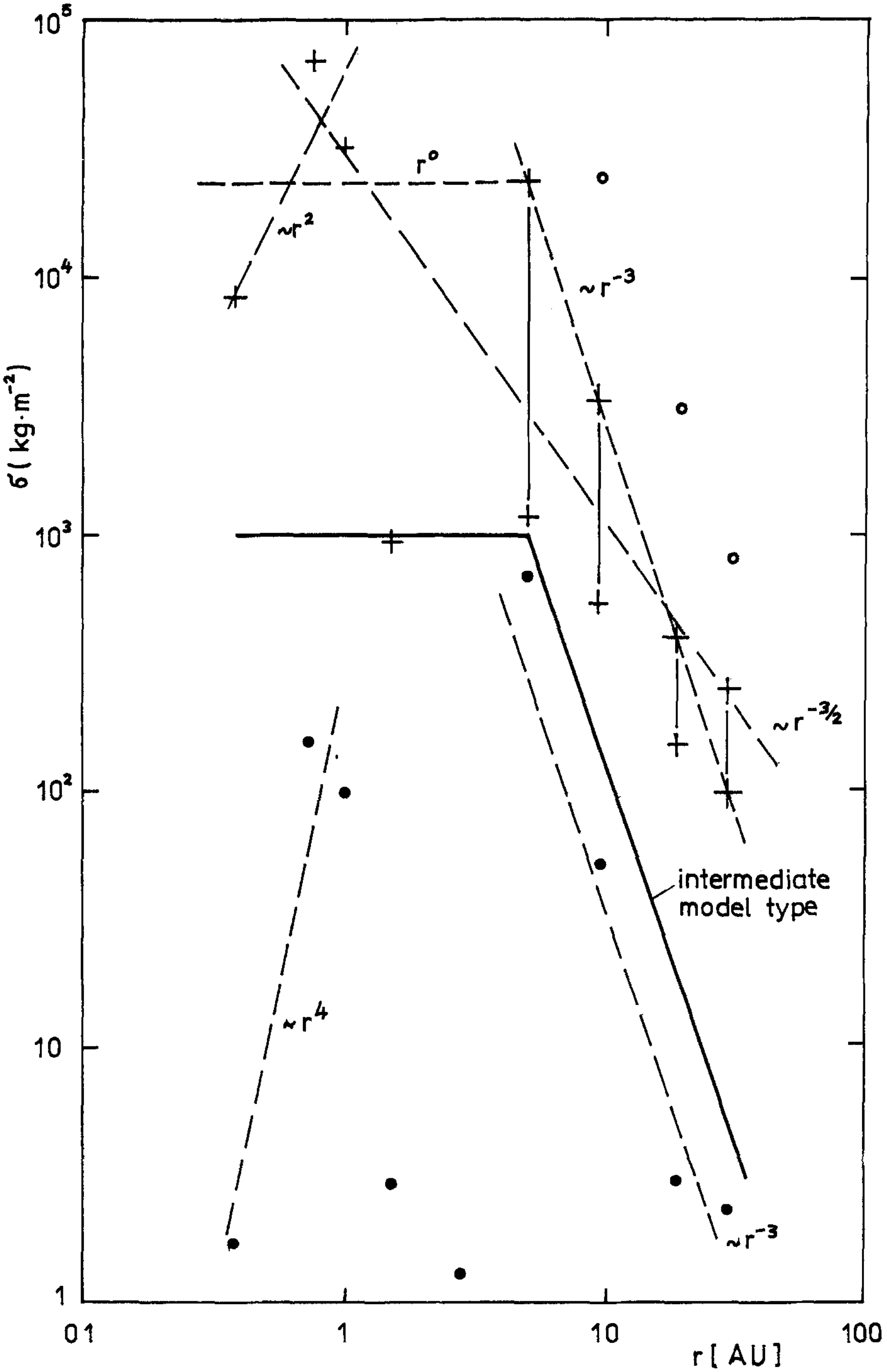

Fig. 1. Preplanetary disk surface densities.

$$v_0 = \left(\gamma \frac{M_c}{r}\right)^{1/2} \tag{10}$$

can be used, defining with $v_0 = \Omega_K r$ the Keplerian angular velocity $\Omega_K = (\gamma M_c/r^3)^{1/2}$.

With (10), Equation (7) can be reduced to

$$\left(\frac{\partial}{\partial t} + \Omega_K \frac{\partial}{\partial \varphi}\right) u_r - 2\Omega_K u_\varphi = -\frac{\partial}{\partial r}\Psi, \tag{11a}$$

$$\frac{1}{2}\Omega_K u_r + \left(\frac{\partial}{\partial t} + \Omega_K \frac{\partial}{\partial \varphi}\right) u_\varphi = -\frac{1}{r}\frac{\partial}{\partial \varphi}\Psi. \tag{11b}$$

The linearized equation of continuity is given with (10), $\delta\sigma = \int dz\, \delta\rho$ and the model-assumption $u_z = 0$ and $\sigma_0 = c_0 r^{-n}$ by

$$\sigma_0\left(\frac{1-n}{r} + \frac{\partial}{\partial r}\right) u_r + \frac{\sigma_0}{r}\frac{\partial}{\partial \varphi} u_\varphi + \left(\frac{\partial}{\partial t} + \Omega_K \frac{\partial}{\partial \varphi}\right)\delta\sigma = 0. \tag{12}$$

Using $\delta\rho = \delta\sigma\delta(z)$, we find that the potential equation for Ψ can be integrated, giving

$$\left.\frac{\partial \Psi}{\partial z}\right|_{z \to +0} = 2\pi\gamma\, \delta\sigma. \tag{13}$$

Away from the disk plane we have

$$\Delta\Psi = 0. \tag{14}$$

Consequently, the z-dependence of the potential can be described by $\exp[-k_z(z)]$, with $k_z > 0$, giving for (13)

$$-k_z \Psi(r, \varphi, z = 0) = 2\pi\gamma\, \delta\sigma, \tag{15}$$

if we neglect the contribution of horizontal forces – which corresponds to the restriction that Ψ may become essential only at those sites, where $\delta\sigma \neq 0$, indicating that this approximation is equivalent to the assumption that the 'Hill-sphere' of a disturbance does not exceed essentially the region of the disturbance. This seems to be an acceptable first approach.

In (15) $k_z > 0$ is determined via Equation (14) with boundary conditions at $z = 0$ by the horizontal scales of $\Psi(r, \varphi, z = 0)$. Equations (11), (12), and (15) will be used for the further discussion of the properties of a preplanetary thin disk of particles influenced by the collective action of their gravitation.

3. Radial Structures

The stability of a thin disk around a massive central body has been discussed by Hunter (1965) and Toomre (1964). Hunter made use of spheroidal coordinates and Legendre functions, and obtained the frequencies of free oscillations as eigenvalues of an infinite matrix; while Toomre applied Fourier–Bessel analysis to obtain equilibrium figures for galactic disks, replacing the galaxies by a finite number of concentric rings for a radially local stability analysis valid for single rings.

In what follows it will be shown that a more realistic radial stability analysis for disks with differential rotation can be carried out by use of Bessel functions. With (11), (12), and (15) with $\partial/\partial t = \omega$, $\partial/\partial\varphi = -k_\varphi$ and $f = \omega - \Omega_K k_\varphi$ we obtain the velocities

$$u_r = D^{-1}\left\{-f\frac{\partial\Psi}{\partial r} + 2\Omega_K\frac{k_\varphi}{r}\Psi\right\}, \tag{16a}$$

$$u_\varphi = D^{-1}\left\{f\frac{k_\varphi}{r}\Psi + \frac{\Omega_K}{2}\frac{\partial\Psi}{\partial r}\right\}, \tag{16b}$$

where $D = f^2 + \Omega_K^2$.

An appropriate potential equation follows by introducing (16) in (12) and taking into account (15); which leads to the equation

$$\frac{\partial^2}{\partial r^2}\Psi + A\frac{1}{r}\frac{\partial}{\partial r}\Psi + B\frac{1}{r^2}\Psi + C\Psi = 0 \tag{17}$$

with

$$A = \frac{(4-n)\Omega_K^2 + (1-n)f^2 - 3f\Omega_K k_\varphi}{f^2 + \Omega_K^2}, \tag{18a}$$

$$B = \frac{k_\varphi^2(7\Omega_K^2 + f^2)f + \Omega_K^3 k_\varphi(2n-3) + f^2\Omega_K k_\varphi(3+2n)}{f(f^2 + \Omega_K^2)}, \tag{18b}$$

$$C = k_z\frac{f^2 + \Omega_K^2}{2\pi\gamma\,\sigma_0}; \tag{18c}$$

where σ_0, Ω_K (and f) are functions of r. Consequently, the radial structure of Ψ depends on the radial dependence of the surface density and via Ω_K on the differential rotation. For axisymmetrical ($k_\varphi = 0$) and stationary ($\omega \ll \Omega_K$) cases follow $A = 4 - n$, $B = 0$ and $C = k_z(M_c/2\pi c_0)r^{n-3}$. The general solution of (17) is given then by a superposition of Bessel functions

$$\Psi = G_1 r^{-\alpha}J_p(\epsilon r^\beta) + G_2 J_{-p}(\epsilon r^\beta) \tag{19}$$

with

$$\alpha = (3-n)/2, \qquad \beta = (n-1)/2,$$

$$\epsilon = \left(\frac{k_z M_c}{2\pi c_0}\right)^{1/2}\frac{2}{n-1}, \qquad p = \frac{3-n}{n-1}.$$

Thus any realistic solution depends directly on the radial behaviour of the surface density σ_0.

Models for surface densities of the preplanetary disk have been discussed intensively by Weidenschilling (1977).

He proposes a $r^{-3/2}$ dependence, giving $n = 3/2$. But, as can be seen from Figure 1, there is a very great uncertainty even in the orders of magnitude for realistic surface densities. In Figure 1 the crosses indicate values from Weidenschilling (1977), resulting from assumed initial solar-composition masses of the terrestrial (proto-) planets, computed from their iron content, and from different models for the outer planets as indicated by the vertical bars. The dots in Figure 1 represent values, as they have been computed simply by distributing the actual planetary masses over the ecliptic plane with the surface density σ_{0n} due to the nth planet (of mass m_n and orbital radius r_n) $\sigma_{0n} = m_n/\pi(r_a^2 - r_i^2)$, with $r_a = (r_{n+1} + r_n)/2$ and $r_i = (r_n + r_{n-1})/2$. These values can be regarded as minimal. Having in mind that the used above mathematical formalism can be applied to non-pressure-dominated disks of gravitationally interacting condensed particles only, the 'equivalent solar-composition' assumption of Weidenschilling (1977), which could be used for an even earlier gas phase, should give to great values. Therefore, as a model-type an 'intermediate model', as indicated in Figure 1 shall be used in this paper.

Realistic solutions for Ψ would involve, of course, a superposition of more than two Bessel functions, as in (19). But to get a first quantitative impression of the gros-characteristics of solutions of (19), the two representative solutions, based on the 'intermediate model' for σ_0 shall be used. Here it should be mentioned too, that the general structure of the solutions of (19) does not depend very sensitively on small variations of n. Therefore, a possible reality of some properties of solutions of (19) can not be used to determine n with great accuracy.

3.1. Special Solutions

Solution I describes for the quasi-stationary 'long-time' regime $\omega \ll \Omega_K$ the axisymmetric radial structure of an originally homogeneous disk with $\sigma_0 = \text{const.}$, or $n = 0$, what can be applied to the inner parts (see Figure 1) of the system, Solution II refers to the long time regime of an axisymmetric disk with radially decreasing density $\sigma_0 \sim r^{-3}$ or $n = 3$, being representative for the outer parts of the system.

Solution I: The above given conditions for this model lead to $A = 4$, $B = 0$, and $C = M_c k_z / 2\pi c_0 r^3$. The corresponding solutions is with $v_0 = (\gamma M_c / r)^{1/2}$

$$\Psi_I = C_1 \left(\frac{k_z}{2\pi\gamma\sigma_0}\right)^{3/2} v_0^{3/2} J_3\left(2v_0 \sqrt{\frac{k_z}{2\pi\sigma_0\gamma}}\right) + \\ + C_2 \left(\frac{k_z}{2\pi\gamma\sigma_0}\right)^{3/2} v_0^{3/2} N_3\left(2v_0 \sqrt{\frac{k_z}{2\pi\gamma\sigma_0}}\right), \tag{20}$$

where J_3 is a Bessel function of first kind and of 3-rd order, while N_3 is a Neumann function (or Bessel function of second kind) of 3-rd order. The characteristic length-scale for the quasiperiodic structures, as described by J_3 and N_3, is given by $L_I = M_c k_z / 2\pi\sigma_0$. For $r \to \infty$ follows $v_0 \to 0$ and therefore $N_3(r \to \infty) \to -\infty$. This shall be excluded for planetogonic models. Therefore, the further discussion shall be based on $C_2 = 0$.

The radial dependence of the density-disturbance $\delta\sigma_I$, determined with $\Delta\Psi_I = 4\pi\gamma\delta\sigma_I$, is given by

$$\delta\sigma_I = \frac{-C_1}{4\pi\gamma} x^{-4}\{J_2(2x^{-1/2}) + x^{-1/2}J_1(2x^{-1/2})\}, \tag{21}$$

where x is a dimensionless radial variable, defined by

$$x = r \cdot 2\pi\sigma_0/Mk_z = r/L_I = v_0^{-2}2\pi\gamma\sigma_0/k_z.$$

With the asymptotic representations for $2x^{-1/2} > 3$ (order of the Bessel functions) it follows that

$$J_2(2x^{-1/2}) \approx \pi^{-1/2}x^{1/4}\cos(2x^{-1/2} - \tfrac{5}{4}\pi), \tag{22a}$$

$$J_1(2x^{-1/2}) \approx \pi^{-1/2}x^{1/4}\cos(2x^{-1/2} - \tfrac{3}{4}\pi); \tag{22b}$$

and as an approximative description for (21)

$$\delta\sigma_I \approx \frac{-C_1 x^{-15/4}}{4\sqrt{2}\pi^{3/2}\gamma}\{(1 + x^{-1/2})\cos 2x^{-1/2} + (1 - x^{-1/2})\sin x^{-1/2}\}. \tag{23}$$

The extrema at $x = x_e$ of this radial density disturbance can be found from

$$(1 - 3x_e)J_2(2x_e^{-1/2}) = 6x_e^{1/2}J_1(2x_e^{-1/2}), \tag{24}$$

or, in the asymptotic approximation, by

$$\tan 2x_e^{-1/2} = \frac{6x_e^{1/2} + 3x_e - 1}{6x_e^{1/2} - 3x_e + 1}. \tag{25}$$

Solutions of (25) are given for the first zero-values in Table I. As can be seen even from Equation (20), there appear quasiperiodic structures, governed by nearly constant velocity-differences. It is interesting to note, that there exist ordering-schemes, describing the planetary distances with a constant velocity-difference as the structure parameter (Litzroth, 1980; Möhlmann, 1981).

Solution II: The coefficients for this case with $n = 3$ are $A = 1, B = 0$, and $C = M_c k_z/2c_0$, giving the solution

$$\Psi_{II} = C_3 J_0\left(r\sqrt{\frac{M_c k_z}{2\pi c_0}}\right) + C_4 N_0\left(r\sqrt{\frac{M_c k_z}{2\pi c_0}}\right), \tag{26}$$

where J_0 and N_0 are zero-order Bessel functions. The characteristic length-scale for the quasi-periodic radial structures is $L_{II} = \sqrt{2\pi c_0/M_c k_z}$. The radial dependence of the connected density disturbance is given then by

$$\delta\sigma_{II} = -\frac{C_3}{4\pi\gamma L_{II}^2}J_0(r/L_{II}) - \frac{C_4}{4\pi\gamma L_{II}^2}N_0(r/L_{II}). \tag{27}$$

Consequently, the extrema of $\delta\sigma_{II}$ can be found at

$$J_1(r/L_{II}) = -C_5 N_1(r/L_{II}), \tag{28}$$

TABLE I

Theoretical and real orbital radii of the satellites of the Sun, Jupiter, Saturn, and Uranus

Theor. model I		Planet. system $L_I = 4.481 \times 10^{13}$ m			Jupiter system $L_I = 1.2347 \times 10^{11}$ m			Saturn system $L_I = 2.084 \times 10^{11}$ m			Uranus system $L_I = 5.3023 \times 10^{10}$ m		
	x_e	$r_e[10^{11}$ m]	$r_r[10^{11}$ m]	Ident.	$r_e[10^8$ m]	$r_r[10^8$ m]	Ident.	$r_e[10^8$ m]	$r_r[10^8$ m]	Ident.	$r_e[10^8$ m]	$r_r[10^8$ m]	Ident.
1	0.267923	118			331			558 L_I from the			142 L_I from the		
2	0.089590							average of			average of		
3	0.043384	19			54			90 S3, S4, S5			23 U1, U2, U3,		U5
4	0.025267												
5	0.016430	7.22	7.79	♃	20.3	18.8	J4	34.2	35.61	S8	8.71		
6	0.011497											[d](5.86	U4)
7	0.008477	3.73	3–5	Aster.	10.5	10.7	J3	17.67	14.81	S7	4.50	4.38	U3
8	0.006500												
9	0.005138	2.26	2.28	♂	6.34	6.71	J2	10.71	12.22	S6	2.72	2.67	U2
10	0.004161												
11	0.003437	1.51	1.50	⊕	4.24	4.22	J1	7.16	[c]		1.82	1.92	U1
12	0.002886												
13	0.002457	1.08	1.08	♀	3.03	[b]		5.12	5.27	S5	1.30	1.30	U5
14	0.002117												
15	0.001842	0.81	[a]	☾?	2.28	2.22	J15	3.84	3.77	S4, S12	0.98		?
16	0.001618												
17	0.001432	0.63	0.58	☿	1.77	1.81	J5	2.98	2.95	S3, S13, S14	0.76		?
18	0.001276												
19	0.001145	L_I from the			1.41	1.34	J14	2.39	2.38	S2	0.61		?
20	0.001032	average of											
21	0.000936	♀,⊕, ♂			1.16	1.18	J16,[e]	1.95	1.86	S1	0.50	0.51	[g]
22	0.000852												
23	0.000779				L_I from the			1.62	1.51	S11, S10			
24	0.000658				average of								
25	0.000658				J1, J2, J3, J4			1.37	1.40	S15, S16, S17,[f]			

[a] possible orbit of the early Moon.
[b] possibly disturbed by Io = J1.
[c] possibly disturbed by Titan = S6.
[d] possibly disturbed by Titania = U3.
[e] outer edge of Jovian Ring.
[f] outer edge of Saturnian A-ring.
[g] outer edge of Uranian rings (ϵ-ring).

where $C_5 = C_4/C_3$ depends on the relative excitation of the modes J_0 and N_0. With the asymptotic representations for $r > L_{II}$ it follows that $\cot(r/L_{II} - \frac{3}{4}\pi) = -C_5$, or

$$\tan \frac{r}{L_{II}} = \frac{1+C_5}{1-C_5} \tag{29}$$

for the quasiperiodic radii of the density extrema. For a sequence of rings at $r = r_n$ this reduces to

$$r_{n+k} = r_n + 2k\pi L_{II} \tag{30}$$

as a law for periodic planetary distances, This seems to be of relevance for the outer planetary system (Litzroth, 1983).

4. Comparison with Real Satellite Systems

As is well known, the four 'developed' systems of massive central body and connected satellite system in the solar system, namely the planetary system and the satellite systems of Jupiter, Saturn and Uranus, exhibit some common or comparable properties (Stiller *et al.*, 1980, 1984). Such 'reconstruction characteristics' are, for every system:

- coplanarity of the orbits,
- circularity of the orbits,
- anisotropy due to predominance of one angular-momenta-orientation,
- quasiperiodicity of orbital radii.

These 'reconstruction characteristica' can be understood as to be caused by a comparable process-sequence of the early evolution of these systems, similar to the "hetegony principle" of Alfvén and Arrhenius (1976). The following conclusions about these early formation-processes shall be derived from the reconstruction-characteristica:

- separation (origin) of the complete matter of every system from a greater structure, as it is implied by the predominance of one angular-momenta-orientation for each system,
- evolution to an early presatellite thin disk,
- comparable gross-structures of the four systems indicate that the process-sequence of their formation was independent of special initial conditions. Therefore, these processes should be describable in a "quasi-stationary" approximation, which does not depend on special initial conditions, which were different for the four systems.

The above discussion encourages to apply the preceding analysis of the evolution of radial structures of thin disks on the description of the formation-processes of satellite systems. The following model of a pre-satellite disk shall be used for the further discussion:

- Keplerian velocity of the original 'ground state' of small particles (grains), as described by (10),
- use of a model-type as indicated by the 'intermediate model' for the undisturbed surface density (see Figure 1),
- restriction to 'quasistationary states', $\omega \ll \Omega_K$,

– restriction to a linear stability analysis, that may indicate those sites with exponentially increase in the density, where in the following non-linear regime ring-like matter accumulations may evolve,

– as a first step, axially symmetric models shall be discussed as predecessors of possible later (non-axisymmetric) 'azimuthal instabilities' of the resulting axisymmetric rings (Möhlmann, 1984; Boss, 1982).

5.1. Inner Systems

Table I and Figure 2 give the relative radii x_e of the first density extrema of $\delta\sigma_I$, as they have been computed from (25), and their possible correlation with real orbital radii r_r of the inner planets and the inner satellites of Jupiter, Saturn, and Uranus.

Here it should be noted, that $r_e = x_e L_I$ follows from $L_I = M_c k_z / 2\pi\sigma_0 = \gamma M_c / x_e v_0^2$. The v_0-values have been found by comparison with actual orbital velocities of appropriate planets and satellites which are mentioned in Table I.

It is remarkable that with, $L_I = 4.5 \times 10^{13}$ m, $k_z \approx L_I^{-1}$ follows $c_0 = M_c / 2\pi L_I^2 = 1.6 \times 10^2 \,\mathrm{kg\,m^{-2}}$, this is in the range of the values according to Figure 1, but very near to the minimal values, limited by the present planetary masses.

5.2. Outer Systems

A basic element of the above-discussed model is the assumption, that planetary (and statellite-) bodies may form at those sites where $\delta\sigma$ has maxima. Then, in the outer systems, the radial distribution of the planets is given by the linear relation (30). The resulting scale length L_{II} for the outer planetary system is given by $L_{II} = 2.4051 \times 10^{11}$ m, giving a radial structure of the outer planetary system, as described by Table II.

TABLE II

	$r_e[10^{11}\,\mathrm{m}]$	$r_r[10^{11}\,\mathrm{m}]$
Saturn	(14.33)	14.33
Uranus	29.44	28.84
Neptune	44.55	45.09
Pluto	59.67	59.66
?	74.78	?
:	:	:
?	120.00	?

It should be noted that the value in brackets has been used as a start value, and that L_{II} has been determined by the average of the differences $\overline{r_n - r_{n-1}} = 15.11 \times 10^{11}$ m.

It is interesting to note that, with $L_{II} = 2.4 \times 10^{11}$ m, $k_z \approx L^{-1}$ and $c_0 = M_c L_{II} / 2\pi$ we obtain surface densities (described in Figure 1 by open circles), which are greater by one order of magnitude than the maximum values of Weidenschilling (1977).

This may indicate an original massive outer band and a separate inner disk of lower surface density.

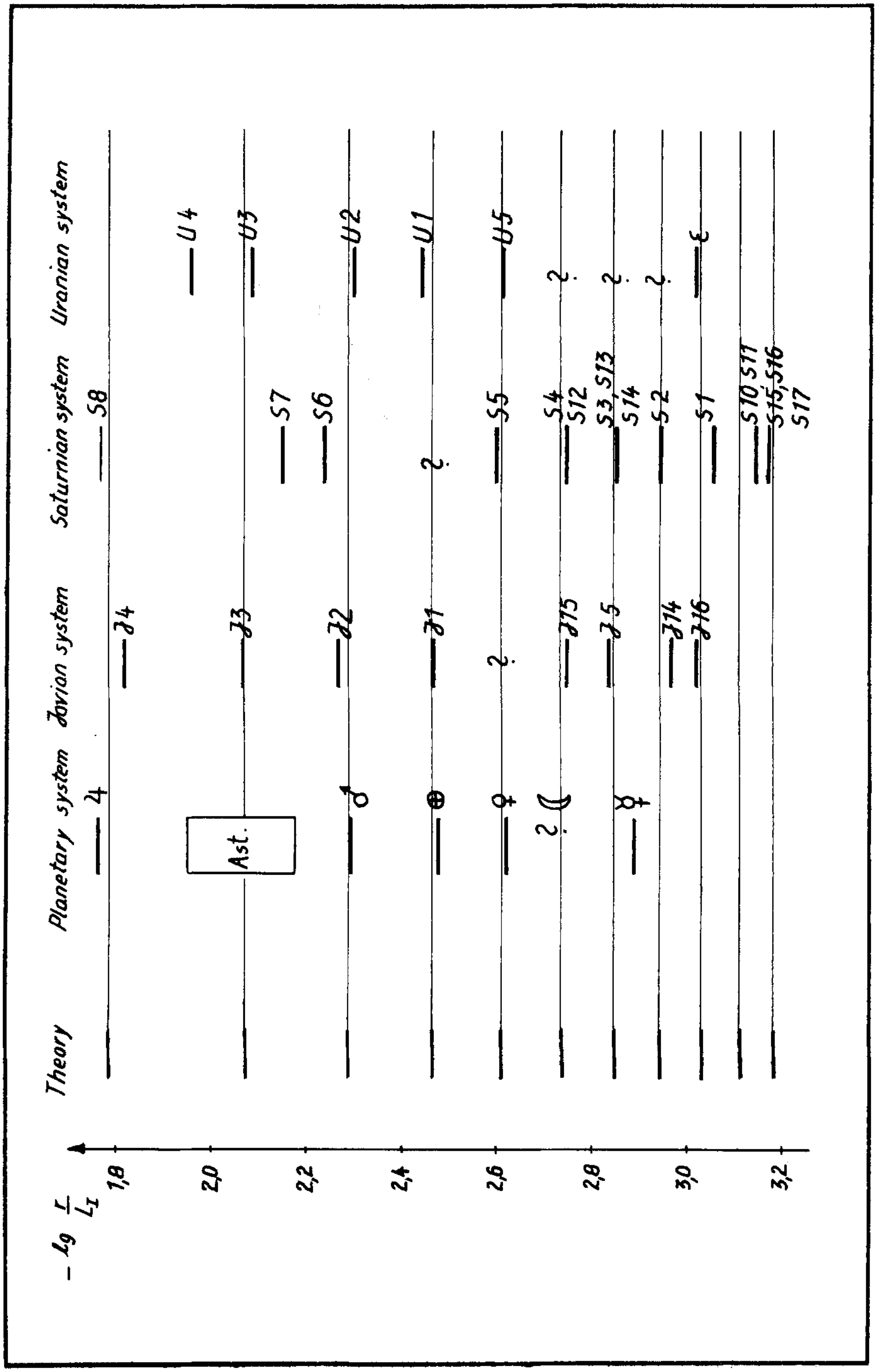

Fig. 2. Comparison of real and computed orbital radii.

As can be seen by comparison with Table I, there are no outer evolved satellite systems of the planets. Oberon could have been influenced by Titania and the Jovian satellites J13, J6, J10, and J7 (with an inclination of about 30°) and the group of retrograde orbiting satellites J8, J9, J11, and J12, probably, have been captured.

The disappearence of outer systems around the planets may be connected with their limited sphere of influence in the solar gravitational field. Consequently, hypothetical earlier outer rings or bands have been destroyed by solar tidal action.

5. Conclusions

The models discussed in the foregoing sections lead to the following planetogonic scheme:

- existence of a preplanetary (pre-satellite) disk of gas, plasma and (condensed) particles with a disk-mass, small compared to that of the central body,
- formation of a thin disk with height H much smaller than its radial extension $R \gg H$, caused by the action of early gravitational, centrifugal and frictional forces and due to large scale magnetic fields,
- radial structurization of this axisymmetric thin disk into rings for sufficiently decreased gas-pressure (temperature), where rings form by nonlinear collective gravitational effects at the sites of the maxima of the (linear) disturbed density – as described mathematically in Section 3,
- later azimuthal instability of these rings with further condensation-caused decrease of the action of pressure and due to collective self-gravitation of the disk and resulting fast growth of planetary bodies ('collective accretion') (Möhlmann, 1984a).

It should be noted here, that this scheme can be applied to origin and early evolution of the planetary system and the satellite systems of Jupiter, Saturn and Uranus.

The mathematical description of the essential physical processes of this scheme, based on a hydrodynamic formulation of the collective structuring effects of the self-gravitation of a particle disk around a massive central body, fits quite well observed radial structures in the orbits of planets and satellites in the solar system. Predictions seem to be possible as well for three yet unknown Uranian satellites, and a new approach to a possible lunar origin between Venus and Mercury or inside the Mercury orbit. It is interesting to note here that, on the basis of investigations of lunar material, Wood and Mitler (1974) concluded that the genesis of lunar matter occurred far inside the actual Earth orbit. Furthermore, the orbital radii derived from this planetogonic approach for possible transplutonian planets could coincide with cometary families.

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