ON A GENERALIZED THERMO-ELASTIC PROBLEM IN AN INFINITE CYLINDER UNDER INITIAL STRESS

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Abstract. In this paper we studied the influence of the initial stress on the propagation of Rayleigh waves in a homogeneous-isotropic, generalized thermo-elastic body, subject to the boundary conditions that the outer surface is traction free. In addition it is subject to linear radiations, adiabatic isothermal transfer conditions. We found that the frequency equation of Rayleigh waves contains a term involving the initial stress and, therefore, the phase velocity of Rayleigh waves changes with respect of this initial stress, when the initial stress, vanishes, the derived frequency equation reduces to that one obtained in classical generalized thermo-elastic case which includes the relaxation time of heat conduction.

1. Introduction

The dynamical problem of a generalized thermo-elastic waves has been discussed by Norwood and Warren (1969), Chattapadhy and Kar (1981), Sukhendu and Addy (1979), and Tomiat and Shindo (1979); Elnagar and Abd-Allah (1985) investigated the dynamical problem of thermo elastic solid. But they all considered initially stress-free media. Besides the Earth, many structural bodies are found to be initially stressed. It is of practical importance to study the effect of the initial stresses on the waves propagated in these bodies. Using the generalized theory of thermo-elasticity proposed by Green and Lindsay (1972), we studied the influence of the initial stress on the propagation of Rayleigh waves, in an infinite cylinder under incremental thermal stress. We found that the frequency equation of Rayleigh waves contains the term involving the initial stress and so the phase velocity of Rayleigh waves changes with respect to this initial stress. When the initial stress vanishes, the derived frequency equation reduces to the one that obtained in classical generalized thermo-elastic case which includes the relaxation time of heat conduction.

2. Formulation of the Problem

Consider a homogeneous and isotropic in a generalized thermo-elastic solid taking in account the time needed for acceleration of the heat flow (infinite circular cylinder of radius R). The axis may be taken along the z-axis, subject to the boundary conditions traction free at $r = R$. Let us suppose that an infinite cylinder under initial stress P and initial temperature T_0. When the temperature of the infinite cylinder is changed incremental thermal stresses τ_{ij} together with incremental strain e_{ij} are produced. Referring the medium to cylindrical polar coordinates (r, θ, z), z being the

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axis of the cylinder and assume U_θ, U_z, T are functions of r, z, and t only. The dynamic equation of motion in the absence of body forces under initial compression P are given (cf. Bolt, 1965) by

$$\frac{\partial s_{rr}}{\partial r} + \frac{\partial s_{rz}}{\partial z} + \frac{1}{r}(s_{rr} - s_{\theta a}) + P\frac{\partial \omega_e}{\partial z} = \rho \frac{\partial^2 U_r}{\partial t^2},$$

$$\frac{\partial s_{rz}}{\partial r} + \frac{\partial s_{zz}}{\partial z} + \frac{1}{r} s_{rz} + \frac{P}{r}\frac{\partial}{\partial r}(r\omega_\theta) = \rho \frac{\partial^2 U_z}{\partial t^2}. \qquad (1)$$

The generalized equation of heat conduction is of the form

$$U\nabla^2 = \rho C_e(\dot{T} + \tau\ddot{T}) + \propto (3\lambda + 2\mu) T_0 \nabla \cdot (\dot{\mathbf{U}} + \tau\ddot{\mathbf{U}}), \qquad (2)$$

where τ represents the time lag needed to establish steady state heat conduction in an element of volume when a temperature gradient is suddenly imposed on that element. It is called the relaxation time. Also $\mathbf{U} = (U_r, 0, U_z)$ is the displacement vector, T is the temperature change about the equilibrium temperature T_0, ρ is the density of the medium, λ and μ are Lamé's constant and $\propto$ is the coefficient of volume expansion.

The stress-strain relations with incremental isotropy under initial stress are given by

$$s_{rr} = (\delta + \mu + P)\frac{\partial U_r}{\partial \bar{r}} + (\delta - \mu + P)\frac{\partial U_z}{\partial z} + (\delta - \mu + P) \times$$
$$\times \frac{U_r}{r} - \frac{\gamma}{x_\theta}(T + \tau\dot{T}),$$

$$s_{\theta\theta} = (\delta + \mu + P)\frac{U_r}{r} + (\delta - \mu + P)\frac{\partial U_r}{\partial r} + (\delta - \mu + P)\frac{\partial U_r}{\partial z} -$$
$$- \frac{\gamma}{x_\theta}(T + \tau\dot{T}),$$

$$s_{zz} = (\delta + \mu)\frac{\partial U_z}{\partial z} + (\delta - \mu)\frac{\partial U_r}{\partial r} + (\delta - \mu)\frac{U_r}{r} - \frac{\gamma}{x_\theta}(T + \tau\dot{T}),$$

$$s_{rz} = \mu\left(\frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r}\right), \qquad (3)$$

where $\delta = \lambda + \mu$, x_θ is the isothermal compressibility and the incremental strain components and the rotation are given by

$$e_{rr} = \frac{\partial U_r}{\partial r}, \quad e_{\theta\theta} = \frac{\partial U_r}{\partial r}, \quad e_{zz} = \frac{\partial U_z}{\partial z},$$

and

$$e_{rz} = \frac{1}{2}\left(\frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r}\right), \quad \omega_\theta = \frac{1}{2}\left(\frac{\partial U_r}{\partial z} - \frac{\partial U_z}{\partial r}\right). \tag{4}$$

Equation (1) with the help of Equations (3) and (4) may be written as:

$$(\delta+\mu+P)\frac{\partial \Delta}{\partial r} + 2\left(\mu + \frac{P}{2}\right)\frac{\partial \omega_\theta}{\partial z} - \frac{\gamma}{x_\theta}\frac{\partial}{\partial r}(T+\tau\dot{T}) = \rho\frac{\partial^2 U_r}{\partial t^2}, \tag{5}$$

$$(\delta+\mu)\frac{\partial \Delta}{\partial z} - \frac{2}{r}\left(\mu - \frac{P}{2}\right)\frac{\partial}{\partial r}(r\omega_\theta - \frac{\gamma}{x_\theta}\frac{\partial}{\partial z}(T+\tau-\dot{T}) =$$
$$= \rho\frac{\partial^2 U_z}{\partial t^2}, \tag{6}$$

where
$$\Delta = \frac{\partial U_r}{\partial r} + \frac{U_r}{r} + \frac{\partial U_z}{\partial z}.$$

By Helmholtz's theorem (cf. Morse and Feshbach, 1953), the displacement U_r and U_z can be written in the form

$$U_r = \frac{\partial \phi}{\partial r} - \frac{\partial \psi}{\partial z},$$
$$U_r = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial r}, \tag{7}$$

where the two functions ϕ and ψ are known in the theory of elasticity, by Lamé's potential representing irrotational and rotational parts of the displacement vector **U**,respectively.

Using the Equations (7), the Equations (5) and (6) reduce to

$$\nabla^2\phi = \frac{\rho}{(\delta+\mu+P)}\frac{\partial^2\phi}{\partial t^2} + \frac{\gamma}{x_\theta(\delta+\mu+P)}(T+\tau\dot{T}), \tag{8}$$

$$\nabla^2\phi = \frac{\rho}{(\delta+\mu)}\frac{\partial^2\phi}{\partial t^2} + \frac{\gamma}{x_\theta(\delta+\mu)}(T+\tau\dot{T}), \tag{9}$$

$$\nabla^2\psi = \frac{\rho}{\mu + (P/2)} \frac{\partial^2\psi}{\partial t^2}, \tag{10}$$

$$\nabla^2\psi = \frac{\rho}{\mu - (P/2)} \frac{\partial^2\psi}{\partial t^2}, \tag{11}$$

respectively. Similar results were obtained by Dey and Addy (1984) and Dey and Chakroborty (1984), while the second reference deriving the constitutive equation for Rayleigh waves in elastic medium under initial stress. Since the initial stress has been taken in the direction of r only, the velocity of by waves will be different in r and z directions. In the absence of P the Equations (8), (9), (10), and (11) reduce to two equations only. Now Equations (8) and (9) represents the compressive wave along the r and z directions, respectively, and Equations (10) and (11) represents the shear wave along those directions, respectively.

Equation (8) represents the longitudinal wave in the direction of r with velocity $c_1 = [(\delta + \mu + P)/\rho]^{1/2}$ and Equation (11) represents the velocity of the shear wave in the direction of r with velocity $c_2 = [(\mu - P/2)/\rho]^{1/2}$. Equation (9) represents the longitudinal wave in the direction of z with velocity $c_1 = [(\delta + \mu)/\rho]^{1/2}$ and Equation (10) represents the shear wave in the direction of z with velocity $c_2 = [(\mu + P/2)/\rho]^{1/2}$.

Since we are considering the propagation of Rayleigh waves in direction of r only, we shall consider the Equations (8) and (11) only for our discussion. Assuming a simple harmonic time-dependent factor $\exp(i\omega t)$ of all the quantities and omitting the factor $\exp(i\omega t)$ throughout we find that Equations (2), (8), and (11) yield a set of differential equations for ϕ, ψ, T, of the form

$$\nabla^2 T = \frac{\rho c_e i\omega}{K} T(1 + i\omega\tau) + \frac{\gamma(3\lambda + 2\mu)}{K} T_0 \nabla^2 \phi \times i\omega (1 + i\omega\tau), \tag{12}$$

$$\nabla^2\phi = \frac{-\rho\omega^2}{\delta + \mu + P} \phi + \frac{\gamma T(1 + i\omega\tau)}{x\theta(\delta + \mu + P)}, \tag{13}$$

$$\nabla^2\psi = \frac{-\rho\omega^2}{\mu P/2} \psi, \tag{14}$$

we can eliminate T or ϕ between (12) and (13) by cross-differentiation to obtain.

$$\nabla^4\phi + \left[\frac{\omega^2}{c_1^2} - \frac{\rho c_e i\omega\tau'}{K}(1 - \epsilon\tau')\right] \nabla^2\phi - \frac{i\omega^3 \rho c_e \tau'}{K c_1^2} = 0, \tag{15}$$

$$\epsilon = \frac{\gamma^2 T_0}{\rho^2 c_1^2 c_e x_\theta}, \quad \gamma = \alpha(3\gamma + 2\mu), \ \tau' = 1 + i\omega\tau.$$

3. Solution of the Equations

General solution of (15) and (14) are given as in Harinath (1977), then if we introduce the inversion of the Fourier transform, which is defined by

$$\phi(r, z, \omega) = \int_0^\infty \hat{\phi}(\eta, z, \omega)\, J_0(\eta r)\, \eta \, d\eta. \tag{16}$$

Substituting (16) into (15) we obtain

$$\left(\eta^2 - \frac{\partial^2}{\partial z^2}\right)\left(\eta^2 \tau \frac{\partial^2}{\partial z^2}\right)\hat{\phi} - \left[\frac{\omega^2}{c_1^2} - \frac{\rho c_e i\omega\tau'}{K}(1-\epsilon\tau')\right] \times \\ \times \left(\eta^2 - \frac{\partial^2}{\partial z^2}\right)\hat{\phi} + \frac{i\omega^3 \rho c_e \tau'}{Kc_1^2}\hat{\phi} = 0. \tag{17}$$

The indicial equation governing (17) is

$$f^4 - \left[\frac{\omega^2}{c_1^2} - \frac{\rho c_e i\omega\tau'}{K}(1+\epsilon\tau')\right] f^2 + \frac{i\omega^3 \rho c_e \tau'}{Kc_1^2} = 0. \tag{18}$$

Putting $\xi_j^2 = \eta^2 - f_j^2$, $R_e(\xi_j) \geq 0$, $j = 1, 2$.

As noted already, the factor $\exp(i\omega t)$ has been omitted in the expression for ϕ, ψ, T. Moreover, f_1, f_2 are the roots of the Equation (18) given by

$$f_1^2, f_2^2 = \left\{\left[\frac{\omega^2}{2c_1^2} - \frac{\rho c_e i\omega\tau}{2k}(1+\epsilon\tau') \pm \left[\frac{\omega^2}{4c_1^2} - \frac{\rho c_e i\omega\tau'}{4K}(1+\epsilon\tau')\right]^2 - \right.\right. \\ \left.\left. - (1+\epsilon\tau')\right]^2 - \frac{i\omega^3 \rho c_e \tau'}{Kc_1^2}\right]^{1/2}\right\}.$$

Then the solution of (17) is:

$$\hat{\phi}(\eta, z, \omega) = A(\eta)\, e^{-\xi_1 z} + B(\eta)\, e^{-\xi_2 z}$$

which leads to:

$$\phi(r, z, t) = \int_0^\infty [A(\eta) e^{-\xi_1 z + i\omega t} + B(\eta) e^{-\xi_2 z + i\omega t}]\, J_0(\eta r)\, \eta \, d\eta. \tag{19}$$

Similarly we can obtain the solution for Equation (14) which leads to:

$$\psi(r, z, t) = \int_0^\infty C(\eta) e^{-\xi_3 z + i\omega t} J_0(\eta r)\, \eta \, d\eta, \tag{20}$$

where

$$\xi_3^2 = \eta^2 - \frac{\omega^2}{c_2^2}, \quad R_e(\xi_3) \geq 0.$$

Also to obtain the temperature deviation T we substitute from (19) into (13) we get

$$T = \frac{\rho x_\theta}{\gamma \tau'} \int_0^\infty [A(\eta)\,(\omega^2 - c_1^2\,\xi_1^2) e^{-\xi_1 z + i\omega t} + \\ + B\,(\eta)\,(\omega^2 - c_1^2\,\xi_2^2) e^{-\xi_2 z + i\omega t}]\, J_0\,(\eta r)\,\eta\,\mathrm{d}\eta. \tag{21}$$

In terms of the potentials ϕ, ψ the stress components s_{rr} and s_{rz} are given by:

$$s_{rr} = (\lambda + P)\,\nabla^2\phi + 2\mu\frac{\partial^2\phi}{\partial r^2} - 2\mu\frac{\partial^2\psi}{\partial r\partial z} - (\lambda+P)\frac{1}{r}\frac{\partial\psi}{\partial z} - \frac{\gamma}{x_\sigma}(T + \tau\dot{T}),$$

$$s_{rz} = 2\eta\,\frac{\partial^2\phi}{\partial r\partial z} - \eta\,\frac{\partial^2\psi}{\partial z^2} + \eta\,\frac{\partial^2\psi}{\partial r^2}. \tag{22}$$

Substituting (19), (20), (21) into (22) we get

$$s_{rr} = \int_0^\infty A(\eta)\left\{(\lambda+P)\left[\eta J_0\,(\eta r)\,(\xi_1^2 - \eta^2) + \frac{\eta^2}{r}\,J_1(\eta r) - \right.\right. \\ \left.\left. - 2\mu\eta^3\,J_0(\eta r) + \frac{2\mu\eta^2}{r}\,J_1(\eta r) - \rho(\omega^2 - c_1^2\,\xi_1^2)\,J_0\,(\eta r)\eta\right\} e^{-\xi_1 z + i\omega t}\,\mathrm{d}\eta + \\ + \int_0^\infty B(\eta)\left\{(\lambda+P)\left[\eta J_0(\eta r)\,(\xi_1^2 - \eta^2) + \frac{\eta^2}{r}\,J_1(\eta r)\right] - \right. \\ \left. - 2\mu\eta^3\,J_0(\eta r) + \frac{2\mu\eta^2}{r}\,J_1\,(\eta r) - \rho(\omega^2 - c_1^2\,\xi_2^2)\,J_0(\eta r)\eta\right\} e^{-\xi_2 z + i\omega t}\,\mathrm{d}\eta - \\ - \int_0^\infty C(\eta)\left[2\mu\,\xi_3^2\,\eta^2\,J_1(\eta r)\,(\lambda+P)\,\frac{1}{r}\,\xi_3 J_0\,(\eta r)\,\eta\right] e^{-\xi_3 z + i\omega t}\,\mathrm{d}\eta,$$

$$s_{rz} = 2\mu \int_0^\infty [A(\eta)\,\xi_1\,e^{-\xi_1 z + i\omega t} \\ + B\,(\eta)\,\xi_2\,e^{-\xi_2 z + i\omega t}]\eta^2\,J_1(\eta r)\,\mathrm{d}\eta - \mu\int_0^\infty C(\eta)\left[\xi_3^2\,J_0\,(\eta r) + \right. \\ \left. + \eta^2\,J_0(\eta r) - \frac{\eta}{r}\,J_1(\eta r) e^{-\xi_3 z + i\omega t}\right]\eta\,\mathrm{d}\eta \tag{23}$$

4. Frequency Equation

In this section we obtained the frequencies equations for the boundary conditions which specify that the outer surface of the cylinder be trraction free, i.e.

$$s_{rr} = 0 \quad \text{at} \quad r = R \tag{24}$$

and subject to either linear radiation adiabatic isothermal heat transfer conditions:

$$\left.\begin{aligned} \frac{\partial T}{\partial r} + hT &= 0, \\ -\frac{\partial T}{\partial r} &= 0, \end{aligned}\right\} \quad T = 0 \quad \text{at} \quad r \,\alpha\, R, \tag{25}$$

where R is the radius of the cylinder and h is a non-negative constant. In the analysis the follows, frequency equations are derived for the linear radiation surface condition and the frequency equations corresponding adiabatic and isothermal surface conditions deduced as limiting cases. The three boundary conditions given by (24) and (25) now suffice to determine the arbitrary functions $A(\eta)$, $B(\eta)$, and $C(\eta)$. Using these conditions leads to the following form:

$$A(\eta)\left\{(\lambda+P)\left[\eta J_0(\eta R)(\xi_1^2-\eta^2)+\frac{\eta^2}{R}J_1(\eta R)\right]-2\mu\eta^3 J_0(\eta R)+\right.$$

$$\left.+\frac{2\mu\eta^2}{R}J_1(\eta R)-\rho(\omega^2-c_1^2\xi_1^2)J_0(\eta R)\eta\right\}e^{-\xi_1 z}+B(\eta)\left\{(\lambda+P)\times\right.$$

$$\times\left[\eta J_0(\eta R)(\xi_2^2-\eta^2)+\frac{\eta^2}{R}J_1(\eta R)\right]-2\mu\eta^3 J_0(\eta R)+$$

$$\left.+\frac{2\mu\eta^2}{R}J_1(\eta R)\times\rho(\omega^2-d_1^2\xi_1^2)J_0(\eta R)\eta\right\}e^{-\xi_2 z}-$$

$$-C(\eta)\left[(2\mu\,\xi_3^2\eta^2 J_1(\eta R)-(\lambda+P)\frac{1}{R}3J_0(\eta R)\eta\right]e^{-\xi_3 z}=0,$$

$$A(\eta)\,[2\mu\,\xi_1\,e^{-\xi_1 z}\eta^2 J_1(\eta R)]\,e^{-\xi_1 z}+B(\eta)\,[2\mu\,\xi_2\,\eta^2 J_1(\eta R)]\,e^{-\xi_2 z}-$$

$$-C(\eta)\,\mu\eta\left[\xi_3^2 J_0(\eta R)+\eta^2 J_0(\eta R)-\frac{\eta}{R}J_1(\eta R)\right]e^{-\xi_3 z}=0,$$

$$A(\eta)\,(\omega^2-c_1^2\xi_1^2)\eta\,[hJ_0(\eta R)-\eta J_1(\eta R)]\,e^{-\xi_1 z}+B(\eta)\times$$

$$\times(\omega^2-c_1^2\xi_2^2)\eta\,[hJ_0(\eta R)-\eta J_1(\eta R)]e^{-\xi_2 z}=0. \tag{26}$$

In order to obtain non-trival solution the following relation, which is another than the frequency equation must be satisfied.

$$\mu\eta\left[\xi_3^2 J_0(\eta R) + \eta^2 J_0(\eta R) - \frac{\eta}{R} J_1(\eta R)\right]\eta\,[hJ_0(\eta R) - \eta J_1(\eta R)] \times$$
$$\times(\lambda+P)\left\{\left[\eta J_0(\eta R)(\xi_1^2-\eta^2) + \frac{\eta^2}{R}J_1(\eta R)\right] - 2\mu\eta^3 J_0(\eta R) + \frac{2\mu\eta^2}{R}J_1\times\right.$$
$$\left.\times(\eta R) - \rho(\omega^2-c_1^2\,\xi_1^2)\,J_0(\eta R)\,\eta\right\}(\omega^2-c_1^2\,\xi_2^2) + \left\{(\lambda+P)\left[\eta J_0(\eta R)\times\right.\right.$$
$$\left.\times(\xi_2^2-\eta^2) + \frac{\eta^2}{R}J_1(\eta R)\right] - 2\mu\eta^3 J_0(\eta R) + \frac{2\mu\eta^2}{R}J_1(\eta R) - \rho(\omega^2-c_1^2\,\xi_2^2\times$$
$$\left.\times J_0(\eta R)\,\eta\right\}(\omega^2-c_1^2\,\xi_1^2) - \left[2\mu\,\xi_3^2\,\eta^2 J_1(\eta R) - (\lambda+P)\frac{1}{R}\xi_3 J_0(\eta R)\eta\right],$$
$$\eta[hJ_0(\eta R) - \eta J_1(\eta R)]\,2\mu\eta^2 J_1(\eta R)\,[\xi_1(\omega^2-c_1^2\,\xi_2^2)-\xi_2(\omega^2-c_1^2\,\xi_1^2)] = 0. \tag{27}$$

It is extremely difficult to obtain roots of this transcendental equation. However, if the radius of the cylinder is assumed to be small that ηR is small compared with one (i.e.) the wavelength is large in comparison with radius of the cylinder. Then $J_0(\eta R) \simeq 1$, $J_1(\eta R) \simeq \eta R/2$ substituting these approximations values into the frequency equation (27) we find

$$\mu\eta^2\,[\xi_3^2 + \eta^2/2]\left[h - \frac{\eta^2 R}{2}\right]\left\{\left[(\lambda+P)\,[\eta(\xi_1^2-\eta^2/2]-\mu\eta^3 -\right.\right.$$
$$\left.-\rho(\omega^2-c_1^2\,\xi_1^2)\eta\right] + \{(\lambda+P)\,[\eta(\xi_2^2-\eta^2/2]-\mu\eta^3-\rho(\omega^2-c_1^2\,\xi_2^2)\eta\}\times$$
$$\left.\times(\omega^2-c_1^2\,\xi_1^2)\right\} - \mu\eta^4 R\left[\mu\eta^3 R\,\xi_3^2 + (\lambda+P)\frac{1}{R} - \xi_3\eta\right]\times$$
$$\times\left[h - \frac{\eta^2 R}{2}\right][\xi_1(\omega^2-c_1^2\,\xi_2^2) - \xi_2\times(\omega^2 - c_1^2\,\xi_1^2)\} = 0. \tag{28}$$

This is the frequency equation of a generalized thermo-elastic Rayleigh waves which has not yet been studied. It is clear from this frequency equation that the phase velocity of Rayleigh wave depends on initial stresses P present in the medium.

The frequency equation (28) contain the initial stresses, when $P = 0$, that is when there is no initial stresses, we get expressions for the frequency equation which is agree with the result by Nayfeh and Nemat-Nasser (1972), the Equation (28) is

$$\mu\eta^2 [\xi_3^2 + \eta^2/2] \left[h - \frac{\eta^2 R}{2}\right] \{[\lambda\eta(\xi_1^2 - \eta^2/2] - \mu\eta^3 -$$
$$- \rho(\omega^2 - c_1^2 \xi_1^2)\} \times (\omega^2 - c_1^2 \xi_2^2) + [\lambda\eta (\xi_2^2 + \eta^2/2) - \mu\eta^3 - \rho(\omega^2 - c_1^2 \xi_2^2)\eta] \times$$
$$\times \rho(\omega^2 - c_1^2 \xi_1^2) - \left[\mu \xi_3^2 \eta^3 R - \frac{\lambda}{R} \xi_3\eta\right]\eta \left[h - \frac{\eta^2 R}{2}\right] \mu\eta^3 R \times$$
$$\times [\xi_1 (\omega^2 - c_1^2 \xi_2^2) - \xi_2(\omega^2 - c_1^2 \xi_1^2)] = 0. \tag{29}$$

This is also the same frequency equation of Rayleigh waves in a generalized thermo-elastic in an infinite cylinder as obtained by Locket (1958) or Day and Addy (1979). In deducing we assumed a convection condition for the temperature on the boundary for thermal insolation $h = 0$ and Equation (28) reduces to

$$- \mu\eta^2 [\xi_3^2 + \eta^2/2] \frac{\eta^2 R}{2} \{(\lambda + P) [\eta(\xi_1^2 - \eta^2/2)] - \mu\eta^3 -$$
$$- \rho(\omega^2 - c_1^2 \xi_1^2)\eta\} (\omega^2 - c_1^2 \xi_2^2) + \{(\lambda + P) [\eta(\xi_2^2 - \eta^2/2)] - \mu\eta^3 -$$
$$- \rho(\omega^2 - c_1^2 \xi_2^2)\eta\} (\omega^2 - c_1^2 \xi_1^2) + \left[\mu \xi_3^2 \eta^3 R - (\lambda + P) \frac{\lambda}{R} \xi_3\eta\right]\eta \frac{\eta^2 R}{2} \times$$
$$\times [\xi_1 (\omega^2 - c_1^2 \xi_2^2) - \xi_2(\omega^2 - c_1^2 \xi_1^2)] = 0. \tag{30}$$

If the temperature vanish on the boundary, $h \to \infty$ and Equation (28) reduces to

$$\mu\eta^2 [\xi_3^2 + \eta^2/2] \left[\{(\lambda + P) [\eta(\xi_1^2 - \eta^2/2)] - \mu\eta^3 - \rho(\omega^2 - c_1^2 \xi_1^2)\eta\} \times \right.$$
$$\times (\omega^2 - c_1^2 \xi_2^2) + \{(\lambda + P) [\eta(\xi_2^2 - \eta^2/2)] - \mu\eta^3 - \rho(\omega^2 - c_1^2 \xi_2^2)\eta\} \times$$
$$\left. \times (\omega^2 - c_1^2 \xi_1^2) \right] - \left[\mu \xi_3^2 \eta^3 R - (\lambda + P) \frac{\lambda}{R} \xi_3\right]\eta \mu\eta^3 R [\xi_1(\omega^2 - c_1^2\xi_2^2) -$$
$$- \xi_2 (\omega^2 - c_1^2 \xi_1^2)] = 0. \tag{31}$$

If we put initial stress $P = 0$ and h, ϵ vanish (i.e.) when there is no coupling between the temperature and the strain fields and we have the frequency equation of (28) which takes the form

$$- \mu\eta^2 [\xi'^2_3 + \eta^2/2] \frac{\eta^2 R}{2} \Bigg[\{\lambda [\eta(\xi'^2_1 - \eta^2/2)] - \mu\eta^3 -$$

$$- \rho(\omega^2 - c_1^2 \xi'^2_1)\eta\} (\omega^2 - c_1^2 \xi'^2_2) + \{(\lambda + P) [\eta(\xi'^2_2 - \eta^2/2)] - \mu\eta^3 -$$

$$- \rho(\omega^2 - c_1^2 \xi'^2_2)\eta\} (\omega^2 - c_1^2 \xi'^2_1) \Bigg] + \left[\mu \xi'^2_3 \eta^3 R - \frac{\lambda}{R} \xi'_3 \eta \right] \frac{\eta^2 R}{2} \mu \times$$

$$\times [\xi'_1 (\omega^2 - c_1^2 \xi'^2_2) - \xi_2(\omega^2 - c_1^2 \xi'^2_1)] = 0. \tag{32}$$

where

$$\xi'^2_1 \xi'^2_2 = \eta^2 - \left\{ \left[\frac{\omega^2}{2c_1^2} - \frac{\rho c_e i\omega\tau'}{2K} \right] \pm \left[\frac{\omega^2}{4c_1^2} - \frac{\rho c_e i\omega\tau'}{4K} \right]^2 - - \frac{i\omega\, \rho c_e \tau'}{K c_1^2} \right]^{1/2} \right\}.$$

It is clear that Equation (32) is the familiar frequency equation of Rayleigh wave in a generalized thermo-elastic in an infinite cylinder in classical case as obtained by Dey and Addy (1979) but in half-space and Tomita and Shindo (1979) but in the magento-thermo-elastic solids, we can say that by using the generalized theory of thermo-elasticity which takes into account the time needed for acceleration of the heat flow. By introducing the relaxation time in the heat conduction equation, the attenuation constant for a simple harmonic wave in an infinite elastic cylinder alter by very small coupled theory of thermo-elasticity.

5. Conclusions

The general form of the wave motion for a generalized thermo-elastic solid under initial stress can be separated into four equations by using Lamé potential (7); considering Equations (8) and (11), we have seen that they have the basic solutions to the generalized thermo-elastic equation of motion for an infinite cylinder. As a results of our discussion to the frequency equation (which contain initial stress), we noticed that when the initial stress vanish it is the same frequency equation of Rayleigh waves in a generalized thermo-elastic medium. Also with $h = 0$, $P = 0$, and $\epsilon = 0$ we obtain the familiar frequency equation of Rayleigh waves in a generalized thermo-elastic in classical case.

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