THE WAVE NATURE AND DYNAMICAL QUANTIZATION OF THE SOLAR SYSTEM

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Abstract. A heuristic model is proposed of the mean distances between the solar-system planets, their satellites and the primaries. The model is based on: (i) the concept of the solar system structure wave nature; (ii) the micro-mega analogy (MM analogy) of the micro- and megasystem structures, and (iii) the oscillator amplitude "quantization" phenomenon, occuring under wave action, discovered on the basis of the classical oscillations theory (Damgov *et al.*, 1990, 1991).

From the equation, describing the charge rotation under the action of an electromagnetic wave, an expression is obtained for the discrete set of probable stationary motion amplitudes. The discrete amplitude values – the "quantization" phenomenon – are defined by the argument values at the extreme points of the N-order Bessel functions. Using this expression, the mean related distances are computed from the solar system planets and the Saturn, Uranian and Jovian satellites to the primaries.

1. Introduction

The commensurability and resonance phenomena of the solar system motion structure – (including planets, asteroids, planets' satellites) – have been an object of detailed discussions and experimental examination in the last years (Molchanov, 1973; Beletsky, 1977; Blehmann, 1981; Davies, 1985; Gorkavy and Fridmann, 1990; Tchetchelnitsky, 1980, 1986; Saslaw, 1987; Shklovsky, 1988). Systematic observations and measurements have been carried out using all available means. A considerable bulk of empiric data for the solar system has been obtained from the "Voyager 2" mission. The understanding of the inevitable resonance character of evolving mature oscillation systems leads to series of interesting concepts about the resonance character of the solar system motion dynamics. One of the most outstanding among them is Molchanov's hypothesis (Molchanov, 1973) about the complete resonance character of the large planets in-orbit motion. A. M. Molchanov has noticed that the mean motion of the nine large planets $n_1, \ldots, n_9$ are related approximately by nine linear homogenous equations

$$k_1^{(j)} n_1 + \cdots + k_9^{(j)} n_9 = 0, \qquad j = 1, \ldots, 9\,,$$

with integer coefficients $k_1^{(j)}, \ldots, k_9^{(j)}$. The mean motions of the Jovian, Saturn and Uranian satellites are related to similar equations.

If asteroids are considered as planets and excluding Pluto, it is established (Thcetchelnitsky, 1980) that the planetary distances obey the following regularity:

$$\frac{a_{k+1}}{a_k} \simeq 1.75 \pm 0.20\,,$$

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i.e., the relation of the semi-major orbital axes of neighbouring planets is almost constant.

Up to now, whole series of phenomena in the solar system structure are considered anomalous or obscure. For example, the revolution periods of the celestial bodies vary within comparatively narrow range of values even if their masses differ considerably.

Those and other phenomena do not fit properly in the pattern of classical celestial mechanics and are actually ignored due to the absence of traditional explanation.

The development of astronautics and modern astrodynamics calls for: (1) an understanding of the anomalies' origin; (ii) filling up the gaps in the existing theoretical models; (iii) clarification of certain phenomena, such as the nature of stability of commensurability, periodicity and resonances and their probable preferability, having in mind that the abovementioned phenomena are observed not only among celestial bodies, but also among artificial satellites, space ferries and orbital stations.

2. Analysis – General Premises

Practically all contemporary scientists come to the conclusion that the new picture of the Universe is based on "closing down the Nature ring" that leads to close interrelations between the microcosm physics and astrophysics. The general data on near-the-Earth, deep space and especially solar system structures cannot be thoroughly interpreted without a general notion of the wave nature of occuring phenomena.

Since 1982, a great interest has arisen in the quantization problem – (in the large) and the Universe megasystem wave structure (Louise, 1982, Wayte, 1982, Greenberger, 1983, Houpis and Mendis, 1983). The concept and the corresponding idea of the megaquantum wave structure of astronomic systems has been considered.

According to the wave Universe concept, the large astronomic systems are considered as wave dynamic systems that are, in a certain sense, analogous to the atomic system (Pippard, 1983). From that point of view the solar system is considered as a wave dynamical system. Its components – the celestial bodies (the Sun, the planets, the satellites and the small celestial bodies) and the interplanetary continuum (interplanetary plasma, electromagnetic fields, etc) are described within a common dynamic substance – field context.

The phenomenologic and dynamic description of such systems is connected with the megaquantum wave dynamics. Assuming that the solar system is a wave dynamic system and hence, the micro-mega-analogy (MM-analogy) is valid, series of deductions have been drawn on the basis of the dynamic isomorphism of the atomic and the solar system structures (Oldershaw, 1982). Different quantization versions have been used – according to Bohr, De Broglie, Sommerfeld, Schrödinger, etc. The essence of all those fundamental works is the substantiation of the existence of megawaves, realizing short-range interactions in a scale, commensurable with the system scale in any Universal megasystem and, in particular, in the solar system, representing an analog to De Broglie's waves for gigantic astronomic systems. However, the common opinion is that the major problem at present is to make out a more detailed identification, phenomenologic and dynamic description of the solar system megaquantum structure and of the analogous systems, constituting of a central body and satellites (Tchetchelnitsky, 1986).

The above-mentioned concepts of the bodies' wave interaction unity in the micro- and macroworld as well as of the basic realization and manifestation behavior closing down in Nature is the essence and grounds for the present work. For the purpose of commensurability, resonance and dynamic quantization studies of the solar system bodies motion structure, different authors use versions of the quantization phenomena within the frames of the quantum mechanics. Herewith, a heuristic model is proposed of the discrete distribution of solar system planets and satellites mean distances from the primaries. That model is based on the solar system structure wave nature concept, as well as on the oscillator amplitude "quantization" phenomenon, occuring under wave action, discovered on the basis of the classical oscillations theory (Damgov 1990,1991, Damgov *et al.* 1990, 1991, Damgov and Grinberg 1991). The analysis of the latter explains and gives analytic grounds for: (i) the existence of a discrete set of stable ("allowed") orbits and a set of resonant, but unstable ("forbidden") orbits of the planets and their satellites; (ii) the phase locking of the in-orbit motion; (iii) the trajectory stability only under exceptional initial conditions and in strictly defined areas, and (iv) the independence of the stable stationary orbits from the planets' and satellites' masses.

3. A Heuristic Model of the Dynamic Quantization of Solar System Bodies Motion Structure

As a rule, the harmonic oscillator is taken as the basis of the field theory interpretations (Pippard, 1983) from point of view of the classical theory of oscillations and the quantum-mechanical approach. The classical approach is of greater heuristic value and efficiency when it comes to the action of fully determined (in the classical sense) forces at an oscillator. This fact, noted by other researchers, is confirmed by the analysis, presented below.

Herewith, as a general model we take a periodical motion, that is the rotation of a charge at which an electromagnetic wave falls along the X-axis.*

The charge motion equation is

* We draw intentionally a simple consideration, so as to outline more vividly the heuristicity and perspectiveness of the presented approach.

$$\ddot{\mathbf{r}} + 2\beta\dot{\mathbf{r}} + \omega_0^2\mathbf{r} = e\mathbf{E}\sin(\nu t - \mathbf{kr}) , \tag{1}$$

where $\mathbf{r}$ is the charge radius-vector; 2β is the dissipation coefficient; e is the charge value; $\mathbf{k}$ is the wave vector; $\mathbf{E}$ is the wave field intensity and ν, t are the frequency and time parameters, respectively.

Plotting the equation along the X-axis, we obtain

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = eE_x \sin(\nu t - kx) , \tag{2}$$

where E_X is the X-axis electrical field component and k is the wave vector modulus.

The solution of Equation (2), describing a linear oscillator under wave action, is written in the form of a quasiharmonic function

$$x(t) = a\sin(\omega t + \alpha) , \tag{3}$$

where $a(t)$ and $\alpha(t)$ are the slowly changing amplitude and phase, $\omega = \nu/N$ is the charge periodic motion frequency, and $N = 1, 2, 3, \ldots$ is a whole number.

By use of the averaging method (Bogoljubov and Mitropolsky, 1974), shortened, these equations are

$$\left.\begin{aligned} \dot{a} &= -\beta a + \frac{eE_xN}{ka\omega} J_N(ka)\sin N\alpha , \\ \dot{\alpha} &= \frac{\omega_0^2 - \omega^2}{2\omega} - \frac{eE_x}{a\omega} J_N'(ka)\cos N\alpha , \end{aligned}\right\} \tag{4}$$

where J_N is the N-order Bessel function of the first kind.

In charge stationary periodic motion mode, i.e., when $\dot{a} = 0$ and $\dot{\alpha} = 0$, Equations (4) can be written as

$$\beta\alpha = \frac{eE_xN}{ka\omega} J_N(ka)\sin N\alpha , \tag{5}$$

$$\omega_0^2 - \omega^2 = \frac{2eE_x}{a} J_N'(ka)\cos N\alpha . \tag{6}$$

According to the amplitude "quantization" phenomenon theory (Damgov *et al.*, 1990, 1991), the role of the phase α is essential whence the excited oscillation synchronization and the adaptive stability maintenance under different perturbing actions are considered. In resonance mode,* condition $\omega = \omega_0$ is accomplished and Equation (6) is valid provided that

$$J_N'(ka) = 0 \quad \text{and} \quad \cos N\alpha = 0 .$$

In order to examine the stationary amplitude and the phase stability, the set of Equations (4) is rewritten, as follows:

* The resonance case is in conformity with the idea about evolutionary mature systems (Tchetchelnitsky, 1980).

$$\dot{a} = f(a, \alpha), \qquad \dot{\alpha} = g(a, \alpha) . \tag{7}$$

The derivatives $\partial f/\partial a$, $\partial f/\partial \alpha$ and $\partial g/\partial a$, $\partial g/\partial \alpha$ obtained for a and α constant values and corresponding to the stationary oscillations are designated by f_a, f_α and g_a, g_α. Hence, the stability condition can be rewritten as follows:

$$\operatorname{Re} P_{1,2} < 0, \quad \text{where} \quad P_{1,2} = \frac{f_a + g_\alpha}{2} \pm \sqrt{\left(\frac{f_a - g_\alpha}{2}\right)^2 + f_\alpha g_a} , \tag{8}$$

$$\frac{f_a + g_\alpha}{2} = -\frac{\beta}{2} - \frac{eE_xN}{2k\omega a^2} \sin N\alpha [J_N(ka) - 2kaJ'_N(ka)] ,$$

$$\frac{f_a - g_\alpha}{2} = -\frac{\beta}{2} - \frac{eE_xN}{2k\omega a^2} \cdot \sin N\alpha \cdot J_N(ka) ,$$

$$f_\alpha g_a = \frac{(eE_xN)^2}{k\omega^2 a^3} \cos^3 N\alpha \cdot J_N(ka)[J'_N(ka) - kaJ''_N(ka)] .$$

The $P_{1,2}$ quantities are derived from the time dependence small deviations of the stationary values of a and α, expressed as:

$$\Delta a = A_1 e^{P_1 t} + A_2 e^{P_2 t}, \qquad \Delta \alpha = B_1 e^{P_1 t} + B_2 e^{P_2 t} ,$$

where A_1, A_2, B_1 and B_2 are constants.

The examination of the stability solutions (Equations (4)) shows, that the stable oscillations amplitude a satisfies condition

$$J'_N(ka) = 0 . \tag{9}$$

Hence, the stationary charge oscillations can be realized for amplitudes a_i, belonging to a strictly defined set of amplitude values. The a_i values are determined by the Bessel function extremes, and using Equation (9) may be presented as

$$ka_i = j_{N,i} , \tag{10}$$

where $J_{N,i}$ is the N-th order Bessel function J_N argument value ka_i at the i-th extremum point.

In conformity with the general premises we assume, that the oscillating charge system under wave action may be used as a general model of description of the solar system. The large Earth orbit semiaxis a_E complies with Equation (9) at $N = 8$ and $i = 5$. Those conditions are assumed as basic: $ka_E = j_{8,5}$.

Further, using expression (10), the following semimajor axes relation is obtained:

$$\frac{a_i}{a_E} = \frac{ka_i}{ka_E} = \frac{j_{N,i}}{j_{8,5}} = \frac{j_{8,i}}{j_{8,5}} . \tag{11}$$

At the same time it should be noted that the solar system planets arrangement

TABLE I

Mean planet distances in the solar system

Planets in the solar system	Data from direct astronomical measurements of planet distances from the Sun, (Allen, 1973) A.U.	Titius–Bode Law (12), (Nieto, 1972) A.U.		Computed planet distances using Equation (11) of the "Oscillator-wave" model	
		k	a_k	i	$\frac{a_i}{a_E} = \frac{j_{8,i}}{j_{8,5}}$
Mercury	0.39	$-\infty$	0.4	1	0.392
Venus	0.72	0	0.7	3	0.723
Earth	1.00	1	1.0	5	1.000
Mars	1.52	2	1.6	9	1.530
Asteroids	2.78	3	2.8	19	2.824
Jupiter	5.2	4	5.2	37	5.132
Saturn	9.55	5	10.0	71	9.474
Chiron (Collwell's object)	13.71			104	13.689
Uranus	19.18	6	19.6	147	19.180
Neptune	30.03	7	38.8	232	30.035
Pluto	39.67	8	77.2	307	39.598

bears regular character, expressed on the whole by the Titius–Bode law (Nieto, 1972), giving the mean planets-to-Sun distances a_k by equation

$$a_k = 0{,}4 + 0{,}3.2^k \,, \tag{12}$$

where a_k is expressed in astronomical units, and $k = -\infty$ for Mercury, $k = 0$ for Venus, $k = 1$ for the Earth, etc. For more than 200 years this law has not found a convincing theoretical explanation.

A better expression of the mean distances is proposed by several authors (Pletser 1986, 1988, Lalaja, 1989; Neuhauser and Feitzinger, 1986, Rawal, 1989) in the form as follows

$$r_n = r_0 \cdot d^n \,, \tag{13}$$

where n is the n-th planet distance from the Sun or n-th satellite distance from the primary, r_0 is a simple normalizing distance, different for each system, d is a constant, characterizing the geometric progression, valid for all systems. The integer n for Mercury is equal to 1, for Venus is equal to 2, etc. Some of the values do not correspond to an existing celestial body, but to a "hole". Expression (13), compared to the earlier Titius–Bode law (12) is less artificial, as the discount value $n = -\infty$ for Mercury is avoided.

The solar system planets mean distances are presented in Table I. For comparison reasons, the direct astronomic measurements data is given parallel to the results, computed by the classical Titius–Bode law (12) and Equation (11) according to the "oscillator–wave" model, described above. All data is expressed in astronomical units (A.U.).

A good correspondence is observed between the computed (Equation (11)) and astronomically measured radii. Especially significant is the correspondence between the computed and measured radii of Neptune and Pluto. The Titius–Bode law determines the mean distances of those two planets with an error of 23% and 49%, respectively.

The computed data of the mean satellite distances from Saturn, Uranus and Jupiter, as well as the mean ring system distances from Saturn, are given in Table II. The calculations are made on the basis of the "oscillator-wave" model, using an expression, similar to Equation (11). For comparison reasons, the data obtained from direct astronomical distance measurements (Allen, 1973) are presented as well. Again, a good correspondence is seen between the calculated and measured mean distances.

The measurement and computed data normalizing is realized relatively to an arbitrary chosen representative body. Obviously, there are no restrictions and any other satellite can be chosen as a representative body for the performance of similar normalizing.

It is necessary to note, that according to the "oscillator-wave" model, there is a certain differentiation between the solar system (Table I) and planet-satellite (Table II) structures. In the first case, the wave action frequency ν and solar system planets revolution frequencies ω_i relation remains invariable ($N = \gamma/\omega_i = 8 = \text{const}$). The differences between the planet mean distances from the Sun are determined by the values of the Bessel function arguments $j_{8,i}$ at the extreme i-th points. In the second case, i.e., for planet satellite systems, the correspondence between the calculation and astronomical measurements data is observed different frequency multiplicities ($N = \nu/\omega_i = \text{vary}$) and the corresponding values of the first Bessel function extremes $j_{N,1}$.

4. Conclusions

The harmonic oscillator obeys the classical laws to a greater extent than any other system. A number of problems, related to harmonic oscillators, have the same solution in classical and quantum mechanics.

Regardless of its simplicity, the "oscillator-wave" model obviousiy reflects a number of processes in the micro- and macroworld. The model is manifested naturally in different material media and harmonizes with the modern ideas about the world into and out of us as a totality of particles and fields. Also, it takes into account the wide extention of the oscillating processes in Nature. In the presence of particle flows and fields of different nature, the model realizes (materializes) widely in Nature in a very natural way. In one way or another, the model has been considered by a number of authors, but the most essential feature of behavior – the "quantization" phenomenon, has escaped their attention.

TABLE II

Mean Saturn, Uranian, Jovian satellite and Saturn-ring-system distances from the primaries in conformity with the proposed "oscillator-wave" model

Satellites	Data from direct astronomical measurements of satellite mean distances from the primaries, (Allen, 1973) 10^{-3} A.U.	Normalized data from direct astronomical measurements	N	Normalized computed mean satellite distances from the primaries in conformity with the "Oscillator-Wave" model
Saturnian satellites				$j_{N,1}/j_{10,1}$
Janus	1.060	0.301	2	0.308
Mimas	1.241	0.352	3	0.357
Enceladus	1.592	0.452	4	0.452
Tethys	1.970	0.550	5	0.545
Dione	2.523	0.716	7	0.729
Rhea	3.524	1.000	10	1.000
Titan	8.166	2.317	25	2.328
Hyperion	9.911	2.812	31	2.838
Iapetus	23.718	6.736	77	6.737
Phoebe	86.580	24.569	284	24.560
Uranian Satellites				$j_{N,1}/j_{10,1}$
Miranda	0.872	0.488	4	0.452
Ariel	1.282	0.718	7	0.728
Umbriel	1.786	1.000	10	1.000
Titania	2.930	1.641	17	1.624
Oberon	3.919	2.104	23	2.152
Jovian Satellites				$j_{N,1}/j_{17,1}$
Amalthea	1.209	0.096	1	0.096
Io	2.819	0.224	3	0.220
Europa	4.489	0.357	5	0.336
Ganymede	7.155	0.568	9	0.563
Callisto	12.585	1.000	17	1.000
Saturn Ring System				$j_{N,1}/j_{14,1}$
Crape or C ring:				
faint	0.498	0.827	11	0.803
Gap: dark	0.595	0.988	13	0.934
Main B ring:				
very bright	0.602	1.000	14	1.000
Cassini division:				
dark	0.782	1.299	18	1.261
Outer A ring:				
moderately bright	0.802	1.332	19	1.326

The "oscillator-wave" model confirms the bodies' wave interaction mechanism in the micro- and macroworld, as well as the closing down and convergence of the basic properties manifestation in Nature. The heuristic importance of the model is drawn on the basis of the remarkably fruitful hypothesis about the solar system wave dynamical structure, propounded recently by a number of researchers. Notwithstanding its simplicity, the proposed model gives a phenomenologic and dynamic description of the megaquantum-resonance structure of the solar system and analogous systems, consisting of a central body and satellites. It seems that the model reconciles two opposite approaches – this of the classical celestial mechanics of N bodies and that of the quantum-mechanical wave system theory. Based on the ideas of the Universe wave nature, the model gives an enough detailed identification of the solar system resonance structure within the frames of the classical theory. The large number of possible resonances are in complete corespondence, e.g., with the fact of existence of numerous resonances in the Jupiter and Saturn satellite systems. The model gives the orbit quantization, independent on the initial dynamic conditions and the planets' and satellites' masses. At the same time, by analogy with the atomic system theory, the model defines the ranges of the initial dynamic conditions, determining the possibility for planet or satellite capture, forming the solar system planet and satellite structure. Hence, the model may trace the space substance zones, determinating in the course of evolution the present structure of the solar system planets and satellites arrangement. A more detailed study of the "oscillator-wave" model, proposed herewith, may reveal all possible stable orbits of the solar system planet and planet-satellite structures, including the fine Saturn ring system structures etc.

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