EXPRESSION OF THE INITIAL POINCARÉ CANONICAL VARIABLES AS FUNCTIONS OF THE NEW IN A NINTH ORDER J–S THEORY

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Abstract. We establish the solution of the ninth order – in masses – canonical J–S equations of motion by Hori–Lie technique – i.e., by expressing the initial Poincaré canonical variables as functions of the new variables through the Hori–Lie canonical transformation. Terms of order higher than 9 in the masses are neglected.

According to Lie theorem (Lie, 1888; and Hori, 1966), we have the change of variables equality

$$f(L_1, L_2, H_1, H_2, P_1, P_2, \lambda_1, \lambda_2, K_1, K_2, Q_1, Q_2) \equiv \sum_{\nu=0}^{\infty} \frac{1}{\nu!} D_S^\nu f(L_1', \ldots, Q_2'), \quad (1)$$

where L_u, λ_u, H_u, K_u, P_u, Q_u; $u = 1, 2$ are the Poincaré canonical variables.

We can write

$$\begin{matrix} f \equiv L_u; & f \equiv H_u; & f \equiv P_u; \\ f \equiv \lambda_u; & f \equiv K_u; & f \equiv Q_u \end{matrix} \quad (u = 1, 2) \quad (2)$$

We shall consider in particular the substitution $f = H_u$; $u = 1, 2$. Then we shall have

$$H_u = \sum_{\nu=0}^{\infty} \frac{1}{\nu!} D_S^\nu H_u', \quad (3)$$

where D_S^ν is the Lie operator. From its definition, we have

$$D_S^0 H_u' = H_u';$$

$$D_S^1 H_u' = (H_u', S), \ldots, D_S^\nu H_u' = (\underbrace{(H_u', S), \ldots, S}_{\nu \text{ times}}). \quad (4)$$

Expanding Equation (3), we obtain

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$$H_u = H'_u + \sum_{\nu=1}^{\infty} \frac{1}{\nu!} D_S^{\nu-1} D_S^1 H'_u =$$

$$= H'_u + \sum_{\nu=1}^{\infty} \frac{1}{\nu!} D_S^{\nu-1} (H'_u, S) =$$

$$= H'_u + (H'_u, S) + \sum_{\nu=2}^{\infty} \frac{1}{\nu!} D_S^{\nu-2} D_S^1 (H'_u, S) . \tag{5}$$

and so on.

Neglecting terms of order higher than the ninth, we get

$$\begin{aligned} H_u = H'_u &+ (H'_u, S) + \tfrac{1}{2}((H'_u, S), S) + \\ &+ \tfrac{1}{6}(((H'_u, S), S), S) + \tfrac{1}{24}((((H'_u, S), S), S), S) + \\ &+ \tfrac{1}{120}(((((H'_u, S), S), S), S), S) + \\ &+ \tfrac{1}{720}((((((H'_u, S), S), S), S), S), S) + \\ &+ \tfrac{1}{5040}(((((((H'_u, S), S), S), S), S), S), S) + \\ &+ \tfrac{1}{40320}((((((((H'_u, S), S), S), S), S), S), S), S) + \\ &+ \tfrac{1}{362880}(((((((((H'_u, S), S), S), S), S), S), S), S), S) . \end{aligned} \tag{6}$$

We have $S = \Sigma_{k=1}^{\infty} S_k$, so in our case

$$S = S_1 + S_2 + \cdots + S_9 . \tag{7}$$

From the definition of a Poisson bracket, we have

$$(H'_u, S_k) = \frac{\partial S_k}{\partial K'_u} \cdot \begin{pmatrix} k = 1, 2, \ldots, \infty \\ u = 1, 2 \end{pmatrix} . \tag{8}$$

Therefore, we can write the formula for H_u as

$$\begin{aligned} H_u = H'_u &+ \frac{\partial S_1}{\partial K'_u} + \frac{\partial S_2}{\partial K'_u} + \frac{\partial S_3}{\partial K'_u} + \frac{\partial S_4}{\partial K'_u} + \frac{\partial S_5}{\partial K'_u} + \frac{\partial S_6}{\partial K'_u} + \frac{\partial S_7}{\partial K'_u} + \frac{\partial S_8}{\partial K'_u} + \frac{\partial S_9}{\partial K'_u} + \\ &+ \frac{1}{2}\left(\frac{\partial S_1}{\partial K'_u}, S_1\right) + \frac{1}{2}\left(\frac{\partial S_1}{\partial K'_u}, S_2\right) + \frac{1}{2}\left(\frac{\partial S_1}{\partial K'_u}, S_3\right) + \frac{1}{2}\left(\frac{\partial S_1}{\partial K'_u}, S_4\right) + \frac{1}{2}\left(\frac{\partial S_1}{\partial K'_u}, S_5\right) + \\ &+ \frac{1}{2}\left(\frac{\partial S_1}{\partial K'_u}, S_6\right) + \frac{1}{2}\left(\frac{\partial S_1}{\partial K'_u}, S_7\right) + \frac{1}{2}\left(\frac{\partial S_1}{\partial K'_u}, S_8\right) + \frac{1}{2}\left(\frac{\partial S_2}{\partial K'_u}, S_1\right) + \frac{1}{2}\left(\frac{\partial S_2}{\partial K'_u}, S_2\right) + \\ &+ \frac{1}{2}\left(\frac{\partial S_2}{\partial K'_u}, S_3\right) + \frac{1}{2}\left(\frac{\partial S_2}{\partial K'_u}, S_4\right) + \frac{1}{2}\left(\frac{\partial S_2}{\partial K'_u}, S_5\right) + \frac{1}{2}\left(\frac{\partial S_2}{\partial K'_u}, S_6\right) + \frac{1}{2}\left(\frac{\partial S_2}{\partial K'_u}, S_7\right) + \end{aligned}$$

$$+\frac{1}{2}\left(\frac{\partial S_3}{\partial K_u'}, S_1\right)+\frac{1}{2}\left(\frac{\partial S_3}{\partial K_u'}, S_2\right)+\frac{1}{2}\left(\frac{\partial S_3}{\partial K_u'}, S_3\right)+\frac{1}{2}\left(\frac{\partial S_3}{\partial K_u'}, S_4\right)+\frac{1}{2}\left(\frac{\partial S_3}{\partial K_u'}, S_5\right)+$$

$$+\frac{1}{2}\left(\frac{\partial S_3}{\partial K_u'}, S_6\right)+\frac{1}{2}\left(\frac{\partial S_4}{\partial K_u'}, S_1\right)+\frac{1}{2}\left(\frac{\partial S_4}{\partial K_u'}, S_2\right)+\frac{1}{2}\left(\frac{\partial S_4}{\partial K_u'}, S_3\right)+\frac{1}{2}\left(\frac{\partial S_4}{\partial K_u'}, S_4\right)+$$

$$+\frac{1}{2}\left(\frac{\partial S_4}{\partial K_u'}, S_5\right)+\frac{1}{2}\left(\frac{\partial S_5}{\partial K_u'}, S_1\right)+\frac{1}{2}\left(\frac{\partial S_5}{\partial K_u'}, S_2\right)+\frac{1}{2}\left(\frac{\partial S_5}{\partial K_u'}, S_3\right)+\frac{1}{2}\left(\frac{\partial S_5}{\partial K_u'}, S_4\right)+$$

$$+\frac{1}{2}\left(\frac{\partial S_6}{\partial K_u'}, S_1\right)+\frac{1}{2}\left(\frac{\partial S_6}{\partial K_u'}, S_2\right)+\frac{1}{2}\left(\frac{\partial S_6}{\partial K_u'}, S_3\right)+\frac{1}{2}\left(\frac{\partial S_7}{\partial K_u'}, S_1\right)+\frac{1}{2}\left(\frac{\partial S_7}{\partial K_u'}, S_2\right)+$$

$$+\frac{1}{2}\left(\frac{\partial S_8}{\partial K_u'}, S_1\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_1\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_2\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_3\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_4\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_5\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_6\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_7\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_2\right), S_1\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_2\right), S_2\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_2\right), S_3\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_2\right), S_4\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_2\right), S_5\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_2\right), S_6\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_3\right), S_1\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_3\right), S_2\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_3\right), S_3\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_3\right), S_4\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_3\right), S_5\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_4\right), S_1\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_4\right), S_2\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_4\right), S_3\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_4\right), S_4\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_5\right), S_1\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_5\right), S_2\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_5\right), S_3\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_6\right), S_1\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_6\right), S_2\right)+\frac{1}{6}\left(\left(\frac{\partial S_1}{\partial K_u'}, S_7\right), S_1\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_1\right)+\frac{1}{6}\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_2\right)+\frac{1}{6}\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_3\right)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_1\Bigr), S_4\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_1\Bigr), S_5\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_1\Bigr), S_6\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_2\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_2\Bigr), S_2\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_2\Bigr), S_3\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_2\Bigr), S_4\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_2\Bigr), S_5\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_3\Bigr), S_1\Bigr)+$$

$$+\frac{1}{6}\Bigl(\frac{\partial S_2}{\partial K'_u}, S_3\Bigr), S_2\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_3\Bigr), S_3\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_3\Bigr), S_4\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_4\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_4\Bigr), S_2\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_4\Bigr), S_3\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_5\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_5\Bigr), S_2\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_2}{\partial K'_u}, S_6\Bigr), S_1\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_1\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_1\Bigr), S_2\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_1\Bigr), S_3\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_1\Bigr), S_4\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_1\Bigr), S_5\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_2\Bigr), S_1\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_2\Bigr), S_2\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_2\Bigr), S_3\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_2\Bigr), S_4\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_3\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_3\Bigr), S_2\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_3\Bigr), S_3\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_4\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_4\Bigr), S_2\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_3}{\partial K'_u}, S_5\Bigr), S_1\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_4}{\partial K'_u}, S_1\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_4}{\partial K'_u}, S_1\Bigr), S_2\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_4}{\partial K'_u}, S_1\Bigr), S_3\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_4}{\partial K'_u}, S_1\Bigr), S_4\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_4}{\partial K'_u}, S_2\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_4}{\partial K'_u}, S_2\Bigr), S_2\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_4}{\partial K'_u}, S_2\Bigr), S_3\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_4}{\partial K'_u}, S_3\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_4}{\partial K'_u}, S_3\Bigr), S_2\Bigr)+$$

$$+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_4}{\partial K'_u}, S_4\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_5}{\partial K'_u}, S_1\Bigr), S_1\Bigr)+\frac{1}{6}\Bigl(\Bigl(\frac{\partial S_5}{\partial K'_u}, S_1\Bigr), S_2\Bigr)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_5}{\partial K_u'}, S_1\right), S_3\right)+\frac{1}{6}\left(\left(\frac{\partial S_5}{\partial K_u'}, S_2\right), S_1\right)+\frac{1}{6}\left(\left(\frac{\partial S_5}{\partial K_u'}, S_2\right), S_2\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_5}{\partial K_u'}, S_3\right), S_1\right)+\frac{1}{6}\left(\left(\frac{\partial S_6}{\partial K_u'}, S_1\right), S_1\right)+\frac{1}{6}\left(\left(\frac{\partial S_6}{\partial K_u'}, S_1\right), S_2\right)+$$

$$+\frac{1}{6}\left(\left(\frac{\partial S_6}{\partial K_u'}, S_2\right), S_1\right)+\frac{1}{6}\left(\left(\frac{\partial S_7}{\partial K_u'}, S_1\right), S_1\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_1\right), S_1\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_1\right), S_2\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_1\right), S_3\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_1\right), S_4\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_1\right), S_5\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_1\right), S_6\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_2\right), S_1\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_2\right), S_2\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_2\right), S_3\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_2\right), S_4\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_2\right), S_5\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_3\right), S_1\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_3\right), S_2\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_3\right), S_3\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_3\right), S_4\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_4\right), S_1\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_4\right), S_2\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_4\right), S_3\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_5\right), S_1\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_5\right), S_2\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_1\right), S_6\right), S_1\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_2\right), S_1\right), S_1\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_2\right), S_1\right), S_2\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_2\right), S_1\right), S_3\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_4\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_5\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_2\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_3\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_4\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_3\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_3\right), S_2\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_3\right), S_3\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_4\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_4\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_5\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_1\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_1\right), S_2\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_1\right), S_3\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_1\right), S_4\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_2\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_2\right), S_2\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_2\right), S_3\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_3\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_3\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_4\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_4\right), S_1\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_4\right), S_1\right), S_2\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_4\right), S_1\right), S_3\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_4\right), S_2\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_4\right), S_2\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_4\right), S_3\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_5\right), S_1\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_5\right), S_1\right), S_2\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_5\right), S_2\right), S_1\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_1}{\partial K_u'}, S_6\right), S_1\right), S_1\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_1\right), S_1\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_1\right), S_2\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_1\right), S_3\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_1\right), S_4\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_1\right), S_5\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_2\right), S_1\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_2\right), S_2\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_2\right), S_3\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_2\right), S_4\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_3\right), S_1\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right) S_3\right), S_2\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_3\right), S_3\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_4\right), S_1\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_4\right), S_2\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_5\right), S_1\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_1\right), S_1\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_1\right), S_2\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_1\right), S_3\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_1\right), S_4\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_2\right), S_1\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_2\right), S_2\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_2\right), S_3\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_3\right), S_1\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_3\right), S_2\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_4\right), S_1\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_3\right), S_1\right), S_1\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_3\right), S_1\right), S_2\right)+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_3\right), S_1\right), S_3\right)+$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_3\right), S_2\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_3\right), S_2\right), S_2\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_3\right), S_3\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_4\right), S_1\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_4\right), S_1\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_4\right), S_2\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_5\right), S_1\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_1\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_1\right), S_3\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_1\right), S_4\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_2\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_2\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_2\right), S_3\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_3\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_3\right), S_2\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_4\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_2\right), S_1\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_2\right), S_1\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_2\right), S_1\right), S_3\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_2\right), S_2\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_2\right), S_2\right), S_2\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_2\right), S_3\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_3\right), S_1\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_3\right), S_1\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_3\right), S_2\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_4\right), S_1\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_4}{\partial K'_u}, S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_4}{\partial K'_u}, S_1\right), S_1\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_4}{\partial K'_u}, S_1\right), S_1\right), S_3\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_4}{\partial K'_u}, S_1\right), S_2\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_4}{\partial K'_u}, S_1\right), S_2\right), S_2\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_4}{\partial K'_u}, S_1\right), S_3\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_4}{\partial K'_u}, S_2\right), S_1\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_4}{\partial K'_u}, S_2\right), S_1\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_4}{\partial K'_u}, S_2\right), S_2\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_4}{\partial K'_u}, S_3\right), S_1\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_5}{\partial K'_u}, S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_5}{\partial K'_u}, S_1\right), S_1\right), S_2\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_5}{\partial K'_u}, S_1\right), S_2\right), S_1\right) +$$

$$+\frac{1}{24}\left(\left(\left(\frac{\partial S_5}{\partial K'_u}, S_2\right), S_1\right), S_1\right) + \frac{1}{24}\left(\left(\left(\frac{\partial S_6}{\partial K'_u}, S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_2\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_3\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_4\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_5\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_3\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_4\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_3\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_3\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_3\right), S_3\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_4\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_4\right), S_2\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_5\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_3\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_4\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_2\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_2\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_2\right), S_3\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_3\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_3\right), S_2\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_4\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_1\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_1\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_1\right), S_3\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_2\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_2\right), S_2\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_3\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_4\right), S_1\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_4\right), S_1\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_4\right), S_2\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_5\right), S_1\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_3\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_4\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_2\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_2\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_2\right), S_3\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_3\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_3\right), S_2\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_4\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_1\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_1\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_1\right), S_3\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_2\right),S_2\right),S_2\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_2\right),S_2\right),S_2\right),S_2\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_2\right),S_2\right),S_3\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_2\right),S_3\right),S_1\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_2\right),S_3\right),S_1\right),S_2\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_2\right),S_3\right),S_2\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_2\right),S_4\right),S_1\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_3\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_3\right),S_1\right),S_1\right),S_2\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_3\right),S_1\right),S_2\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_3\right),S_2\right),S_1\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_3\right),S_1\right),S_1\right),S_3\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_3\right),S_1\right),S_3\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_3\right),S_3\right),S_1\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_3\right),S_1\right),S_2\right),S_2\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_3\right),S_2\right),S_1\right),S_2\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_3\right),S_2\right),S_2\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_4\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_4\right),S_1\right),S_1\right),S_2\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_4\right),S_1\right),S_2\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_4\right),S_2\right),S_1\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u},S_5\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u},S_1\right),S_1\right),S_1\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u},S_1\right),S_1\right),S_1\right),S_2\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u},S_1\right),S_1\right),S_1\right),S_3\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u},S_1\right),S_1\right),S_1\right),S_4\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u},S_1\right),S_1\right),S_2\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u},S_1\right),S_1\right),S_2\right),S_2\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_3\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_3\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_3\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_4\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_2\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_3\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_2\right), S_2\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_2\right), S_2\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_2\right), S_3\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_3\right), S_1\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_3\right), S_1\right), S_2\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_3\right), S_2\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_4\right), S_1\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_2\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_3\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_1\right), S_2\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_1\right), S_2\right), S_2\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_1\right), S_3\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_2\right), S_1\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_2\right), S_1\right), S_2\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_2\right), S_2\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_3\right), S_1\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_3\right), S_1\right), S_1\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_3\right), S_1\right), S_1\right), S_2\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_3\right), S_1\right), S_2\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_3\right), S_2\right), S_1\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_4\right), S_1\right), S_1\right), S_1\right) + \frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_2\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_1\right),S_2\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_2\right),S_1\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_3\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_1\right),S_3\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_3\right),S_1\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_2\right),S_2\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_2\right),S_1\right),S_2\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_1\right),S_2\right),S_2\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_2\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_2\right),S_1\right),S_1\right),S_2\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_2\right),S_1\right),S_2\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_2\right),S_2\right),S_1\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_3\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_4}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_4}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_2\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_4}{\partial K_u'},S_1\right),S_1\right),S_2\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_4}{\partial K_u'},S_1\right),S_2\right),S_1\right),S_1\right)+$$

$$+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_4}{\partial K_u'},S_2\right),S_1\right),S_1\right),S_1\right)+\frac{1}{120}\left(\left(\left(\left(\frac{\partial S_5}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_2\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_3\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_4\right)+$$

$$
\begin{aligned}
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_2\right), S_1\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_2\right), S_2\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_2\right), S_3\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_3\right), S_1\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_3\right), S_2\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_4\right), S_1\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right), S_1\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right), S_2\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right), S_3\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_2\right), S_1\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_2\right), S_2\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_3\right), S_1\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_3\right), S_1\right), S_1\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_3\right), S_1\right), S_2\right) + \\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_3\right), S_2\right), S_1\right) +
\end{aligned}
$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_4\right), S_1\right), S_1\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_1\right), S_2\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_1\right), S_3\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_2\right), S_1\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_2\right), S_2\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_3\right), S_1\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_2\right), S_1\right), S_1\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_2\right), S_1\right), S_2\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_2\right), S_2\right), S_1\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_3\right), S_1\right), S_1\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_1\right), S_1\right), S_2\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_1\right), S_2\right), S_1\right) +$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_2\right), S_1\right), S_1\right) +$$

$$
\begin{aligned}
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_4\right), S_1\right), S_1\right), S_1\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_1\right), S_1\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_1\right), S_2\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_1\right), S_3\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_2\right), S_1\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_2\right), S_2\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_3\right), S_1\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_2\right), S_1\right), S_1\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_2\right), S_1\right), S_2\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_2\right), S_2\right), S_1\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_3\right), S_1\right), S_1\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_1\right), S_1\right), S_1\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_1\right), S_1\right), S_2\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_1\right), S_2\right), S_1\right)+\\
&+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_2\right), S_1\right), S_1\right)+
\end{aligned}
$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_3\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_1\right), S_1\right), S_1\right), S_2\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_1\right), S_1\right), S_2\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_1\right), S_2\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_2\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_4\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_2\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_3\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_2\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_2\right), S_2\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_3\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right), S_2\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_1\right), S_2\right), S_2\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_1\right), S_3\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_2\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_2\right), S_1\right), S_1\right), S_2\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_2\right), S_1\right), S_2\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_2\right), S_2\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_1\right), S_3\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_1\right), S_1\right), S_1\right), S_2\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_1\right), S_1\right), S_2\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_1\right), S_2\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_2\right), S_2\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K_u'}, S_3\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_2\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_2\right),S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_1\right),S_2\right),S_1\right),S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_1\right),S_2\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_3}{\partial K_u'},S_2\right),S_1\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{720}\left(\left(\left(\left(\left(\frac{\partial S_4}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_1\right),S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_1\right),S_2\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_1\right),S_3\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_2\right),S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_2\right),S_2\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_1\right),S_3\right),S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_2\right),S_1\right),S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_2\right),S_1\right),S_2\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K_u'},S_1\right),S_1\right),S_1\right),S_2\right),S_2\right),S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_3\right), S_1\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right), S_1\right), S_2\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right), S_2\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_2\right), S_1\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_3\right), S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_1\right), S_1\right), S_2\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_1\right), S_2\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_2\right), S_1\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_2\right), S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_3\right), S_1\right), S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_1\right), S_1\right), S_2\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_1\right), S_2\right), S_1\right) +$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_2\right), S_1\right), S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_2\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_2\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_3\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_2\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_2\right), S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_2\right), S_1\right), S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{5040}\left(\left(\left(\left(\left(\left(\frac{\partial S_3}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{40320}\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{40320}\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_2\right)+$$

$$+\frac{1}{40320}\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_2\right), S_1\right)+$$

$$+\frac{1}{40320}\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_2\right), S_1\right), S_1\right)+$$

$$+\frac{1}{40320}\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_2\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{40320}\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_2\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{40320}\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_2\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{40320}\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_2\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{40320}\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_2}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right)+$$

$$+\frac{1}{362880}\left(\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial K'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right). \qquad (9)$$

To get the formula for L_u, P_u, we replace in formula (9) H_u, H'_u by L_u, L'_u, P_u, P'_u and K'_u by λ'_u, Q'_u respectively. The formula for K_u is given by

$$K_u = K'_u - \frac{\partial S_1}{\partial H'_u} - \frac{\partial S_2}{\partial H'_u} - \dots - \frac{\partial S_q}{\partial H'_u} - \frac{1}{2}\left(\frac{\partial S_1}{\partial H'_u}, S_1\right) - \frac{1}{2}\left(\frac{\partial S_1}{\partial H'_u}, S_2\right) - \dots$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$-\frac{1}{362880}\left(\left(\left(\left(\left(\left(\left(\left(\frac{\partial S_1}{\partial H'_u}, S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right), S_1\right) \qquad (10)$$

(Kamel, 1989).

To acquire the expressions for λ_u, Q_u we just replace K_u, K'_u by λ_u, λ'_u; Q_u, Q'_u and H'_u by L'_u, P'_u.

Referring to (Kamel, 1989), we can find that the new Poincaré canonical variables $L'_u, H'_u, \dots, Q'_u$; $u = 1, 2$ could be inversely expressed as functions of the initial Poincaré canonical variables $L_u, H_u, \dots, Q_u$; $u = 1, 2$ by just replacing S by $-S$ and permuting L_u, H_u, P_u and L'_u, H'_u, P'_u; λ_u, K_u, Q_u and λ'_u, K'_u, Q'_u.

For a J–S theory the above formulas are applicable on condition that terms of degree 3 and higher in H, K, P, Q are taken into account. No critical terms of degree 0, 1, 2 in H, K, P, Q are detected in the expansion of the J–S Hamiltonian function, up to the fifth order in the masses.

This might be generalized to be true for any order in the masses, when corresponding calculations are implemented.

In this case, there is no influence of the critical terms and we have

$$\begin{aligned} &L_u \equiv L'_u, \qquad \lambda_u \equiv \lambda'_u \\ &H_u, = H'_u, \qquad K_u \equiv K'_u \qquad u = 1, 2 \\ &P_u \equiv P'_u, \qquad Q_u \equiv Q'_u \end{aligned} \tag{11}$$

i.e., the new Poincaré canonical variables are equivalent to the initial ones.

References

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