LOVE NUMBERS OF THE MOON AND OF THE TERRESTRIAL PLANETS

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Abstract. In the IERS Standards (1989), for the Moon the adopted value of the tide Love number, k_2, is equal to 0.0222. In this paper using the latest geodetic parameters of the Moon a group of internal structure models are constructed for this celestial body (see Table V), then the dependence of the Moon's core size on calculated value of k_2 is explored. The obtained results indicate that the second degree Love number, $k_2 = 0.02664$, of the lunar model 91-04 is near its observed value (0.027 ± 0.006). This implies that the Moon may possess an outer core of 660 km radius and of 300 kbar mean rigidity. With the same method the static Love numbers from degree 2 to 30 are computed for the terrestrial planets – Mercury, Venus, and Mars (see Table VII), and the influence of some parameters (such as the rigidity) of the outer core on low degree Love numbers is discussed. Finally, the likely range of the second degree Love numbers is determined for the terrestrial planets (see Table XI). It seems that if low degree Love numbers of a terrestrial planet can be detected in the future space explorations, there is some possibility to improve the planetary internal structure model. For example, as soon as space techniques yield an observed value of $k_2 > 0.10$ for Mercury, there will be reason to anticipate that a partly melted iron core exists in this planet.

1. Introduction

Tidal friction plays an important role in the spin evolution of some celestial bodies of the solar system. It has been responsible for significantly altering the primordial rotation rates of Mercury, Venus, and the Earth; and generally alters the spin angular velocity of all the natural satellites toward a value which is synchronous with its orbital mean motion.

The tide generating potential due to a perturbing body can be developed in the sum of spherical harmonics W_n. The additional tidal potential, ΔV_n, caused by the deformation of the perturbed body is directly proportional to the spherical harmonic, W_n, of degree n, that is

$$\Delta V_n = K_n(r) W_n \,, \tag{1}$$

where $K_n(r)$ is an auxiliary function of the radius r. At the surface of a celestial body ($r = a$), $K_n(a)$ is equal to k_n which is one of the dimensionless tidal Love numbers of degree n.

Apparently, when we consider the tidal response features of a celestial body, it is necessary to estimate its Love numbers.

Generally, it is a perfect approximation to consider the Moon as a homogeneous elastic spheroid. We can then estimate its second degree Love numbers by means

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TABLE I

k_2 values from different methods

	Bodri (1987)	Kelvin's formula	
	k_2	$\bar{\mu}$ (kbar)	k_2
Mercury	0.184	750	0.0965
Venus	0.411	1450	0.254
Mars	0.447	1050	0.0712
Moon	0.0281	670	0.0219

of some simplified models (e.g., Harrison, 1963; Cheng and Toksöz, 1978; Zhang and Shen, 1988).

As regards the terrestrial planets, since their rheological parameters – rigidity $\mu(r)$, Lamé constant $\lambda(r)$, density $\rho(r)$, and gravity $g(r)$ in the planetary interior are not determined reliably, we run up against difficulties in deriving theoretically their Love numbers. Hence, either the Love numbers for a homogeneous incompressible spherical body are calculated approximately according to Kelvin's formula (e.g. Goldreich and Peale, 1968), or the low degree Love numbers of a celestial body are estimated by means of a certain simplified model of this body (e.g. Szeto, 1983; Zhang and Shen, 1988). Not long ago Bodri (1987) computed second degree Love numbers (h_2, k_2, l_2) for radially heterogeneous compressible models of the terrestrial planets.

In Table I we display these two sorts of the Love numbers k_2 for the Moon and terrestrial planets. Here $\bar{\mu}$ is the mean rigidity of a celestial body. It is well known that the Earth's mean rigidity, is about equal to 1460 kbar. These values of k_2 in Table I are derived by assuming that rigidities of the terrestrial planets are similar to the Earth's. As for the Moon we adopted the value of $\bar{\mu}$ = 670 kbar (Zhang and Shen, 1988). Comparison between the results calculated by using Bodri's method and Kelvin's formula shows that there is evident discrepancy. Apparently, it is necessary to find out how to explain this discrepancy.

In this paper we consider again some problems dealing with calculation of the Love numbers for the Moon and terrestrial planets. The major objective of this paper is to try to find out some physical parameters influencing obviously the low degree Love numbers. Section 2 presents the parametric models of internal structure for the Moon and terrestrial planets. Section 3 describes the calculated results of the Love numbers. Section 4 discusses the dependence of the Moon's core size on the second degree Love numbers in some detail. Section 5 outlines the influence of the outer core rigidities of the terrestrial planets on the low degree Love numbers, and determines the likely range of the second degree ones for each terrestrial planet. Finally, we suggest that one should detect the Love numbers, h_2 and k_2, of a terrestrial planet in the future space explorations, so as to check whether the adopted internal structure model of this planet is reasonable.

TABLE II

Geodetic parameters of the terrestrial planets and the Moon

	a (10^3 km)	$\bar{\rho}$ (g cm^{-3})	$m/m_{\oplus}$	I/ma^2
Mercury	2.440	5.42628	0.05527	–
Venus	6.05145	5.24531	0.8150	–
Mars	3.38992	3.933497	0.10745	0.365–0.374
Moon	1.73753	3.34398	0.01230	0.390395

2. The Parametric Models of the Internal Structure for the Moon and the Terrestrial Planets

The planets of the solar system have been the objects of space exploration for more than 20 years. Due to recent achievements in planetary geodesy, at present important geodetic parameters of these planets are provided with high accuracy (e.g., Bills and Synnott, 1987). The geodetic parameters of the terrestrial planets and the Moon adopted in this study can be found in Table II.

In Table II a is average radius of a celestial body, $\bar{\rho}$ the average density, m and $m_{\oplus}$ the masses of a planet (or the Moon) and the Earth, respectively, and I/ma^2 the dimensionless moment of inertia (I is the mean moment of inertia). In the case of constructing the internal structure model of a celestial body, we regard $\bar{\rho}$ as a constrained condition. In addition, we have also another constrained condition, I/ma^2, for the Moon and Mars.

On the other hand, one can devote oneself to determine the internal structure of the terrestrial planets from a general knowledge about these celestial bodies. Bills and Ferrari (1977) constructed the lunar interior model and gave the profiles of the density, $\rho(r)$, and seismic velocities, $V_p(r)$ and $V_s(r)$, for this model. Siegfried II and Solomon (1974), Zharkov (1983), and Johnston and Toksöz (1977) discussed the task involves the construction of internal structure models for Mercury, Venus, and Mars, respectively.

The major objective of this paper is to explore the influence of the radial variations of certain physical parameters in the interior of a celestial body on the low degree Love numbers. Therefore, when we construct the parametric models of the internal structure, we make three simplifying assumptions as follows.

First, let us assume that the interior of a planet is in the hydrostatic equilibrium state, and further the bulk modulus $K(r)$, and the pressure $p(r)$, satisfy the linear relation (Cook, 1975) in the outer core and the mantle of this celestial body: namely,

$$K(r) = K_0 + bp(r) , \tag{2}$$

where both K_0 and b are two positive constants. We consulted those of the values of the Earth's models and adopted both K_0 and b values that are listed in Table III for the terrestrial planets and the Moon.

From Equation (2) we derive easily the state equation of interior materials,

TABLE III

Values, K_0 and b, used in construction of internal structure models

	Core		Mantle	
	K_0 (10^3 kbar)	b	K_0 (10^3 kbar)	b
Mercury	2.0	3.35	2.3	3.25
Venus	2.0	3.35	2.3	3.25
Mars	2.0	4.0	2.3	3.25
Moon	1.2	4.5	1.1	8.0

$$p(r) = \frac{K_0}{b}\left[\left(\frac{\rho(r)}{\rho_u}\right)^b - 1\right], \tag{3}$$

where ρ_u is the density at $p = 0$. Meantime we have also

$$K(r) = \lambda(r) + \tfrac{2}{3}\mu(r) . \tag{4}$$

Next, let us assume that the rigidity, $\mu_c(r)$, in the outer core of a planet submits to Roche's law, it may then be written as

$$\left.\begin{aligned} \mu_c(r) &= \mu_0\left[1 - c_0\left(\frac{r}{a}\right)^2\right], \\ c_0 &= \frac{5}{3}\left(\frac{a}{r_1}\right)^2\left(1 - \frac{\bar{\mu}_c}{\mu_0}\right); \end{aligned}\right\} \tag{5}$$

where μ_0 is the rigidity at the planetary centre, $\bar{\mu}_c$ the mean rigidity of the outer core, and r_1 the core radius.

As regards the rigidity, $\mu_m(r)$, in the mantle we adopted its average value in the mantle of the Earth's model as

$$\mu_m(r) = \tfrac{2}{3}\lambda_m(r) . \tag{6}$$

Last, in the calculations of this paper we shall not consider the interior temperature affecting the rheological parameters. It is evident that this simplified assumption may affect the physical parameters of Venus. However, for other terrestrial planets there is no influence on the obtained results in substance.

When we have made above-mentioned simplifying assumptions and adopted a series of the values of central density, ρ_0, central rigidity, μ_0, and mean rigidity, $\bar{\mu}_c$, of the outer core, we can set up a group of internal structure models by solving the Emden equation (e.g., Shen and Zhang, 1988; Zhang, 1990).

In Table IV two different models were listed for each celestial body, and they correspond, respectively, to two extreme cases of the central density of this body. There is reason to believe that the profile of the density of a real planet would stand the range between the two extreme models. These models divided the

TABLE IV

Parameters for the planetary models

Zone	Parameter	Model					
		Mercury		Venus		Mars	
		90-01	90-02	90-01	90-02	90-01	90-02
Inner core	$\rho(0)$ ($g\,cm^{-3}$)	9.000	6.500	13.000	10.500	7.600	5.400
	$\mu(0)$ (kbar)	1100	800	1600	1200	1250	1050
	r_0 (km)	220.0	220.0	545.0	545.0	305.0	305.0
	$\rho(r_0)$ ($g\,cm^{-3}$)	8.986	6.494	12.927	10.455	7.584	5.394
Outer core	r_1 (km)	1803.6	2189.0	3166.4	3349.1	1588.6	2039.4
	r_1/a	0.739	0.897	0.523	0.553	0.469	0.602
	$\rho(r_1)$ ($g\,cm^{-3}$)	7.983	5.858	10.323	8.664	7.156	5.111
		3.622	3.900	4.989	5.141	3.793	3.790
	$\bar{\mu}_c$(kbar)	800	600	1200	1000	1000	800
Mantle	r_2 (km)	2374.0	2374.0	5967.4	5967.4	3305.9	3305.9
	$\rho(r_2)$ ($g\,cm^{-3}$)	3.505	3.856	3.738	3.919	3.510	3.570
Shell	$\rho(r_2)$ ($g\,cm^{-3}$)	2.85	2.85	2.85	2.85	2.85	2.85
	d_s (km)	66.0	66.0	84.0	84.0	84.0	84.0
Total	I/ma^2	0.3322	0.3690	0.3317	0.3444	0.3657	0.3742
	$\bar{\mu}$(kbar)	922.4	669.7	1643.2	1428.9	1102.9	1019.5

interior of a planet into four zones – inner core of r_0 radius, outer core of radius from r_0 to r_1, mantle of radius from r_1 to r_2, and shell of d_s thickness.

As for the Moon, we did not distinguish the core into inner and outer regions, but divided the shell into two layers. Therefore, the lunar models comprised also four zones – core of r_1 radius, mantle of radius from r_1 to r_2, lower shell of d_1 thickness, and upper shell of d_2 thickness. In Table V we displayed four different lunar models which have respective radius r_1 of the core (the range of r_1 value is from 380 km to 660 km).

In Tables IV and V for each model we listed also the calculated values of I/ma^2 and the mean rigidity $\bar{\mu}$. The latter can be derived from the equation

$$\bar{\mu} = 3 \int_0^1 \mu(s) s^2 \, ds \,, \tag{7}$$

where $s = r/a$ is the relative radius.

In order to numerically calculate the tidal Love numbers with the step-variable Runge–Kutta method, taking the model 90-01 of the terrestrial planets as an example we fitted the numerical values of physical parameters to a three-order polynomial in relative radius, s, by stepwise regression analysis, and in Table VIa displayed the coefficients of the polynomial for each zone. Similarly, for the lunar models, 91-01 and 91-04, we listed the coefficients of the polynomial in Table VIb.

As to the seismic speeds V_p and V_s in Tables VIa and VIb, we have

TABLE V

Parameters for the lunar models 91-01–04

Zone	Parameter	Model			
		91-01	91-02	91-03	91-04
Core	$\rho(0)$ (g cm^{-3})	7.070	5.293	4.506	4.236
	$\mu(0)$ (kbar)	650.00	450.00	350.00	350.00
	r_1 (km)	378.78	483.03	597.71	663.74
	$\rho(r_1)$ (g cm^{-3})	7.003	5.244	4.442	4.180
	$\bar{\mu}_c$(kbar)	600.00	400.00	300.00	300.00
Mantle	$\rho(r_1)$ (g cm^{-3})	3.432	3.422	3.411	3.406
	r_2 (km)	1677.53	1677.53	1677.53	1677.53
	$\rho(r_2)$ (g cm^{-3})	3.312	3.307	3.302	3.302
Shell	$\rho(r_2)$ (g cm^{-2})	2.90	2.90	2.90	2.90
	d_1 (km)	40.00	40.0	40.00	40.00
	ρ_s (g cm^{-3})	2.639	2.715	2.766	2.742
	d_2 (km)	20.00	20.00	20.00	20.00
Total	I/ma^2	0.39030	0.39044	0.39053	0.39051
	$\bar{\mu}$(kbar)	535.85	535.79	535.71	535.88

$$\left.\begin{aligned} V_p^2 &= \frac{1}{\rho(r)}[\lambda(r) + 2\mu(r)]\,, \\ V_s^2 &= \frac{\mu(r)}{\rho(r)}\,. \end{aligned}\right\} \tag{8}$$

3. The Calculated Results of the Love Numbers

The reduction of the problem of elastic deformations of a spherical body to a system of six differential equations of the first-order was described in a lot of papers (e.g., Melchior, 1983). In this paper the manner of computation is similar to that adopted by Zhang (1991a,b).

The system of six linear differential equations in the Runge–Kutta form is as follows (cf. Zhang, 1991b):

$$\frac{\mathrm{d}X_i(r)}{\mathrm{d}r} = \sum_{j=1}^{6} F_{i,j}(n, r, \rho, \mu, \lambda, g)X_j(r), \qquad i = 1, 2, \ldots, 6; \tag{9}$$

where n is the harmonic degree, $X_1(r), X_2(r), \ldots$, and $X_6(r)$ are the auxiliary functions defined by Molodensky (cf. Zhang, 1991a), and $g(r)$ is the gravity which can be calculated from the equation

$$g(r) = \frac{4\pi G}{r^2}\int_0^r \rho(r)r^2\,\mathrm{d}r\,, \tag{10}$$

TABLE VIa

Parametric models of the terrestrial planets

	Parameter	Inner core	Outer core	Mantle	Shell
Mercury 90-01	r (km)	0.0:220.0	220.0:1803.6	1803.6:2374.0	2374.0:2440.0
	$\rho(r)$ (g cm^{-3})	9.0000 −1.7774 * s^2 −0.0567 * s^3	9.0029 −0.0397 * s −1.6105 * s^2 −0.2725 * s^3	4.0399 −0.6203 * s +0.0732 * s^2	2.85
	$V_p(r)$ (km s^{-1})	7.3588 −1.8945 * s^2 −0.0673 * s^3	7.3676 −0.1197 * s −1.4038 * s^2 −0.7396 * s^3	12.1046 −2.0820 * s +0.2565 * s^2	5.50
	$V_s(r)$ (km s^{-1})	3.4960 −1.1051 * s^2 −0.0669 * s^3	3.5098 −0.1833 * s −0.3696 * s^2 −1.0576 * s^3	6.4707 −1.1140 * s +0.1378 * s^2	3.00
Venus 90-01	r (km)	0.0:545.0	545.0:3166.4	3166.4:5967.4	5967.4:6051.4
	$\rho(r)$ (g cm^{-3})	13.0000 −8.9979 * s^2	13.0251 −0.4177 * s −6.7626 * s^2 −4.3858 * s^3	6.1132 −2.0693 * s −0.3423 * s^3	
	$V_p(r)$ (km s^{-1})	10.6165 −7.9932 * s^2 −0.3136 * s^3	10.6380 −0.3546 * s −6.1419 * s^2 −3.6094 * s^3	17.4792 −6.6269 * s −0.9147 * s^3	5.50
	$V_s(r)$ (km s^{-1})	3.5082 −1.4390 * s^2 −0.2515 * s^3	3.5222 −0.2282 * s −0.2759 * s^2 −2.2207 * s^3	9.3430 −3.5423 * s −0.4889 * s^3	3.00
Mars 90-01	r (km)	0.0:305.0	305.0:1588.6	1588.6:3305.9	3305.9:3389.9
	$\rho(r)$ (g cm^{-3})	7.6000 −1.9382 * s^2 −0.0679 * s^3	7.6017 −0.0318 * s −1.7490 * s^2 −0.4573 * s^3	3.9993 −0.4260 * s −0.0791 * s^3	2.85
	$V_p(r)$ (km s^{-1})	8.3498 −3.7913 * s^2 −0.1809 * s^3	8.3562 −0.1147 * s −3.1203 * s^2 −1.5425 * s^3	11.9705 −1.4315 * s −0.2553 * s^3	5.50
	$V_s(r)$ (km s^{-1})	4.0555 −2.4393 * s^2 −0.4339 * s^3	4.0651 −0.1696 * s −1.4986 * s^2 −2.1419 * s^3	6.3985 −0.7652 * s −0.1364 * s^3	3.00

TABLE VIb

Parametric models of the Moon

	Parameter	Core	Mantle	Lower shell	Upper shell
Luna 91-01	r (km)	0.0–378.8	378.8–1677.5	1677.5–1717.5	1717.5–1737.5
	$\rho(r)$ (g cm^{-3})	7.0700 −1.4140 * s^2	3.4493 −0.0740 * s −0.0059 * s^2 −0.0674 * s^3	2.90	2.639
	$V_p(r)$ (km s^{-1})	5.5556 −0.0028 * s −2.9505 * s^2 −0.8207 * s^3	8.5091 −0.5862 * s −0.1762 * s^2 −0.4384 * s^3	6.50	5.50
	$V_s(r)$ (km s^{-1})	2.9245 −0.0051 * s −2.7416 * s^2 −0.8207 * s^3	4.5483 −0.3133 * s −0.0942 * s^2 −0.2344 * s^3	3.50	3.00
Luna 91-04	r (km)	0.0–663.7	663.7–1677.5	1677.5–1717.5	1717.5–1737.5
	$\rho(r)$ (g cm^{-3})	4.2360 −0.3812 * s^2	3.43830 −0.0666 * s −0.0220 * s^2 −0.0575 * s^3	2.90	2.742
	$V_p(r)$ (km s^{-1})	6.65639 −0.0057 * s −1.8641 * s^2 −0.2800 * s^3	8.5087 −0.4861 * s −0.3737 * s^2 −0.3269 * s^3	6.50	5.50
	$V_s(r)$ (km s^{-1})	2.8535 −0.0272 * s −2.2323 * s^2 −1.1936 * s^3	4.5481 −0.2599 * s −0.1997 * s^2 −0.1748 * s^3	3.50	3.00

where G is the gravitational constant. In this paper we take $G = 6.6726 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$.

We computed the Love numbers from degree 2 to 30 for the terrestrial planets (the results of model 90-01 are listed in Table VII), and those from degree 2 to 25 for four lunar models.

In order to compare with other researcher's results, using the six-layer lunar model constructed by Bills and Ferrari (1977) (simplified as BF-6) we completed also the same computation. For the model BF-6 the algorithm used in the determination of the values of $\rho(r)$, $V_p(r)$ and $V_s(r)$ is based on a cubic spline. In Table VIII we displayed the calculated results for the lunar models of 91-01, 91-04, and BF-6. In Figure 1 the Love numbers of degree n for the two models of 91-04 and BF-6 were shown as a function of harmonic degree n.

For later applications, Table IX listed the second degree Love numbers and their combinations $\delta_2(= 1 + h_2 - \frac{3}{2}k_2)$ and $\gamma_2(= 1 + k_2 - h_2)$. For comparison data of three Earth's models (Zhang, 1991a, b) are also displayed in Table IX.

TABLE VII

Love numbers of the terrestrial planets

	Mercury 90-01			Venus 90-01			Mars 90-01		
n	h_n	k_n	l_n	h_n	k_n	l_n	h_n	k_n	l_n
2	0.1230	0.0644	0.0288	0.3372	0.1672	0.0809	0.1037	0.0562	0.0279
3	0.0698	0.0235	0.0060	0.1909	0.0623	0.0238	0.0602	0.0224	0.0080
4	0.0473	0.0114	0.0019	0.1385	0.0339	0.0121	0.0439	0.0126	0.0037
5	0.0347	0.0064	0.0009	0.1127	0.0222	0.0074	0.0353	0.0082	0.0021
6	0.0270	0.0040	0.0006	0.0972	0.0160	0.0050	0.0298	0.0058	0.0014
7	0.0220	0.0028	0.0005	0.0865	0.0122	0.0035	0.0259	0.0043	0.0010
8	0.0186	0.0020	0.0004	0.0784	0.0097	0.0026	0.0230	0.0034	0.0008
9	0.0162	0.0015	0.0004	0.0718	0.0079	0.0020	0.0207	0.0027	0.0006
10	0.0144	0.0012	0.0004	0.0664	0.0065	0.0016	0.0189	0.0022	0.0005
15	0.0101	0.0006	0.0003	0.0488	0.0032	0.0008	0.0136	0.0011	0.0003
20	0.0083	0.0003	0.0002	0.0391	0.0019	0.0005	0.0111	0.0006	0.0002
25	0.0073	0.0002	0.0002	0.0332	0.0012	0.0004	0.0098	0.0003	0.0002
30	0.0068	0.0002	0.0001	0.0290	0.0009	0.0004	0.0090	0.0003	0.0002

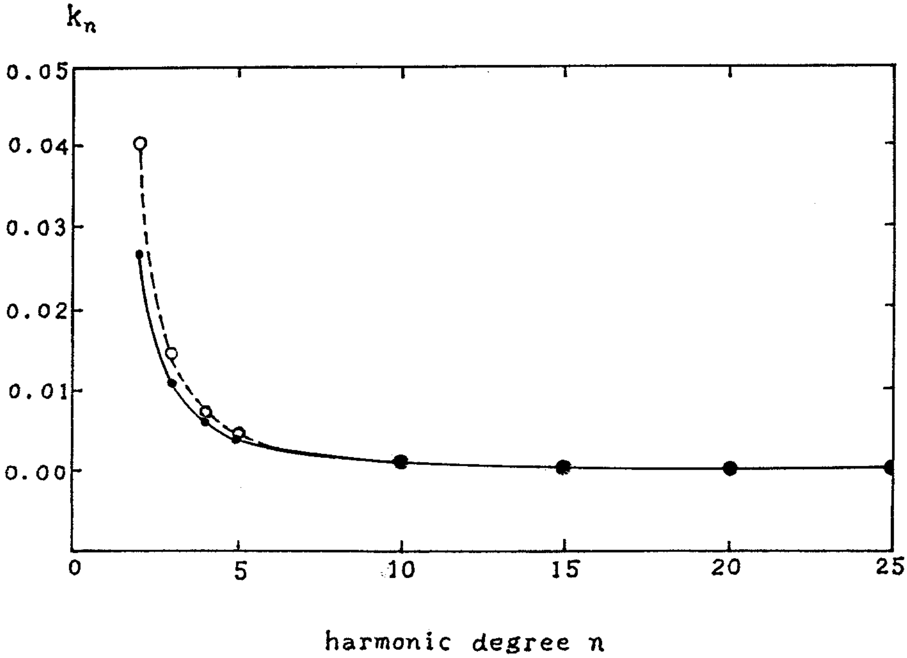

Fig. 1. The Love numbers k_n as functions of the degree n for the models Luna 91-04 (solid line) and BF-6 (dashed line).

TABLE VIII

Love numbers of the Moon

	Luna 91-01			Luna 91-04			BF-6		
n	h_n	k_n	l_n	h_n	k_n	l_n	h_n	k_n	l_n
2	0.0424	0.0244	0.0125	0.0465	0.0266	0.0123	0.0711	0.0405	0.0146
3	0.0261	0.0106	0.0037	0.0262	0.0106	0.0035	0.0362	0.0146	0.0027
4	0.0196	0.0062	0.0016	0.0190	0.0060	0.0016	0.0233	0.0072	0.0009
5	0.0159	0.0041	0.0009	0.0153	0.0039	0.0009	0.0167	0.0043	0.0004
6	0.0134	0.0029	0.0005	0.0129	0.0028	0.0005	0.0129	0.0028	0.0003
7	0.0116	0.0022	0.0004	0.0112	0.0021	0.0004	0.0105	0.0020	0.0002
8	0.0102	0.0017	0.0003	0.0099	0.0016	0.0003	0.0089	0.0015	0.0002
9	0.0092	0.0013	0.0002	0.0088	0.0013	0.0002	0.0077	0.0011	0.0001
10	0.0083	0.0011	0.0001	0.0080	0.0011	0.0002	0.0068	0.0009	0.0001
15	0.0057	0.0005	0.0001	0.0055	0.0005	0.0001	0.0045	0.0004	0.0001
20	0.0044	0.0003	0.0000	0.0043	0.0003	0.0000	0.0036	0.0002	0.0001
25	0.0036	0.0002	0.0000	0.0036	0.0002	0.0000	0.0031	0.0002	0.0000

TABLE IX

Second degree Love numbers and δ_2, γ_2 for the Moon and terrestrial planets

Model	h_2	k_2	l_2	δ_2	γ_2
Luna 91-01	0.04242	0.02440	0.01248	1.00582	0.98198
Luna 91-02	0.04324	0.02485	0.01241	1.00597	0.98161
Luna 91-03	0.04542	0.02604	0.01233	1.00635	0.98063
Luna 91-04	0.04650	0.02664	0.01229	1.00654	0.98014
Mercury 90-01	0.12297	0.06435	0.02878	1.02644	0.94139
Mercury 90-02	0.17499	0.09823	0.04738	1.02764	0.92324
Venus 90-01	0.33715	0.16718	0.08090	1.08638	0.83003
Venus 90-02	0.38443	0.19677	0.08596	1.08928	0.81234
Mars 90-01	0.10367	0.05618	0.02788	1.01940	0.95251
Mars 90-02	0.12484	0.06918	0.03097	1.02107	0.94434
EARTH 1066A	0.60981	0.30093	0.08501	1.15842	0.69112
1066B	0.61304	0.30148	0.08387	1.16118	0.68808
PREM	0.60380	0.29859	0.08404	1.15591	0.69480

4. The Dependence of the Moon's Core Size on the Second-Degree Love Numbers

In the IERS Standards (1989) a series of lunar gravitational parameters are listed. One of these parameters is the Love number k_2 which takes 0.0222 as its applying value. Williams *et al.* (1987) got $k_2 = 0.027 \pm 0.006$ by analysing LLR data between 1969/8 and 1986/2. Meantime, from Table I of this paper, if we consider the Moon as a homogeneous elastic spheroid and adopt the lunar mean rigidity of $\bar{\mu} = 670$ kbar, the calculated value of 0.0219 is obtained by using Kelvin's formula. However, it is of interest to explore how the Love number, k_2, vary as a function of Moon's core size.

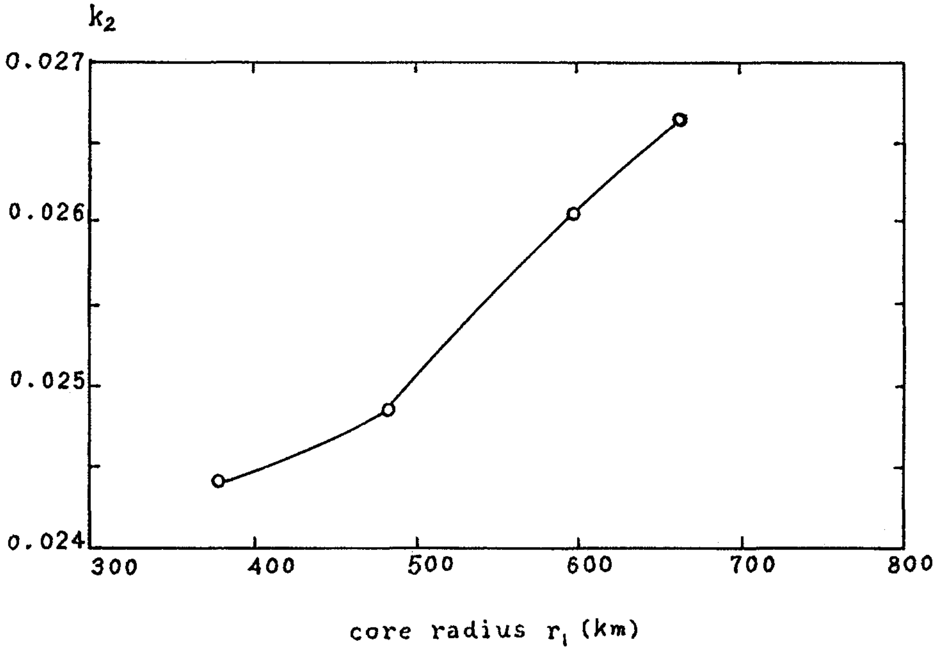

Fig. 2. Relation between the k_2 value and the r_1 radius of the core for the lunar models.

In Section 2 of this paper, we constructed four different lunar models which have individual radius r_1, i.e., 378.78, 483.03, 597.71, and 663.74 km, of the core (see Table V). The range of the mean rigidity, $\bar{\mu}$, of four models is 535.7 ~ 535.8 kbar, and that of the mean moment of inertia, I/ma^2, is 0.3903 ~ 0.3905. Therefore, there is reason to consider that the dynamical characteristics of these models, except different core size, ought to be similar to each other.

The calculated results (see Table IX) indicate that the Love number k_2 increases with increasing radius of the core. In the case of the invariance of the lunar mean rigidity, the k_2 values as a function of the core radius, r_1, are illustrated in Figure 2.

Apparently, the calculated values of k_2 for four lunar models constructed in this paper conform to the observed value within the limits of error of measurement (± 0.006). Comparatively speaking, the k_2 values for lunar models 91-03 and 91-04 are more close to the observed value. This implies that the Moon may possess an outer core of about 600 km radius and of 300 kbar mean rigidity. Simultaneously, the value of $\bar{\mu} \simeq 536$ kbar is taken as the mean rigidity of the whole Moon. This conclusion agrees with the previous estimate about the Moon's core size (Zhang and Shen, 1988). On the contrary, the calculated value of k_2 for the lunar model of BF-6 (see Table VIII) is not close to the observed value.

TABLE X

Influence of the core rigidity on the Love numbers for the terrestrial planets

Rigidity (kbar)						
$\bar{\mu}$	$\bar{\mu}_c$	h_2	k_2	l_2	δ_2	γ_2
Mercury 90-01						
841.6	600	0.14859	0.07813	0.03274	1.03140	0.92954
801.2	500	0.16633	0.08769	0.03543	1.03479	0.92136
760.9	400	0.18933	0.10013	0.03884	1.03914	0.91079
720.5	300	0.22041	0.11698	0.04334	1.04495	0.89656
Venus 90-01						
1539	1000	0.35458	0.17597	0.08121	1.09063	0.82139
1410	100	0.48761	0.24326	0.08318	1.12271	0.75565
1398	10	0.51044	0.25487	0.08341	1.12814	0.74443
1396	0.1	0.51299	0.25618	0.08343	1.12873	0.74318
1396	0.01	0.51286	0.25611	0.08344	1.12870	0.74325
Mars 90-01						
1102	1000	0.10390	0.05630	0.02789	1.01945	0.95240
1010	100	0.15544	0.08441	0.03059	1.02882	0.92897
1000	10	0.16725	0.09089	0.03115	1.03091	0.92364
999	0.1	0.16874	0.09171	0.03122	1.03118	0.92297
999	0.01	0.16875	0.09172	0.03122	1.03118	0.92296

5. The Range of the Second-Degree Love Numbers for the Terrestrial Planets

Since our understanding of planetary interior is still far from complete, the internal structure models constructed in this paper may possess some considerable uncertainty. Even though we cannot obtain the Love numbers with high accuracy as for the Earth (or the Moon), the range of the second degree Love numbers for the terrestrial planets may be yet determined by means of our knowledge of these bodies.

It is well known that for the Earth the presence of a fluid core ($\mu_c = 0$) makes an obvious influence on the Love numbers. With the current popular data we cannot still affirm whether a fluid (or partly melted) core exists in a terrestrial planet or not. In Section 2 we have taken the μ_c values in the range $600 \sim 1200$ kbar. In order to explore the influence of the core state for a terrestrial planet on the low-degree Love numbers, for the model 90-01 of Venus and Mars taking the $\bar{\mu}_c$ values from 10^{-2}–10^3 kbar we obtained a series of the results which were given in Table X. The tabulated data show that its rate of change is similar to that of the Earth's model (Zhang, 1991a).

As regards Mercury it is generally believed that its iron core is largely solid. Taking the range of core rigidities, $\bar{\mu}_c$, from 300–600 kbar we derived the results which were listed also in Table X. For Mercury the k_2 values as a function of the $\bar{\mu}_c$ values were illustrated in Figure 3.

With the data in Tables IX and X we may determine the range of the second-degree Love numbers for Mercury, Venus, and Mars, and displayed these results

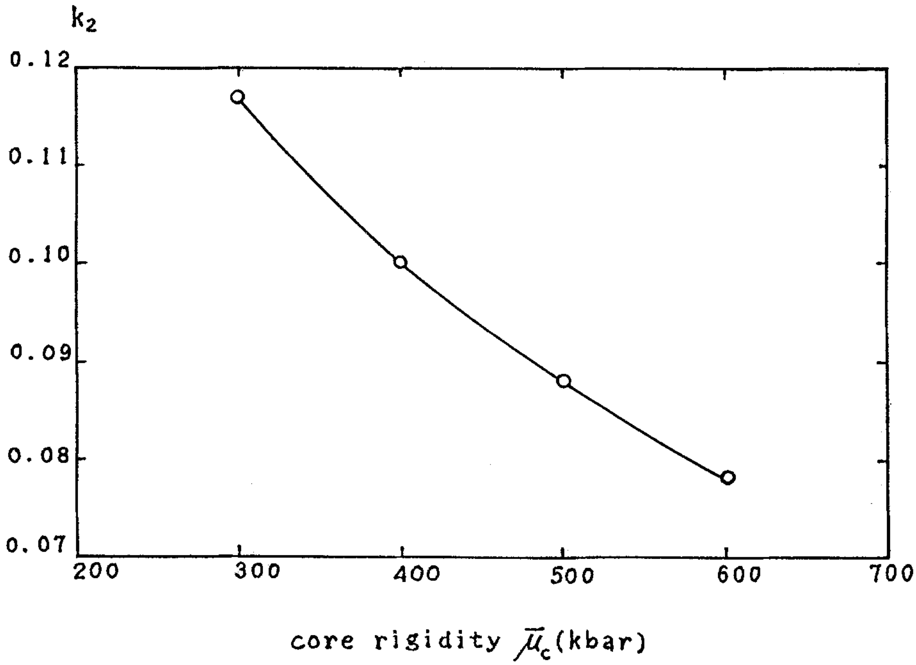

Fig. 3. Dependence of the k_2 value on $\bar{\mu}_c$ for Mercury.

TABLE XI

The range of the second degree Love numbers for the terrestrial planets

	h_2	k_2	l_2
Mercury	0.125–0.190	0.0645–0.100	0.0288–0.0388
Venus	0.350–0.540	0.175–0.270	0.0760–0.0835
Mars	0.104–0.169	0.0563–0.0917	0.0279–0.0312

in Table XI. The tabulated data obviously show that the values of Bodri (1987) are not to fall in this range. In particular, the calculated values of k_2 for Mars and Venus (see Table I) are too large. In order to obtain a k_2 value approximate to that of Bodri (1987), it is necessary to decrease the mean rigidities, $\bar{\mu}$, to 400, 780, and 125 kbar for Mercury, Venus, and Mars, respectively. But the calculated results in this paper (see Table X) indicate that so low mean rigidities for the terrestrial planets seem incredible.

For the purpose of comparison with other models of planetary interiors, the use of five-layer Cytherean model with a fluid core (PVM) constructed by Zharkov (1983) was computed also the low degree Love numbers of this model, PVM, and the following results obtained: $h_2 = 0.5370$, $k_2 = 0.2670$, and $l_2 = 0.0770$. Apparently, these values are to fall in the range shown by Table XI.

Since the low degree Love numbers, h_2 and k_2, are specially sensitive to the state of materials in the outer core of a planet; that is to say, if the Love number,

k_2, of a terrestrial planet can be detected in the future space explorations, there is a possibility to improve this planetary model. In particular, by comparing the observed value with the calculated value of k_2 we can check whether or not a fluid (or partly melted) core exists in this planet.

6. Conclusions

(1) In this paper the calculated value of k_2 for the lunar model 91-04 equals 0.02664, which is near its observed value (0.027). This implies that the Moon may possess an outer core of 660 km radius and of 300 kbar mean rigidity.

(2) The likely range of the values of k_2 for Venus is $0.18 \sim 0.27$. Data from current space explorations have not yet demonstrated that Venus possesses a considerably large fluid core. Therefore, the upper-limit of second degree Love numbers, k_2, may be considered as about equal to 0.27.

(3) It is generally believed that the iron core of Mercury is largely solid, then there is no possibility for Mercury that the formula $k_2 > 0.10$ hold true. Otherwise, there is reason to anticipate that a partly melted iron core exists in this planet.

(4) The range of the values of k_2 for Mars is $0.06 \sim 0.09$, which is about equal to 0.3 times that of the Earth. This is obviously different from the calculated result (see Table I of this paper) of Bodri (1987).

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