FLUX AND POLARIZATION REFLECTED FROM A NONCONSERVATIVE RAYLEIGH-SCATTERING PLANETARY ATMOSPHERE

TÕNU VIIK

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Abstract. Assuming that the Rayleigh-scattering atmosphere of a planet is homogeneous and plane-parallel we estimate the influence of non-conservativeness of the atmosphere and the state of polarization of the incident radiation upon the phase curves both for flux and polarization.

1. Introduction

About twenty years ago the optical properties of the clouds of Venus were under careful scrutiny. It appeared that the measured polarization was a very sensitive function of wavelength (Coffeen and Gehrels, 1969), thus allowing to rule out many speculations about the physical constituents of the clouds. Assuming that the atmosphere of Venus is Rayleigh-scattering, Horák (1950) had made the first calculations based on the results of Chandrasekhar (1947).This work was continued by Horak and Little (1965). These calculations showed clearly that the major contribution to the polarization reflected from Venus in the range of 3400 to 9900 Å was from aerosols rather than molecules as was later proved by *in situ* experiments.

Horak (1950) employed the integration over the visible crescent of Venus thus generalizing the exact method given by Schoenberg (1929) for local scalar reflection according to the laws of Lambert and Lommel–Seeliger. Since Horak (1950) had to extensively interpolate between the tabular values his calculations were in considerable error as has been shown by Kattawar and Adams (1971) who had used the invariant-imbedding approach (Adams and Kattawar, 1970) and the Monte Carlo method. However these errors could not change the verdict on the hypothesis of the molecular atmosphere.

Though the problem of the physical constituents of the clouds of Venus is comfortably settled it is of considerable interest to determine the flux and polarization of the radiation reflected from an imaginary planetary atmosphere assuming the non-conservativeness and linearly polarized incident flux.

This task turned out to be a rather easy one after we had compiled a package of FORTRAN subroutines to solve the vector equation of transfer (Viik, 1989). The solution is based on the method of discrete ordinates by Chandrasekhar (1960) and on the kernel approximation method (Viik *et al.*, 1985).

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2. Formulation of the Problem

We consider a planetary atmosphere being illuminated by a parallel beam of radiation, the state of polarization of which depends on the components of its flux

$$\mathbf{F} = (F_l, F_r, F_U, F_V)^T$$

Natural light is described by the set $F_l = F_r = 0.5$, $F_U = F_V = 0$. Linearly polarized light is described either by $F_l = 1$, $F_r = F_U = F_V = 0$ or by $F_l = F_U = F_V = 0$, $F_r = 1$.

Usually the following assumptions are made:

(1) The planetary atmosphere is homogeneous and locally plane-parallel since the geometrical thickness of the atmosphere is very small compared with the radius of the planet.

(2) The atmosphere is horizontally homogeneous.

(3) The distance between the planet and the source of illumination is infinitely large (incident beam is parallel).

(4) The atmosphere is bounded by a Lambert bottom with albedo A.

We computed the flux, the polarization and the albedo of the radiation reflected from such an atmosphere.

Since Horak (1950) was the first to discuss this problem we shall adopt his notation as has also been done by others (Kattawar and Adams, 1971; de Rooij, 1985). Let ρ be the planetary radius, r – the distance between the Sun and planet, R – the distance between the Sun and the Earth and Δ – the distance between the Earth and planet. Phase angle α is the angle Sun-planet-Earth. In Figure 1 P is an arbitrary point on the planetary surface, ζ is the longitude and η is the colatitude.

We set up a local coordinate system at the point P choosing the outward normal as the z-axis, x-axis is chosen so that the azimuthal angle ϕ_0 for the incident solar ray is zero. Having defined x and z, the y follows from the assumption of a right-handed coordinate system.

At the point P the angles of incidence and reflection are defined as follows

$$\mu_0 = \cos\beta = \sin\eta\cos(\zeta - \alpha), \tag{1}$$

$$\mu = \cos\theta = \sin\eta\cos\zeta, \tag{2}$$

where β and θ are acute angles.

According to the formulae of spherical trigonometry the azimuthal angle of the reflected ray is

$$\cos\varphi = (\cos\theta\cos\beta - \cos\alpha)(\sin\theta\sin\beta)^{-1}, \tag{3}$$

$$\sin\varphi = \sin\alpha\cos\eta(\sin\theta\sin\beta)^{-}. \tag{4}$$

In order to find the flux and polarization over a planet's disc we have to rotate the Stokes vector by the angle ϕ, where

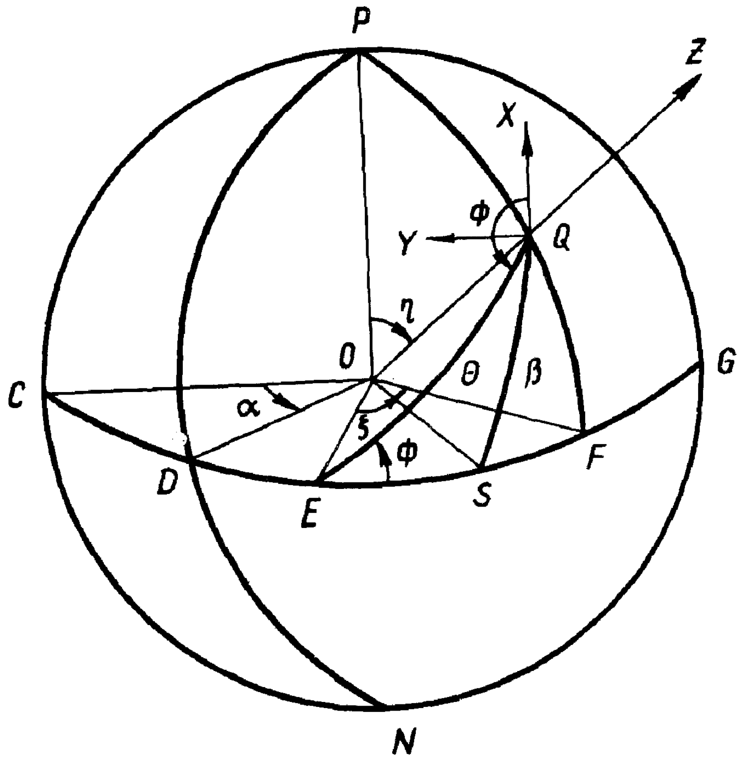

Fig. 1. Planetary coordinates.

$$\cos \phi = \sin \eta \sin \zeta (\sin \theta)^{-1}, \tag{5}$$

$$\sin \phi = \cos \eta (\sin \theta)^{-1}; \tag{6}$$

respectively, the matrix of linear transformation is the following (de Rooij, 1985)

$$L(\phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\phi & \sin 2\phi & 0 \\ 0 & -\sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The next step is to find the differential surface area $d\sigma$ at the point P

$$d\sigma = \rho^2 \sin \eta \, \mathrm{d}\eta \, \mathrm{d}\zeta. \tag{7}$$

To obtain the flux density and polarization we integrate the Stokes vector over the visible crescent

$$\mathbf{D} = (\rho/\Delta)^2 \mathbf{j}(\alpha) = (\rho/\Delta)^2 \int_0^{\pi} \mathrm{d}\eta (\sin \eta)^2 \int_{\alpha - \pi/2}^{\pi/2} \mathrm{d}\zeta \mathbf{L}(\phi) \mathbf{I}(0, \eta, \zeta, \varphi) \cos \zeta, \tag{8}$$

where $\mathbf{I}$ is the Stokes vector at the point Q.

3. Numerical Results

There are at least three methods to numerically integrate Equation (8), the first of them was elaborated by Horák (1950) and the others by de Rooij (1985). Since our method to find the Stokes vector (Viik, 1989) does not depend on the prescribed set of μ and μ_0 values (we need no interpolation) thus we may exploit the method by Horak (1950). Hence we make the following transformation

$$\xi = (2 \sin \zeta + \cos \alpha - 1)(\cos \alpha + 1)^{-1}, \tag{9}$$

$$\psi = \cos \eta. \tag{10}$$

Then

$$\mathbf{j}(\alpha) = 1/2(\cos \alpha + 1) \cdot \int_{-1}^{+1} \mathrm{d}\psi (1 - \psi^2)^{1/2} \int_{-1}^{+1} \mathrm{d}\xi \mathbf{L}(\phi)\mathbf{I}(0, \psi, \xi, \varphi), \tag{11}$$

or in scalar notations,

$$j_I(\alpha) = 1/2(\cos \alpha + 1) \cdot \int_{-1}^{+1} \mathrm{d}\psi (1 - \psi^2)^{1/2} \int_{-1}^{+1} \mathrm{d}\xi I(0, \psi, \xi, \varphi), \tag{12}$$

$$j_Q(\alpha) = 1/2(\cos \alpha + 1) \cdot \int_{-1}^{+1} \mathrm{d}\psi (1 - \psi^2)^{1/2} \cdot \int_{-1}^{+1} \mathrm{d}\xi [I(o, \psi, \xi, \varphi) \cos 2\phi + Q(0, \psi, \xi, \varphi) \sin 2\phi]. \tag{13}$$

Horák (1950) and Kattawar and Adams (1971) performed the first integral by a quadrature employing Chebyshev polynomials of the second kind and the second integral by Legendre–Gauss quadrature. We used for both integrals Legendre–Gauss quadrature of the order of M=6 while the order for the discrete ordinate method was N=8.

To test the scheme of numerical integration we, too, compared the exact results computed for Lambert's law $I = \mu_0$: namely,

$$j(\alpha) = 2/3 \cos \alpha (\cos \alpha \sin \alpha + \pi - \alpha) + 1/3 \sin \alpha (1 - \cos 2\alpha), \tag{14}$$

with the results obtained by the numerical quadrature. The maximum relative error for $M = 6$ was 2×10^{-3} and for $M = 12 - 3 \times 10^{-4}$. It is clear that the flux reflected by a planetary atmosphere is larger if the atmosphere is less absorbing. Figure 2 shows that at smaller phase angles this difference is more pronounced than at large phase angles. The flux decreases rapidly in the region of phase angles from ~10° to ~70°. Then the decrease continuous at a slower pace up to ~140° where it again begins to decrease more rapidly. This behaviour is the same for all albedos of single scattering. We were especially interested in how does the non-

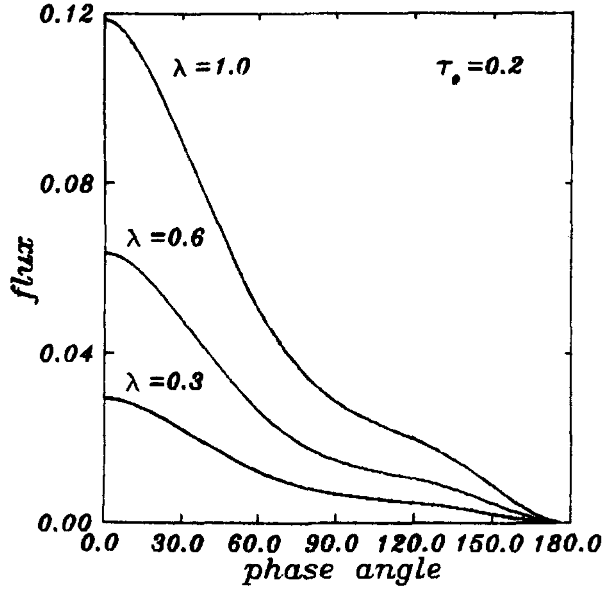

Fig. 2. Flux versus phase angle for a Rayleigh-scattering atmosphere as a function of optical thickness.

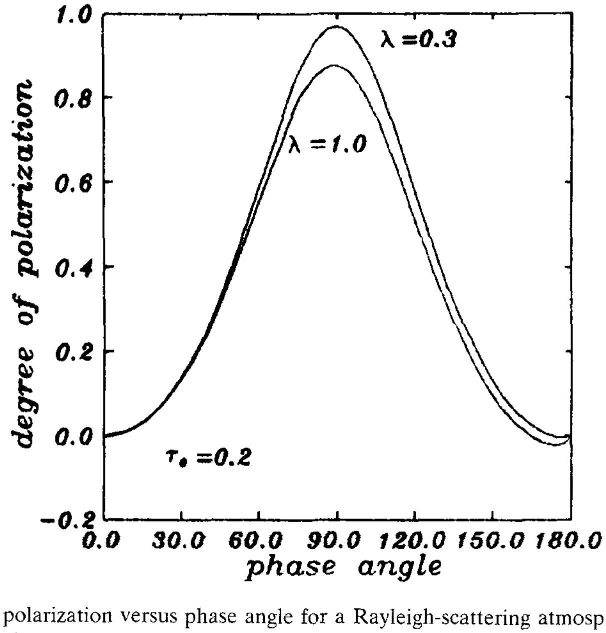

Fig. 3. Degree of polarization versus phase angle for a Rayleigh-scattering atmosphere as a function of the polarization of the incident radiation ($F_l = 0.5$, $F_r = 0.5$).

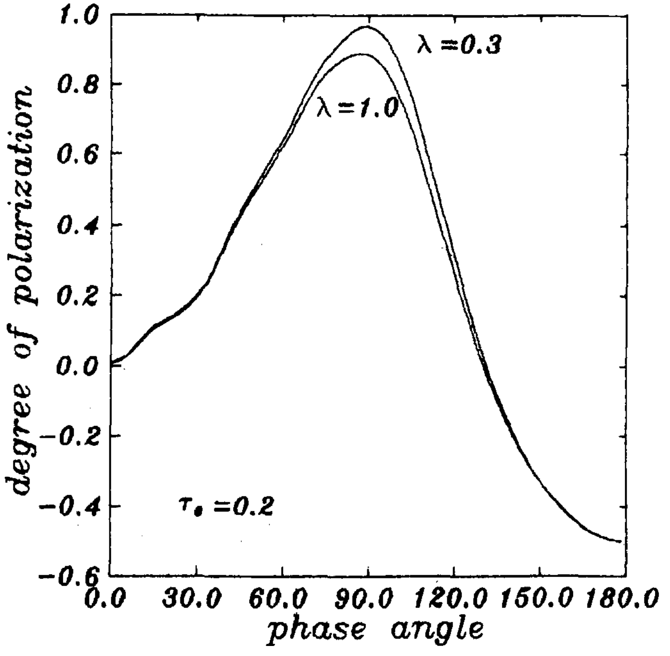

Fig. 4. Same as Figure 3 only $F_l = 1$ and $F_r = 0$.

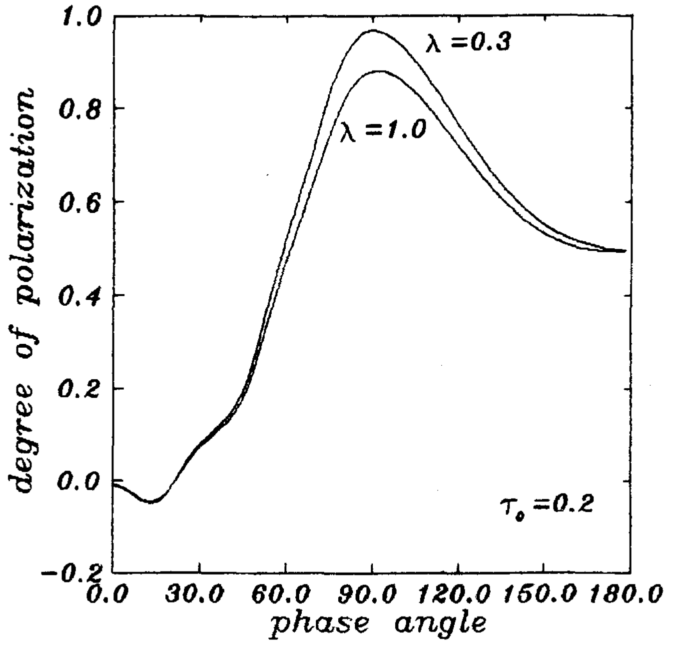

Fig. 5. Same as Figure 3 only $F_l = 0$ and $F_r = 1$.

conservativeness influence the degree of polarization of the reflected radiation. It appears that this influence is not very large since the degree of polarization for $\lambda = 0.3$ and $\lambda = 1.0$ differs at most 10% in the region of phase angles from 80° to 110° only, the degree of polarization being larger for smaller λ (Figure 3). Approximately the same may be observed also in the cases when the incident radiation is not natural but linearly polarized either in l or in r direction (Figures 4 and 5).

If the incident radiation is not polarized, the degree of polarization of the reflected radiation is positive almost in the whole region of phase angles. Only at very large phase angles there exists a small negative polarization. One is inclined to suppose that if the incident radiation is linearly polarized in l direction (in the plain of the main meridian) the negative polarization will prevail for all phase angles. This is not the case since the degree of polarization becomes negative only at approximately 130° if the optical thickness is 0.2 (Figure 4). And vice versa, if the incident radiation is linearly polarized in r direction there exists a small region of negative polarization from 0° to 20° (Figure 5).

Kattawar and Adams (1971) pointed out that for ground albedos other than zero the polarization phase curves are skewed toward larger phase angles since the unpolarized radiation from the surface will have less of a chance to escape for larger phase angles. The nonconservative atmospheres show the same trend as observed in Figure 6.

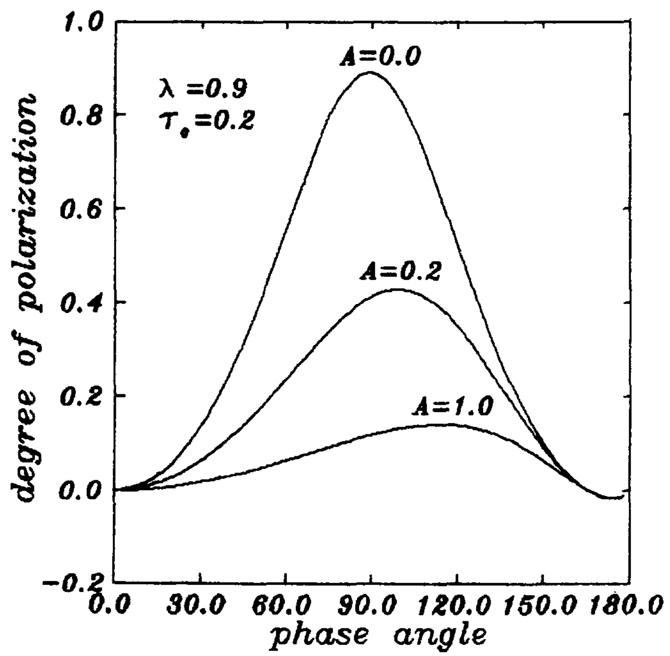

Fig. 6. Degree of polarization versus phase angle for a Rayleigh-scattering atmosphere as a function of the ground albedo.

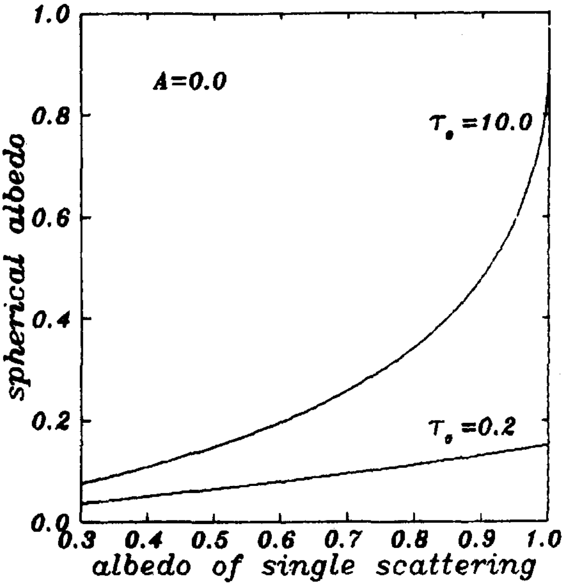

Fig. 7. Spherical albedo versus albedo of single scattering as a function of the optical thickness of the atmosphere.

In Figure 7 we give tne dependence of the spherical albedo of a planet on the albedo of single scattering in its atmosphere. For thicker atmospheres the spherical albedo increases steeply toward larger λ-s.

4. Conclusions

The preceding calculations and discussion have concerned effects associated with non-conservativeness of the Rayleigh-scattering planetary atmosphere and with the different states of polarization of the incident radiation.

It is concluded that, in our model, the effects are rather pronounced, thus allowing to better understand the physical situation in real planetary atmospheres.

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