ON PENETRATION DEPTH OF THE SHOEMAKER-LEVY 9 FRAGMENTS INTO THE JOVIAN ATMOSPHERE

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Abstract. The actual penetration depth of the Shoemaker-Levy 9 fragments into the Jovian atmosphere is still an open question. From fundamental equations of meteoric physics with variable cross-section, a new analytic model of energy release of the fragments is presented. In use of reasonable parameters, a series of results are calculated for different initial mass of the fragments. The results show that the largest fragment explodes above pressure levels of 3 bars and does not penetrate into the H_2O cloud layer of the Jovian atmosphere, and that airburst of smaller fragments occur even above the upper cloud layer.

1. Introduction

On 16–22 July, 1994, fragments of the comet Shoemaker-Levy 9 (SL-9) impacted the Jupiter. None of the pre-crash models is entirely correct which is not surprising for such an unprecedented event (Chapman, 1994). The actual penetration depth of the SL-9 fragments into the Jovian atmosphere is still an open question. Proposed scenarios range from far above the cloud layer to penetrated down into the atmospheric levels of several bars or more. The penetration depth should be related to size of the body entering the atmosphere (Boehnhardt and Schulz, 1995). The mass and size of the SL-9 fragments are still uncertain, with estimated sizes ranging from 0.64–4.06 km (Weaver *et al.*, 1995). Direct relation between these still uncertain quantities has not yet been demonstrated, thus a key role for the description of the entry phenomena is played by the energy deposition in the atmosphere versus altitude, a relation which has been not established so far (Boehnhardt and Schulz, 1995).

In principle, the impact of the SL-9 fragments on the Jupiter is a particular problem of meteoric physics as that of meteoroids on the atmosphere of the Earth, i.e. the fragments are decelerated, ablated, and finally exploded as airburst. However, it may be more complex in case of the SL-9 fragments impact on the Jupiter, because parametres of the SL-9 and the Jovian atmosphere as well as interaction between them have yet uncertainly been known. Some models for the impacts could be established under some reasonable assumptions. There are various models, and some results were presented. For example, Sekanina (1993) extrapolated the classical equations of meteor physics and showed that the extremely high ablation rate leads to disintegration of 10^{16} g fragment at pressure level about 0.3 bar. Takata *et al.* (1994) used a smooth particle hydrodynamics (SPH) model to compute that

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energy release will be gradual and penetrate to several hundred bars. Boslough *et al.* (1994) used a Eulerian model and shown maximum energy release at 30 bars. Yabe *et al.* (1994) used a Eulerian code and found that 3 km impactor release nearly all of their energy above 10 bars. Mac Low and Zahnle (1994) used both analytic model with variable cross-section (pan-cake) and numerical model of the astrophysical hydrocode ZEUS and found consistent energy release profiles and its peak at pressure of order 10 bars.

In this paper, a new analytic model is presented, in which analytic formula of energy release rate of the fragments is derived from fundamental equations of meteoric physics. Using reasonable cross-sections and other parameters, a series of results are calculated for different initial mass of the fragments.

2. Problem of Meteoric Physics for Impact of SL-9 on Jupiter

The motion of the SL-9 fragments in the Jovian atmosphere is essentially a particular problem of meteoric physics. The fundamental equations of meteoric physics are that of deceleration and ablation (mass-loss), which are written as follows (see Ceplecha and Borovicka, 1992),

$$M\frac{\mathrm{d}V}{\mathrm{d}t} = -\Gamma S\rho V^2 \tag{1}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\Gamma\sigma S\rho V^3 \tag{2}$$

Here, M and V are mass and velocity of a fragment at an arbitrary point (or time) on its trajectory. S is its cross-section. Γ and σ are drag and ablation coefficients, and ρ is density of the Jovian atmosphere. For isothermal atmosphere, a good approximation is

$$\rho = \rho_0 \exp(-Z/H) \tag{3}$$

Here, ρ_0 is the density at height $Z = 0$, H is the scale height. Assuming that inclination of the trajectory to the vertical is a constant of θ, then,

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = -V\cos\theta \tag{4}$$

Under the condition of constant coefficients Γ and σ, the relation between its mass and velocity can be derived from (1) and (2) as follows,

$$M/M_\infty = \exp(\sigma V^2/2 - \sigma V_\infty^2/2) \tag{5}$$

Here, M_∞ and V_∞ are the initial (outside atmosphere) values of the mass and the velocity of the fragment. Assuming that the fragment deforms quasi-statically and expands laterally as aerodynamic pressure overcomes its material strength. Then its effective radius R increases as a function of depth Z according to,

$$R\frac{\mathrm{d}^2R}{\mathrm{d}Z^2} \approx \frac{4\Gamma\rho\sec^2\theta}{\delta} \tag{6}$$

Here, δ is its density. Mac Low and Zahnle (1994) found that an approximate solution to question (6) is,

$$R(Z) \approx H\sec\theta(4\Gamma\rho/\delta)^{1/2} \tag{7}$$

However, the solution does not satisfy the condition $R(Z\rightarrow\infty) = R_\infty$, as Field and Ferrara (1995) pointed out. In this paper, we take a better approximation,

$$S = \pi R^2 = \pi[R_\infty^2 + R^2(Z)] = \pi[R_\infty^2 + R_1^2\exp(-Z/H)] \tag{8}$$

with $R_1 \approx H$ sec $\theta(4\Gamma\rho_0/\delta)^{1/2}$. The relation between its velocity and height can be derived from Equations (1), (4), (5) and (8) as follows,

$$\begin{aligned} E_i\left(\frac{\sigma V_\infty^2}{2}\right) - E_i\left(\frac{\sigma V^2}{2}\right) &= \frac{\pi\Gamma\rho_0 H}{M_\infty\cos\theta}\exp\left(\frac{\sigma V_\infty^2}{2}\right) \\ &\quad\times\left[2R_\infty^2\exp\left(-\frac{Z}{H}\right) + R_1^2\exp\left(-\frac{2Z}{H}\right)\right] . \end{aligned} \tag{9}$$

Here, $E_i(x)$ is the exponential integral function,

$$E_i = \int_\infty^x \frac{e^x}{x}\mathrm{d}x \tag{10}$$

The relation of energy loss rate versus velocity and height of the fragment are derived from above equations as follows,

$$\begin{aligned} \frac{\mathrm{d}E}{\mathrm{d}t} &= \frac{V^2}{2}\frac{\mathrm{d}M}{\mathrm{d}t} + MV\frac{\mathrm{d}V}{\mathrm{d}t} = -\pi\Gamma\rho_0 V^3\left(\frac{\sigma V^2}{2} + 1\right) \\ &\quad\times\left[R_\infty^2\exp\left(-\frac{Z}{H}\right) + R_1^2\exp\left(-\frac{2Z}{H}\right)\right] \end{aligned} \tag{11}$$

or energy release rate per unit height is,

$$\begin{aligned} \frac{\mathrm{d}E}{\mathrm{d}Z} &= (\pi\Gamma\rho_0\cos\theta)V^2\left(\frac{\sigma V^2}{2} + 1\right) \\ &\quad\times\left[R_\infty^2\exp\left(-\frac{Z}{H}\right) + R_1^2\exp\left(-\frac{2Z}{H}\right)\right] \end{aligned} \tag{12}$$

The relation between velocity V_* and height Z_* at the maximum value of $\mathrm{d}E/\mathrm{d}Z$ can be derived as follows,

$$\frac{\pi\Gamma\rho_0 H}{M_\infty\cos\theta}\frac{[R_\infty^2\exp(-Z/H)+R_1^2\exp(-2Z/H)]}{[R_\infty^2\exp(-Z/H)+2R_1^2\exp(-2Z/H)]} = \frac{\sigma V_*^2+2}{4(\sigma V^2+1)}\exp\left(\frac{\sigma V_*^2}{2}+\frac{\sigma V_\infty^2}{2}\right). \tag{13}$$

Meanwhile, V_* and Z_* also satisfy Equation (9). Therefore, solutions of V_* and Z_* can be obtained by combination of Equations (13) and (9). Corresponding, the time-scale of the maximum energy release rate $(\mathrm{d}E/\mathrm{d}t)_*$ is,

$$\tau = E_*/(\mathrm{d}E/\mathrm{d}t)_* \tag{14}$$

If the time-scale τ is less than 1 second, the fragment will explode at the height Z_* (airburst). Therefore, Z_* represents the penetration depth of the SL-9 fragments.

3. Parameter Selection and Calculation Results

Based on observation data of the Jovian atmosphere (Gehrels, 1976), we select $\rho_0 = 1.52\times10^{-4}$ g cm^{-3} and $H = 30$ km at $Z = 0$ of the pressure $P_0 = 1$ bar, $\Gamma = 0.85$, $V_\infty = 60$ km s^{-1} and $\theta = 45°$ are selected as Zahnle and Mac Low (1994). According to properties of the meteoroid material (Ceplecha and Borovicka, 1992), we select that $\sigma = 0.10$ s^2 km^{-2} and $\sigma = 0.75$ g cm^{-3} for the regular cometary material and $\sigma = 0.08$ s^2 km^{-2} and $\delta = 1.00$ g cm^{-3} for the dense cometary material. The diameters of the SL-9 fragments range from 0.64–4.06 km (Weaver *et al.*, 1995). Considering above data, a series of calculations are made for 10^{14}, 10^{15}, 10^{16}, 10^{17} g of M_∞ and the results are shown in Table I. Profiles of $V(Z)$ and $M(Z)/M_\infty$, $\mathrm{d}E(Z)/\mathrm{d}t$ for $M_\infty = 10^{14}$ g and 10^{16} g and $Z_*(M_\infty)$ are shown in Figures 1–3. According to (7), it is found that $R_1 = 0.9645$ km for $\delta = 1.0$ g cm^{-3} and $R_1 = 1.1137$ km for $\delta = 0.75$ g cm^{-3}. The numerical solution of Equations (9) and (13) are as follows: $V_* = 59.076$ km s^{-1} for $\delta = 1.0$ g cm^{-3} and $V_* = 59.225$ km s^{-1} for $\delta = 0.75$ g cm^{-3}.

The calculated results show a distinct maximum at height Z_* in profiles of energy release of the SL-9 fragment (Figure 2), and the fragments explode in very short time-scale ($\tau \leq 0.02$ seconds). The airburst height of the largest fragment is above pressure level of 2.5 bars, i.e. it does not penetrate into H_2O cloud layer, and the airbursts height of small fragment is even above the cloud top.

4. Discussion and Conclusion

The results of our analytic model are obtained under some approximation, which should be discussed as follows.

Table I

M_∞ (g)	E_∞ (erg)	δ (g/cm^3)	R_∞ (km)	Z_* (km)	P_* (bar)	M_* (g)	$(\mathrm{d}E/\mathrm{d}t)_*$ (erg/s)	τ (sec)
10^{13}	1.80×10^{26}	1	0.134	90.4	0.0492	9.853×10^{10}	3.71×10^{26}	0.0047
		0.75	0.147	99.4	0.0364	1.225×10^{11}	3.57×10^{26}	0.0060
10^{14}	1.80×10^{27}	1	0.288	59.7	0.1367	9.85×10^{11}	3.39×10^{27}	0.0051
		0.75	0.317	63.3	0.0991	1.23×10^{12}	3.20×10^{27}	0.0066
10^{15}	1.80×10^{28}	1	0.620	30.3	0.3639	9.85×10^{12}	3.12×10^{28}	0.055
		0.75	0.683	40.8	0.2565	1.23×10^{13}	2.99×10^{28}	0.0072
10^{16}	1.80×10^{29}	1	1.337	2.8	0.9122	9.85×10^{13}	2.81×10^{29}	0.0061
		0.75	1.471	13.9	0.6286	1.23×10^{14}	2.75×10^{29}	0.0079
10^{17}	$1.80 * 10^{30}$	1	2.879	-21.3	2.157	9.85×10^{14}	2.63×10^{30}	0.0066
		0.75	3.169	12.4	0.6622	1.23×10^{15}	1.07×10^{30}	0.0203

1. The fundamental theory of meteoric physics is successful to study meteoric phenomena in the Earth (Bronshten, 1983; Ceplecha and Borovicka, 1992). Although numerical models developed in recent years can provide some details for profiles of the energy release as well as fireball or plume, but they involve other rather uncertain assumptions, in particular the ablation have not been considered enough. For example, equivalent ablation coefficient in the model of Takata *et al.* (1994), σ = 0.01–0.001 s^2 km^{-2} is too small even for ordinary chondrites (σ = 0.017 s^2 km^{-2}, Ceplecha and Borovicka, 1992). As far as profiles of energy release and penetration depth, the results of analytic model may be consistent well with numerical model as shown by Mac Low and Zahnle (1995). Therefore, good analytic model is useful to establish the relation between penetration depth and impactor's size.

2. More strictly, gravitational term $-g$ cos θ/M must be added to the right side of Equation (1) and integral must be made from the height where aerodynamic pressure begin to overcome material strength of the fragment instead of $Z = \infty$. However, the gravitational term is not significant to the results (Sekanina, 1993). The tensile strength of the comet is $T/\delta \sim$ 1000 to 3000 dyn cm g^{-1} (Sekanina, 1993). Even taking $T = 10^4$ dyn cm^{-2}, height for aerodynamic pressure, $P_s = \Gamma\rho V^2 = T$, can be estimated as $Z' > 390$ km. Acceleration and ablation of the SL-9 fragments occur mainly below $Z'' < 150$ km, the differences of integrals from Z' and from $Z = \infty$ are ignored. The ablation coefficient σ depends mainly on the property of the impactor (Ceplecha and Borovicka, 1992) and also on its velocity and mass, but the dependence of σ with V is not remarkable near V = 60 km s^{-1}, and the dependence of σ with M for large M is very weak (Bronshten, 1983). Therefore, it is a good approximation that σ is taken as constant. The drag coefficient, Γ, is dependent mainly on the shape of the impactor. Since the tensile strength of the comet SL-9 is weak, it is easy for the SL-9 fragments to deform into "pancake"

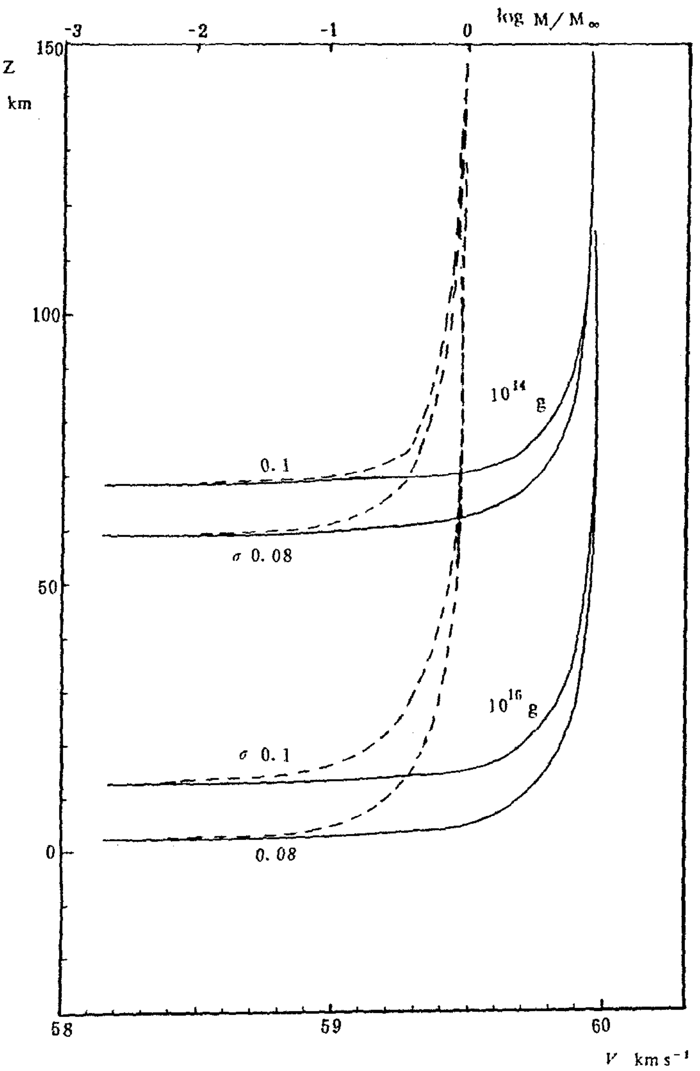

Figure 1. Decelertion ~ height (black) and mass ~ height (dash).

shape, in particular, deceleration of the fragments occur in denser atmospheric layer, therefore $\Gamma = 0.85$ is a good approximation. Similarly, the isothermal atmospheric model and the scale height $H = 30$ km are also good approximations for the height range of the interaction between the SL-9 fragments and Jovian atmosphere. Our analytic model and calculated results may represent better the actual case of the impact of the SL-9 on Jupiter. Small variations in the parameters have essentially no effect on the result.

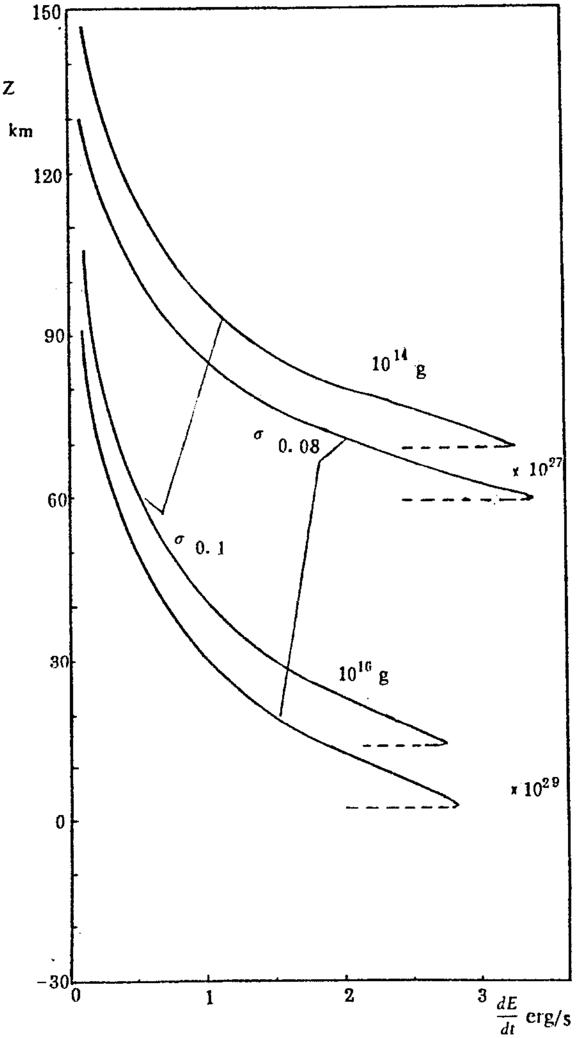

Figure 2. Rate of energy release ($\mathrm{d}E/\mathrm{d}t$) ~ height.

3. Among the SL-9 fragments, G is one of the largest fragments with upper limit of the diameter 4.06 km (Weaver *et al.*, 1995). If its density δ=0.75 g cm^{-3} or 1.0 g cm^{-3}, then its mass is 2.628×10^{16} g or 3.504×10^{16} g, and its airburst heights is at pressure level of 1.306 bars or 1.034 bars by our model, i.e. it only penetrates into the NH_4SH cloud layer and is impossible into H_2O cloud layer. In fact, NH_3, S_2, CS_2, CS, H_2S and S^+ were observed in spectra near the G impact site by Hubble Space Telescope. Many of the sulfur-containing molecule may be derived from a sulfur-bearing parent molecule native to Jupiter. If so, the fragment must have penetrated NH_4SH cloud, but oxygen-containing molecules including

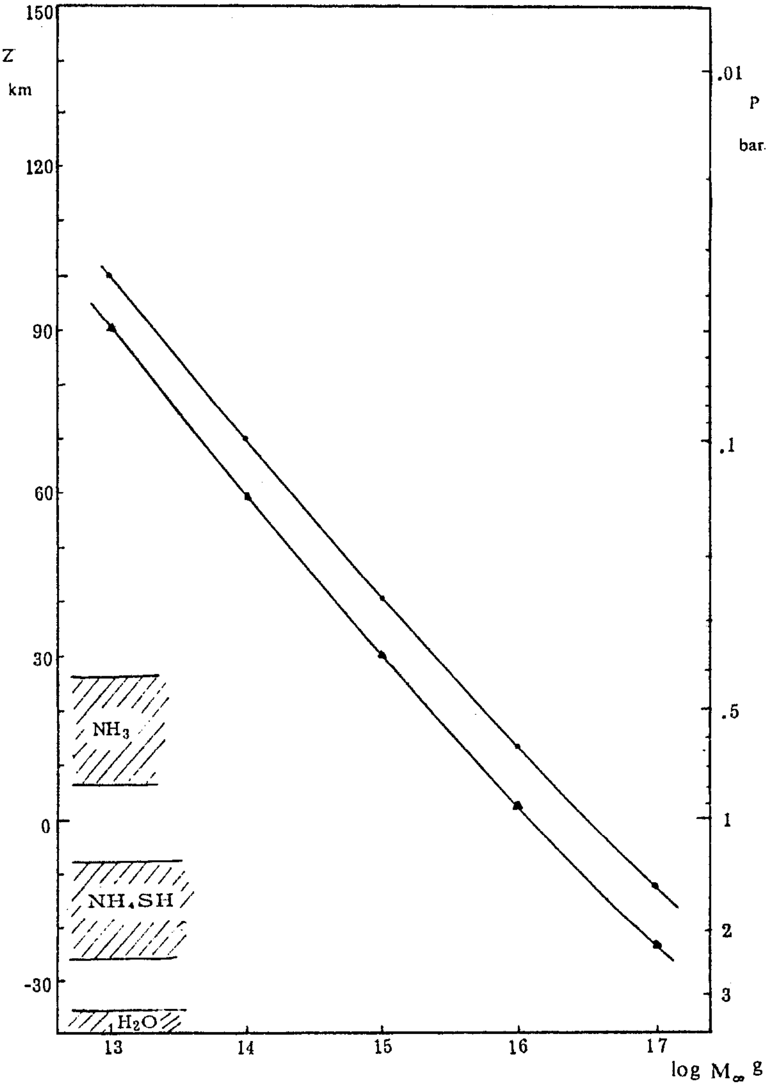

Figure 3. The initial mass of the SL-9 fragment ~ airburst height.

H_3O, were conspicuous by their absence (Noll *et al.*, 1995). These indicate that the largest fragment indeed penetrate into NH_4SH cloud but did fail to reach H_2O cloud and itself was virtually devoid of water.

In summary, we come to conclusion that the SL-9 fragments are of dense cometary material and the largest fragment penetrated into NH_4SH cloud layer but did not reach H_2O cloud layer, small fragments of mass less than 10^{14} g did not even reach the upper cloud layer.

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