NON-LINEAR r.f. HEATING OF IONOSPHERIC PLASMA

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Abstract. The non-linear heating of electrons in the ionospheric plasma due to high-power radio wave propagation has been investigated through an integro-differential equation derived from Boltzmann velocity-moment equations. Various processes appropriate to the situation under study are taken into account. The numerical solution of the derived equation is presented graphically.

1. Introduction

Interaction of electromagnetic waves within the ionospheric plasma during high power radio wave propagation has been investigated by Ginzburg (1960), Lin (1961), Papa (1965), El-Khamy *et al.* (1970), Chakrabarti and Ram (1974), Graham and Fejer (1976), Kumar and Rao (1976), De (1976), Aggarwal *et al.* (1979), Chakrabarti and De (1982), Ram (1982), Sulzer *et al.* (1982), De *et al.* (1987), De *et al.* (1987), using various techniques. Different effects of such interaction are well-known.

During high power wave propagation, the average energy imparted to the electrons by the field gradually heats up the medium and eventually a new equilibrium temperature is reached. The heating is non-linear in nature. This non-linearity arises due to the slow rate of transfer of energy from electrons to heavy particles in the medium. The slow rate is associated with the small magnitude of the ratio of the electron mass to the mass of the heavy particles. The mean free path of the electrons in the medium is quite large due to which they extract a considerable amount of energy from the field. Under the circumstance, the electrons get heated and the complex dielectric constant of the medium becomes a function of the electric field. The non-linearity of the electromagnetic process will be pronounced the more the amplitude of the electric field exceeds that of the plasma field.

For a weakly ionised plasma like ionosphere, the effective collision frequency and ionisation frequency depend on the temperature of the electrons through the average electron velocity. In this presentation, the temperature variation of electrons under the specified circumstance has been investigated with the help of an integro-differential equation derived from Boltzmann's velocity-moment equations. The influences of the geomagnetic field, photoionisation and recombination process towards the phenomenon have been taken into consideration through the basic equations. The numerical solution of the derived equation is presented graphically.

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2. Mathematical Formulation

The plasma may be assumed to be neutral, unbounded and at static equilibrium. The ions are stationary and the electron component of the plasma only affects the plasma waves. The physical situation, as stated earlier, may be represented by the following velocity-moment Boltzmann equations

$$\frac{\partial N}{\partial t} = (\nu_i - \nu_a)N - \alpha N^2, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{e}{m}\mathbf{E}(t) - \nu_e(T_e)\mathbf{v} - \frac{e}{m}\mathbf{v} \times \mathbf{H} - \frac{\nabla p}{m} + \frac{\eta}{m}\nabla^2\mathbf{v}, \tag{2}$$

$$\frac{3}{2}\frac{\partial}{\partial t}(NK\,T_e) + eN\mathbf{v} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + G_{\text{eff}}(T_e)\nu_e(T_e) + ,$$

$$+\frac{3}{2}\delta\nu_e(T_e)NK(T_e - T) - \nabla \cdot \mathbf{q} - \chi\nabla^2 T + Q_i\frac{\partial N}{\partial t} = 0, \tag{3}$$

where

$\mathbf{E}(t)$ = applied electric field,
$\mathbf{H}$ = geomagnetic field,
ν_i = ionisation frequency,
ν_e = effective collision frequency of the electrons with heavy particles,
ν_a = electron-neutral attachment frequency,
$\mathbf{v}$ = average electron velocity,
Q_i = ionisation energy of the medium,
N = electron density,
$\delta = \dfrac{2m}{m'}$,
m' = mass of the heavy particle,
α = electron-ion recombination coefficient,
$\mathbf{q}$ = total energy flow due to electron drift when the heavy neutral and ion-components are assumed to have no net drift velocity
$= -\lambda(T_e)\nabla T_e$,
T = equilibrium plasma temperature,
T_e = electron temperature,
$\lambda(T_e)$ = effective coefficient of electron energy conduction,
$\lambda = K_T\left(1 - \dfrac{\mu\tau'}{\sigma^0 K_T}\right)$,
K_T = coefficient of electron energy conduction at constant electron density,
μ = coefficient of electron energy conduction due to d.c. electric field,
η = coefficient of viscosity,

σ^0 = d.c. electric conductivity,
τ' = current flow coefficient due to thermal gradients at constant electron pressure $P(=NKT_e)$,
G_{eff} = effective fraction energy transfer per collision;

the other symbols have their usual significance.

The expressions for ionisation frequency (ν_i) and effective collision frequency (ν_e) appropriate to the stated circumstance will be considered from the works of Papa (1965) and Datta *et al.* (1981), respectively, which can be written as

$$\nu_i(T_e) = n\left(\frac{8KT_e}{\pi m}\right)^{1/2}(\pi a_0^2)\exp\left(-\frac{Q_i}{KT_e}\right); \tag{4}$$

where

n = neutral particle number density,
a_0 = Bohr radius,

and

$$\nu_e = \nu_{en} + \nu_{ei} =$$
$$= n[aT_e^{1/2} + bT_e + cT_e^{3/2} + \mathrm{d}T_e^2] + \frac{8\pi}{3}\frac{Z^2e^4}{(2\pi m)^{1/2}}\frac{N}{(KT_e)^{3/2}}\ln\Lambda, \tag{5}$$

where

$$\Lambda = \frac{K^{3/2}}{1.78Ze^3}\frac{T_e^{3/2}}{(2\pi N)^{1/2}}\left(\frac{2T_i}{T_e + T_i}\right)^{1/2}$$

and

T_i = ion temperature;

a, b, c, and d are the constants as explained in Aggarwal and Setty (1980). From (2), the expression for $\mathbf{v}$ can be obtained as

$$\mathbf{v} = -\exp(-A)\frac{e}{m}\int_0^t \mathbf{E}(t')\exp\left\{A + \int_t^{t'} M\,\mathrm{d}t''\right\}\mathrm{d}t', \tag{6}$$

where

$$A = \int_0^t \left\{(\mathbf{v}\cdot\nabla) + \nu_e(T_e) + \frac{\eta k^2}{m} + \frac{C}{m}\right\}\mathrm{d}t',$$

$$M = \begin{pmatrix} 0 & H_z & -H_y \\ -H_z & 0 & H_x \\ H_y & -H_x & 0 \end{pmatrix},$$

C = Reynold number.

Substituting $\partial N/\partial t$ from (1) in (3), one obtains

$$\frac{\partial T_e}{\partial T} + \left[n\left(\frac{8KT_e}{\pi m}\right)^{1/2}(\pi a_0^2)\exp\left(-\frac{Q_i}{KT_e}\right) - \nu_a - \alpha N + \delta\nu_e(T_e) - \right.$$
$$\left. - \frac{2k^2K_T}{3NK}\left(1 - \frac{\mu\tau'}{\sigma^0 K_T}\right)\right]T_e = \delta\nu_e(T_e)T + \frac{2\chi k^2 T}{3NK} -$$
$$- \frac{2Q_i}{3K}\left\{n\left(\frac{8KT_e}{\pi m}\right)^{1/2}(\pi a_0^2)\exp\left(-\frac{Q_i}{KT_e}\right) - \nu_a - \alpha N\right\} -$$
$$- \frac{2}{3NK}G_{\text{eff}}(T_e)\nu_e(T_e) + \frac{2e^2}{3mK}\mathbf{E}\cdot\mathbf{v}. \tag{7}$$

Using the expression of $\mathbf{v}$ from (6), we find that Equation (7) yields

$$\frac{\partial}{\partial t}\left(\frac{T_e}{T}\right) + \left[n\left(\frac{8KT_e}{\pi m}\right)^{1/2}(\pi a_0^2)\exp\left(-\frac{Q_i}{KT_e}\right) - \nu_a - \alpha N + \right.$$
$$\left. + \delta\nu_e(T_e) - \frac{2k^2K_T}{3NK}\left(1 - \frac{\mu\tau'}{\sigma^0 K_T}\right)\right]\left(\frac{T_e}{T}\right) =$$
$$= \delta\nu_e(T_e) + \frac{2\chi k^2}{3NK} - \frac{2Q_i}{3KT} \times$$
$$\times \left\{n\left(\frac{8KT_e}{\pi m}\right)^{1/2}(\pi a_0^2)\exp\left(-\frac{Q_i}{KT_e}\right) - \nu_a - \alpha N\right\} -$$
$$- \frac{2}{3NKT}G_{\text{eff}}(T_e)\nu_e(T_e) + \frac{2e^2}{3mKT}\mathbf{E} \times$$
$$\times \left[\exp(-A)\int_0^t \mathbf{E}(t')\exp\left\{A + \int_t^{t'} M\,\mathrm{d}t''\right\}\mathrm{d}t'\right]. \tag{8}$$

If we transform $\delta\nu_e(T_e)t$ as τ and let $T_e/T = \theta$ (the normalized electron temperature), Equation (8) yields

$$\frac{\partial\theta}{\partial\tau} + \frac{1}{\delta\nu_e(T_e)}\left[n\left(\frac{8KT_e}{\pi m}\right)^{1/2}(\pi a_0^2)\exp\left(-\frac{Q_i}{KT_e}\right) - \nu_a - \alpha N + \right.$$
$$\left. + \delta\nu_e(T_e) - \frac{2k^2K_T}{3NK}\left(1 - \frac{\mu\tau'}{\sigma^0 K_T}\right)\right]\theta =$$
$$= 1 + \frac{2\chi k^2}{3NK\delta\nu_e(T_e)} - \frac{2Q_i}{3KT\delta\nu_e(T_e)} \times$$
$$\times \left\{n\left(\frac{8KT_e}{\pi m}\right)^{1/2}(\pi a_0^2)\exp\left(-\frac{Q_i}{KT_e}\right) - \nu_a - \alpha N\right\} -$$
$$- \frac{2}{3NKT\delta\nu_e(T_e)}G_{\text{eff}}(T_e)\nu_e(T_e) + \frac{2e^2}{3mKT\delta\nu_e(T_e)}\mathbf{E} \times$$
$$\times \left[\exp(-A)\int_0^t \mathbf{E}(t')\exp\left\{A + \int_t^{t'} M\,\mathrm{d}t''\right\}\mathrm{d}t'\right] \tag{9}$$

Let a linearly polarized electric field be applied suddenly at the time $t = 0$ such that

$$\left.\begin{aligned} \mathbf{E}(t) &= 0, && \text{for} \quad t < 0 \\ &= E_0 \sin \omega t, && \text{for} \quad t > 0 \end{aligned}\right\}. \tag{10}$$

Using (10), the Equation (9) becomes

$$\begin{aligned} \frac{\partial \theta}{\partial \tau} &+ \frac{1}{\delta v_e(T_e)}\left[n\left(\frac{8KT_e}{\pi m}\right)^{1/2} (\pi a_0^2) \exp\left(-\frac{Q_i}{KT_e}\right) - v_a - \right. \\ &\left. - \alpha N + \delta v_e(T_e) - \frac{2k^2 K_T}{3NK}\left(1 - \frac{\mu \tau'}{\sigma^0 K_T}\right)\right]\theta = \\ &= 1 + \frac{2\chi k^2}{3NK\delta v_e(T_e)} - \frac{2Q_i}{3KT\delta v_e(T_e)} \times \\ &\times \left\{ n\left(\frac{8KT_e}{\pi m}\right)^{1/2} (\pi a_0^2) \exp\left(-\frac{Q_i}{KT_e}\right) - v_a - \alpha N \right\} - \\ &- \frac{2}{3NKT\delta v_e(T_e)} G_{\text{eff}}(T_e) v_e(T_e) + \frac{2e^2}{3mKT\delta v_e(T_e)} E_0 \sin \omega t \cdot \\ &\left[\exp(-A) \int_0^t E_0 \sin \omega t' \exp\left\{ A + \int_t^{t'} M \, \mathrm{d}t'' \right\} \mathrm{d}t' \right]. \end{aligned} \tag{11}$$

The integro-differential Equation (11) will be used to determine the variation of normalized electron temperature within the medium under the modified condition.

3. Discussion

The variation of normalised electron temperature (T_e/T) in the E-region of the ionosphere has been computed numerically, and the results shown graphically on Figure 1. The rapid increase of electron temperature with time as depicted in the

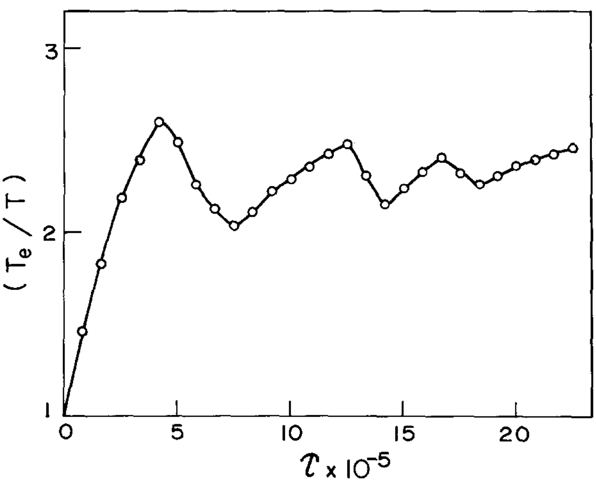

Fig. 1. Variation of normalised electron temperature with time.

figure is due to the dependence of collision frequency on the electron temperature that causes an increase in the average electron velocity. The temperature-dependent terms on the right hand side of the Equation (11) tend to reduce $d\theta/d\tau$ and decelerate the temperature rise until an equilibrium temperature is ultimately reached (Ginzburg, 1964).

The rate of energy transfer from the wave to the medium particles can also be computed through the integro-differential Equation (11).

The physical data are taken from CIRA 1972. The other parametric values are taken from the ionospheric observations over Calcutta (lat. 22°58′ N; long. 88°34′ E).

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