MAXIMUM NON-GRAVITATIONAL ACCELERATION DUE TO OUT-GASSING COMETARY NUCLEI

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Abstract. Maximum possible acceleration due to out-gassing from cometary nuclei is calculated for H_2O and $CO(N_2)$ molecules. It is found that the maximum excess velocity at great distance is 0.18 km s^{-1} so that excess velocities less than this value are compatible with the non-gravitational acceleration due to non-symmetric out-gassing. On the other hand, Comet 1975*q* and comet 1955*V* have excess velocities 0.81 and 0.80 km s^{-1} respectively. These comets may be regarded as the candidates for possible interstellar comets.

1. Introduction

Usually, a comet is newly discovered while it is within $3 \sim 4$ astronomical units from the sun. From a number of observations over its position, an osculating orbit is determined. One can then calculate the original orbit which the comet had while at great distance from the sun, allowing for planetary perturbations. If the original orbit is a hyperbola, the excess velocity at great distance may be genuine, indicating that the comet is of interstellar origin, or it may be due to the so-called non-gravitational force.

Among the original orbits calculated by Marsden, Sekanina and Everhart (1978, to be designated MSE) there are some that have originally hyperbolic orbits. Consensus amongst astronomers appears to be that the excess velocities are due to the so-called non-gravitational force. There are, however, some comets whose original orbits are highly hyperbolic. For comet 1976I, (1975*q*) the original value of $1/a$ is -734×10^{-6} AU^{-1}, which corresponds to an excess velocity 0.81 km s^{-1} at great distance. For comet 1955V, the original value of $1/a$ is -727×10^{-6} AU^{-1} which corresponds to an excess velocity 0.8 km s^{-1}. If these excess velocities were due to the non-gravitational force, it would have to be such as to accelerate the comets by the amount required by observation.

The object of the present paper is to calculate an upper bound on the excess velocities on the presumption that they are due to non-symmetric out-gassing from comets. One should then be able to discuss whether or not the excess velocities of observed comets can be interpreted in terms of the non-gravitational force.

2. Non-Gravitational Acceleration

The gravitational force acting on a comet nucleus is composed of two components (Marsden, 1982). One of these is radial, while the other is in the orbital plane

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and is directed 90° ahead of the comet in true anomaly. These forces vary with the heliocentric distance. The paramcters which specify the magnitudes as well as their dependence on the distance have been determined for some of the periodic comets and these parameters sometime vary with time.

For long-period comets of one apparition only, it is not easy to determine the parameters. However, the nature of the non-gravitational force is ascribed to nonspherical out-grassing of volatile molecules from a comet nucleus.

Squires and Bird (1961) discussed a simple model in which gaseous molecules are ejected toward the Sun. In this model a cometary nucleus is represented by a disk of radius b, (which is taken equal to the nuclear radius) and the repulsive force due to the out-grassing is taken as $\pi p b^2$ where p is the gas pressure appropriate to the surface temperature T, which is calculated by balancing the incident energy to the heat loss due to re-radiation and sublimation of gas molecules. They derived an equation

$$\left.\frac{1}{a}\right|_{\mathrm{app}} - \left.\frac{1}{a}\right|_{\mathrm{real}} = -\frac{K_0}{Rk} l_n \frac{R}{q}, \tag{2.1}$$

where a and q are the semi-major axis and perihelion distance, respectively, R is the distance where the effect due to out-gassing becomes significant; K_0 is the gaussian constant and k/K_0 is the ratio of the repulsive force due to the out-gassing to the gravitational pull of the Sun. *Apparent a* means the value of the semi-major axis of the comet for heliocentric distance $<R$ and *real a* stands for the value of a at distances $>R$. Interesting as the idea may be, the above equation appears to predict a wrong sign of the effect of out-gassing on the change of the reciprocal of the semi-major axis. (For the discussion, see Section 3). Squires and Bird concluded that the change in $1/a$ may be as much as $0.0005\ \mathrm{AU}^{-1}$ if the nuclei have masses 6×10^{11} g and radii 300 m.

Although the model of the nucleus considered by them is too small and no longer acceptable, their discussion is important in that it first presented a quantitative relation between the shift in $1/a$ and the out-gassing of volatile molecules.

3. Basic Relations

Here we develop and derive basic relations between the non-gravitational force and the variations in the orbital elements. Let $\mathbf{x} = (x, y, z)$ be the heliocentric coordinates of a comets, and let $\mathbf{a} = (a_x, a_y, a_z)$ be the non-gravitational acceleration. The equations of motion of the comet then are

$$\ddot{\mathbf{x}} + GM_{\odot}\mathbf{x}/|\mathbf{x}|^3 = \mathbf{a}\,.$$

Multiplying the equation by $\dot{\mathbf{x}}$ and integrating from aphelion to perihelion we obtain the relation

$$\left[\frac{1}{2}\dot{\mathbf{x}}^2 - \frac{GM_\odot}{|\mathbf{x}|}\right]_{-\infty}^{0} = \int_{-\infty}^{0} \mathbf{a}\cdot\dot{\mathbf{x}}\,\mathrm{d}t\,.$$

Since the semi-major axis, a is related to the energy by the relation

$$-GM_\odot/2a = \tfrac{1}{2}\dot{\mathbf{x}}^2 - GM_\odot/r\,,$$

one obtains that

$$\left.\frac{1}{a}\right|_{\mathrm{aph}} = \left.\frac{1}{a}\right|_{\mathrm{peri}} + \frac{2}{GM_\odot}\int_{-\infty}^{0} \mathbf{a}\cdot\dot{\mathbf{x}}\,\mathrm{d}t\,. \tag{3.1}$$

Consider, as a special example, $\mathbf{a} = \epsilon GM_\odot \mathbf{x}/|\mathbf{x}|^3$, where ϵ is less than unity. One immediately obtains the result that

$$1/a_{\mathrm{aph}} = 1/a_{\mathrm{peri}} + 2\epsilon[1/Q - 1/q]\,, \tag{3.2}$$

where Q denotes the apherion distance. Since $Q > q$, $1/a_{\mathrm{aph}}$ is less than $1/a_{\mathrm{peri}}$ for $\epsilon > 0$ and *vice versa*. In the case discussed by Squires and Beard (1961), ϵ is positive so that $1/a_{\mathrm{aph}}$ is less than $1/a_{\mathrm{peri}}$, while the equation (2.1) of Squires *et al.* yields an opposite conclusion.

In reality, the direction of the out-gassing may not be directed toward the sun, because the rotation of the cometary nucleus introduces some asymmetry. We do not here wish to discuss how the rotation introduces an asymmetry. Rather, we wish to estimate an *upper limit* on the change in $1/a$ owing to the out-gassing.

It is clear that the largest contribution to the change $\Delta(1/a)$ would be obtained if the acceleration $\mathbf{a}$ were parallel to the velocity, $\dot{\mathbf{x}}$. It is also clear that the absolute acceleration cannot be greater than the case where all of the sulimating molecules leave the nuclear surface in one and the same direction. In other words,

$$|\mathbf{a}| < \frac{1}{M_c}\int mQv\,\mathrm{d}S\,,$$

where Q is the number of sublimating molecules per unit area per unit time, m and v are the mass and velocity of the sublimating molecules, respectively and where integration is over the total surface of the nucleus with mass, M_c. In the next section, we proceed to estimating the maximum possible acceleration for 2 types of volatile molecules.

4. Results

The maximum possible acceleration has been calculated in the following manner. First, the gas production rate is calculated for each area of the cometary surface for given heliocentric distances as the comet approaches the sun. The method employed is the one given in Yabushita (1987). The nucleus rotates with a period of 50 hr and the axis is perpendicular to the orbital plane. The sublimation rate is designated by Q. Because Q depends on the latitude of the comet, Q ought to be averaged over the surface by weighing the factor $\cos l$, where l is the latitude. The average is denoted by $\bar{Q}$. This is multiplied by the velocity of the comet, $|\dot{\mathbf{x}}|$ and the product $\bar{Q}|\dot{\mathbf{x}}|$ is summed over the orbit as the comet approaches from great distance to perihelion. To calculate the total acceleration, the velocity of the sublimating molecules ought to be specified. We take the velocity to be one-half of the Maxwellion velocity which corresponds to the surface temperature T

$$v = \frac{1}{2} v_M = \frac{1}{2}\left(\frac{3kT}{m}\right)^{1/2},$$

where k is the Boltzmann constant. For cometary surface composed of CO, the surface temperature at the sunward side varied from 90 K to 10 K depending on the heliocentric distance. We take $T = 100$ K, whence $v = 1.49 \times 10^4$ cm/sec.

The numerical computation yielded the result that the sum of $\bar{Q}|\dot{\mathbf{x}}|$ taken over the orbit $= 5.10 \times 10^{27}$ mol cm^{-1} hr^{-1} for a parabolic orbit with $q = 1$ AU. When this is multiplied by vm, the maximum acceleration per unit surface of the nucleus can be obtained. In other words,

$$\delta E = S \times mv \int \bar{Q}|\dot{\mathbf{x}}|\, \mathrm{d}t\,, \tag{4.1}$$

where S is the total surface of the comets. If the excess velocity due to the non-gravitational force is denoted by δV,

$$\frac{1}{2}\frac{4\pi}{3}\rho R^3(\delta V)^2 = 4\pi R^2 \times mv \int \bar{Q}|\dot{\mathbf{x}}|\, \mathrm{d}t\,;$$

so that

$$(\delta V)^2 = \frac{6mv}{\rho R}\int_{-\infty}^{0} \bar{Q}|\dot{\mathbf{x}}|\, \mathrm{d}t\,. \tag{4.2}$$

Inserting the appropriate numerical values, one finally obtains

$$\delta V = 1.23 \times 10^4 \left(\frac{10\ \mathrm{km}}{R}\right)^{1/2}\left(\frac{0.5}{\rho}\right)^{1/2} \mathrm{cm/sec}\,.$$

This is the maximum excess velocity due to the non-gravitational force, when sublimating molecules are CO.

Next, we consider the case where sublimating gases mainly consist of H_2O molecules. For a parabolic orbit with $q = 1.5$ AU, the integral of the quantity $\bar{Q}\dot{\mathbf{x}}$ from great distance to perihelion is calculated at 1.20×10^{27} mol cm^{-1} hr^{-1}. As

TABLE I

Numerical coefficient f which appears in Equation (4.3)

	$CO(N_2)$	H_2O
$q = 0.5$ AU	1.79×10^4	1.31×10^4
1.0 AU	1.23×10^4	8.46×10^3
1.5 AU	0.97×10^4	6.22×10^3

with the case of CO, we take the velocity of sublimating molecules to be $\frac{1}{2}v_M$. From the numerical calculation, the surface temperature has been found to vary from 120 K to 180 K. Taking the upper limit, it is seen that

$$\delta V = 6.22 \times 10^3 \left(\frac{10\ \mathrm{km}}{R}\right)^{1/2} \left(\frac{0.5}{\rho}\right)^{1/2} \mathrm{cm\ s^{-1}}\,.$$

Table I gives the numerical value of the coefficient f when δV is written in the form

$$\delta V = f\left(\frac{10\ \mathrm{km}}{R}\right)^{1/2} \left(\frac{0.5}{\rho}\right)^{1/2} \mathrm{cm\ s^{-1}}\,. \tag{4.3}$$

5. Discussion and Conclusions

Following the work of Squires and Bird (1961), we have calculated the maximum possible acceleration due to non-symmetric out-gassing from a comet as it approaches the sun from aphelion. Two different types of outgassing molecules have been considered, CO (or N_2) and H_2O. CO or N_2 is the most volatile of molecules known to exist in comets, while H_2O is the least volatile. The non-gravitational acceleration is written in the form of Equation (4.3) while the numerical coefficient f therein is given in Table I. It has been shown that for reasonable values of nuclear radius and density, the acceleration ranges from 0.06 km s^{-1} to 0.18 km s^{-1}, depending on the perihelion distance and molecular species. On the other hand, the excess hyperbolic velocity at great distance is 0.81 km s^{-1} for comet 1976I. It is thus seen that the excess velocity of 1976I is too large to be compatible with acceleration due to the non-symmetric out-gassing from cometary nuclei. In Table II, we list those comets such that their excess velocities at great distance are not consistent with the non-gravitational accelerations.

It may be pointed out that the maximum acceleration has been derived in the present paper assuming that all of the sublimating molecules leave the nucleus in the direction apposite to the Sun. Hence, the derived acceleration overestimates the real non-gravitational acceleration by a large factor. It is therefore surprising that there are as many as 15 comets for which the calculated original orbits are

TABLE II

List of comets such that their original $1/a$ values are not consistent with the out-gassing. Original $1/a$ values taken from MSE

Comet	$(1/a)_{\text{original}} \times 10^6$ (AU^{-1})	q (AU)	δV (km $\cdot$ s^{-1})
Class I			
1985IV	−172	0.192	0.39
1957III	−98	0.316	0.29
1899I	−109	0.326	0.31
1932VII	−56	1.647	0.22
1898VIII	−71	2.285	0.25
Class II			
1975XI	−56	0.219	0.22
1911IV	−74	0.303	0.26
1960II	−135	0.504	0.35
1975*q*	−734	0.864	0.81
1955V	−727	0.885	0.80
1940III	−124	1.062	0.33
1968VI	−82	1.160	0.27
1971V	−142	1.233	0.35
1959III	−448	1.251	0.63
1980II	−75	1.814	0.26

not compatible with elliptical orbits modified by the non-gravitational acceleration. These comets may very well be of interstellar origin.

References

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