THE SECULAR ACCELERATION OF THE MOON DETERMINED FROM LUNAR LASER RANGING DATA

XU HUAGUAN, JIN WENJING and HUANG CHENGLI
Shanghai Observatory, Chinese Academy of Sciences,
80 Nandan Road, Shanghi 200030, P.R. China

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Abstract. Lunar Laser Ranging data covering the interval from August 1969 to December 1987 were used to determine the seculer acceleration in the mean longitude of the Moon ($\dot{n}$). In our analysis, the DE200/LE200 planets and lunar ephemerides were adopted for calculating the theoretical distance between the observing station and reflector. The method of stepwise regression was used in the processing of the data and the value of $-25''.4 \pm 0''.1/\text{cy}^2$ was obtained by a weighted least squares fit.

Our result is in good agreement with that derived by other authors using various methods. The uncertainty $\sigma(\dot{n})$ estimated from LLR data would be decreasing rapidly with increasing the data span. The high precision obtained in this paper is mainly due to the longer span and higher measuring accuracy of data.

Key words: Lunar laser ranging, secular acceleration

1. Introduction

The fact of a secular acceleration in the mean longitude of the Moon had been early verified by measuring the angular position of the Moon.

The phenomenon is important to analyse and research following problem: the secular variation of the earth rotation, the tidal friction in the Earth–Moon system, the evolution of the lunar orbit.

To derive the secular acceleration value $\dot{n}$ is considerably different type of data have been used, such as: lunar occultation, ancient solar and lunar eclipse, transits of Mercury, numerical tidal model, satellite measurements of the tides, Lunar Laser Ranging (LLR).

The secular acceleration $\dot{n}$ corresponds to a linear increase about 3.5 $\sim$ 3.7 cm/yr in the mean distance of Earth-Moon. This means that the range data is sensitive to this phenomenon of secular acceleration.

It is significant to determine the value of $\dot{n}$ by LLR data. Since August 1969, the observing station of LLR has been developped from one to three, meanwhile to range four retro-reflectors becomes routinely observation. The accuracy of ranging distance has been greatly improved from 25 cm to 5 cm. A data span longer than 20 years has been obtained.

In this paper, using the observing data of 18 years from global four LLR stations, the value of $\dot{n}$ was obtained. Our results were compared with those derived by others.

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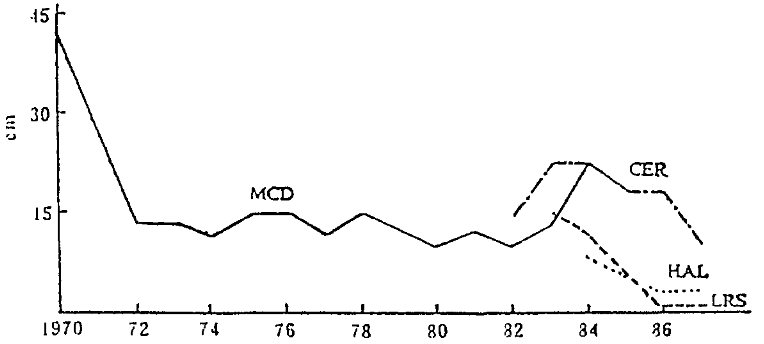

Figure 1. Precisions of normal points for each station.

2. Data used and accuracy of normal points

Total data of 6153 LLR normal points acquired during Aug. 1969–Dec. 1987 at the four stations were used in this study and 23 bad points were rejected.

The internal consistence of the data for each reflector and every year at four stations are listed in Table I and Figure 1.

The stations and reflector codes used are

Station	Reflector
MCD – McDonald 2.7 m	TRA – Tranquility
LRS – MLRS	FRM – Fra Manro
HAL – Haleakala	HAD – Hadley
CER – CERGA	LK_2 – Lunakhod 2

It is clear that the internal accuracy of the data has been improved from 3 ns to 0.1 ns.

3. Solutions

3.1. Reference frame

The barycentric reference frame is adopted and the planetary and lunar ephemerid-ces DE200/LE200, as well as the lunar librations LLB200 integrated simultaneously with the ephemeris were usedin our processing of the LLR data.

3.2. Observing equation

In the reduction of LLR, the observing equation can be written as follows:

$$\Delta\tau = \tau_o - \tau_c = \sum_{i=1}^{n} \left(\frac{\partial\tau_c}{\partial k_i}\right) \delta k_i + R_n + \varepsilon \tag{1}$$

Table I
Status of lunar laser ranging for each station during 1970–1987 (units: ms)

Station	Year	TRA		FRM		HAD		LK_2		Average	
		N	E	N	E	N	E	N	E	N	E
	1970	54	2.8							54	2.8
	1971	84	2.2	74	2.0	73	1.4			231	1.9
	1972	54	1.1	58	1.0	247	0.9			359	0.9
	1973	72	0.9	85	0.9	277	0.9	1	1.4	435	0.9
	1974	39	0.8	53	0.8	211	0.8	24	0.8	327	0.8
	1975	35	1.1	45	1.1	249	0.9	32	1.1	361	1.0
	1976	38	1.1	32	0.7	229	1.0	16	1.2	315	1.0
	1977	14	0.7	13	1.2	201	0.8	14	0.7	242	0.8
MCD	1978	14	0.9	21	0.09	166	1.0	21	1.0	222	1.0
	1979	22	0.8	38	0.7	199	0.8	15	0.8	274	0.8
	1980	31	0.7	57	0.7	200	0.7	4	0.7	292	0.7
	1981	19	0.6	31	0.7	143	0.8	5	0.7	198	0.8
	1982			1	0.7	11	0.7			12	0.7
	1983					28	0.9			28	0.9
	1984					74	1.5			74	1.5
	1985					50	1.2			50	1.2
	Average	476	1.4	508	1.1	2358	0.9	132	0.9		
	1983					44	1.2			44	1.2
	1984			8	0.8	228	0.8			236	0.8
LRS	1985					147	0.4			147	0.4
	1986	3	0.2	16	0.1	53	0.1			72	0.1
	1987	10	0.2	22	0.1	88	0.1			120	0.1
	Average	13	0.1	460.2	560	0.5					
	1982					59	1.0			59	1.0
	1983					232	1.5			232	1.5
	1984	13	1.3	15	1.2	310	1.6	17	1.2	355	1.5
CER	1985	53	1.2	48	1.3	545	1.2	75	1.2	721	1.2
	1986	4	1.2	12	1.3	100	1.2	8	1.3	124	1.2
	1987	1	1.2	13	0.8	28	0.7	14	0.6	56	0.7
	Average	71	1.2	88	1.2	1274	1.3	114	1.2		
	1984					6	0.6			6	0.6
	1985	9	0.3	10	0.4	188	0.4	5	0.3	212	0.4
HAL	1986	8	0.3	6	0.2	103	0.2	8	0.4	1256	0.2
	1987	6	0.2	4	0.2	126	0.2	1	0.3	137	0.2
	Average	23	0.3	20	0.3	423	1.3	14	0.4		

Note: N and E denote the number and rms of normal points, respectively.

where τ_o is the observational time delay; τ_c is the theoretical time delay; $\partial\tau_c/\partial k_i$ is the partial derivative of the time delay with respect to a series of adopted constant k_i; Δk_i is the correction of adopted constant in the reduction of LLR; R_n is the truncation error caused by developing as one order equation and ε the accidental error.

3.3. Partial Derivative

For an elliptical orbit, the partial derivative of the radial distance of the Moon with respect to mean anomaly M approximately:

$$\frac{\partial r}{\partial M} = a \cdot e \cdot \sin M \tag{2}$$

$$M = M_0 + \dot{M}_0(t - t_0) + \frac{1}{2}\ddot{M}_0(t - t_0)^2 \tag{3}$$

where a, e denote semi-major axis and eccentricity, t_0 is the reference epoch J2000.0

The secular acceleration in mean anomaly and mean longitude only differ by 1.5%, so the approximate expression $\dot{n} \approx \ddot{M}$ is used.

Finally, one may write approximately

$$\frac{\partial \rho}{\partial n} \cong \frac{1}{2} a \cdot e \cdot \sin M (t - t_0)^2 \tag{4}$$

3.4. Method

Using the weighted least-square adjustment, $\dot{n}$ is estimated simultaneously with about 60 parameters: observatory coordinates, selenographic coordinates of reflectors, lunar gravitational potential parameters, orbital elements of the Moon and the Earth, parameters of Moon's free librations and so on.

The method of stepwise regression and F-test were used in the processing of LLR data in order to overcome the uncertainty of solution due to the correlation between the parameters. The rejection of data were performed according to 3σ criterion.

Finally, a series of estimated parameters were observed.

4. Result and Discussion

4.1. Result

Analysing the whole data set spanning the interval from Aug. 1969 to Dec. 1987, we find the average secular acceleration of the moon in mean longitude to be

$$\dot{n} = -25''.4 \pm 0''.1/\text{cy}^2. \tag{5}$$

4.2. Comparison

Our result and the corresponding results derived by other authors were listed in Table II. It is in good agreement with those obtained by other authors using various methods.

Table II
Comparison of $\dot{n}$ values

$\dot{n}$ (″/cy^2)	Tech.	Span of data	Author
-26.9 ± 2.0	(1)	279 yr (1677–1973)	Morrison and Word (1975)
-27.2 ± 1.7	(1) and (2)		Mueller (1976)
-27.3 ± 5.2	(3)		Lambeck (1977)
-27.4 ± 3.0	(3)		Goad *et al.* (1978)
-25.0 ± 3.0	(3)	252d, 312d, 382d	Felaentreger *et al.* (1978)
-24.6 ± 5.0	LLR	7 yr (1969–1976)	Calame and Mulholland (1978)
-23.8 ± 4.0	LLR	8 yr (1969–1977)	Williams *et al.* (1978)
-25.3 ± 1.2	LLR	13 yr (1969–1982)	Dickey *et al.* (1983)
-25.4 ± 0.1	LLR	18 yr (1969–1987)	This paper

Note: (1) Transits of mercury; (2) ancient eclipses; (3) satellite measurements of the tide.

4.3. Discussion

(1) It is now commonly thought that the acceleration is due to the effect of tidal friction. Dissipational tidal friction in the oceans is known to provide the largest contribution to the deceleration in the lunar mean longitude. Further, only the second degree components of the ocean tide N_2 contribute significantly to this secular decay.

(2) Table II illustrates that the uncertainty $\sigma(\dot{n})$ estimated from LLR data has been decreasing rapidly with increasing the data span.

(3) The accuracy of our solutions is better than those derived using LLR by other authors because the span of data used in our analysis is longer than those used by them. On the other hand, the data have been weighted according to their error so that the more recent data have more influence.

(4) As we know, given equally spaced and weighted data over a time span T, the formal error would be expected to decrease roughly as $T^{-5/2}$, where T^{-2} results from a non-zero $\dot{n}$ and $T^{-1/2}$ comes from the statistics of evenly distributed measurements of equal accuracy.

The expectation of further decrease in the error of $\dot{n}$ with future improvement in the observation accuracy and increases in the data span is clear.

(5) The determination of $\dot{n}$ will be of great significance to realize the secular variation of earth rotation and study the evolutions of Earth–Moon system.

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