STATISTICAL NATURE OF PERIHELION PASSAGES OF LONG-PERIOD COMETS

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Abstract. The distributions of annual flux and time interval of perihelion passages of long-period comets are examined statistically. The former is found to be well represented by the Poisson distribution, while the latter by the exponential one. This means that the perihelion passages of long-period comets are an almost random phenomenon, at least within the time scale of a few hundred years, even if the samples are affected by observational selection and/or the effect of physical disruption.

It is well known that random and rare events like radioactive decay, occurrence of traffic accident or flow of cars can be suitably treated by the Poisson and the exponential distributions (Meyer, 1975). In this short note, a similar treatment has been applied to the data of perihelion passages of long-period comets (period: $P \geqslant 200$ yr). The data are taken from Marsden's orbital catalogue of comets (1979). It has been shown that behavior of orbital evolution of long-period comets is rather different from orbit to orbit (Nakamura, 1981). Hence, in order to see the effects of the difference of orbital evolution and observational selection, the following 12 sets of samples are analysed separately (q: perihelion distance, i: inclination):

TABLE I

Sample number		$0° \leqslant i < 50°$	$50° \leqslant i < 130°$	$130° \leqslant i \leqslant 180°$
all q	1800–1978	75	252	106
	1850–1978	63	217	91
$q \leqslant 2$ AU	1800–1978	64	224	87
	1850–1978	53	189	73

TABLE II

Mean annual flux		$0° \leqslant i < 50°$	$50° \leqslant i < 130°$	$130° \leqslant i \leqslant 180°$
all q	1800–1978	0.42	1.42	0.60
	1850–1978	0.49	1.70	0.71
$q \leqslant 2$ AU	1800–1978	0.36	1.26	0.49
	1850–1978	0.41	1.47	0.57

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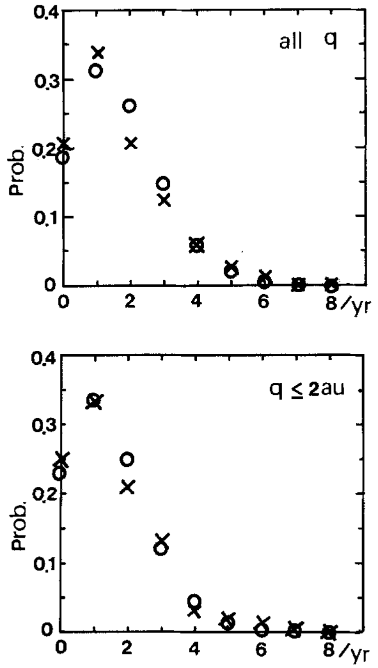

Fig. 1. Distributions of annual flux of perihelion passages for 1850–1978 and 50° $\leqslant i < 130°$. Crosses and circles indicate observed and theoretical (Poisson) values, respectively.

Since the time span of observation is shorter than 200 yr, there is no possibility that returns of the same comets contribute to the present statistics. For the above each set, the distributions of annual number and time interval of perihelion passages are examined.

In Table II are given the mean annual fluxes. Figure 1 shows two examples of the distributions of annual number of perihelion passage. Poisson distributions are fitted to the data with the same mean annual numbers as those in Table II. Differences between the observed distributions and the corresponding theoretical ones are analysed by χ-squares test. It is found that except for one data set, every difference is statistically insignificant. The difference for the data set of 1800–1978, 0° $\leqslant i < 50°$ and all q has been shown to be statistically significant at the level of 5%. However, this is probably due to observational selection, because the corresponding data set with $q \leqslant 2$ AU shows no marked difference. Thus, on the whole, it might be safely said that annual flux of perihelion passages of long-period comets can be well characterized by the Poisson distribution.

Next in Table III are given the mean values of time intervals of successive perihelion passages. Figure 2 shows the observed distribution of time interval for 1850–1978, 50° $\leqslant i < 130°$ and $q \leqslant 2$ AU, at a step of 100 days. The straight line in this figure is a fitted exponential distribution. The distribution of the form

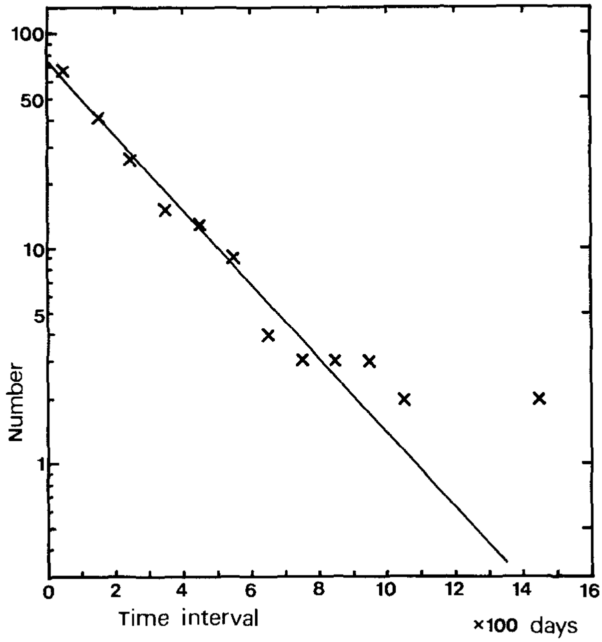

Fig. 2. Time interval distribution of perihelion passage for the data of 1850–1978, 50° $\leqslant i <$ 130° and $q \leqslant 2$ AU. As for a straight line in this figure, see text.

TABLE III

Mean time interval (day)		$0° \leqslant i < 50°$	$50° \leqslant i < 130°$	$130° \leqslant i \leqslant 180°$
all q	1800–1978	811	260	607
	1850–1978	749	198	512
$q \leqslant 2$ AU	1800–1978	997	287	751
	1850–1978	873	251	619

$$N \int_t^{t+100} \frac{1}{T_0} \exp\left(-t/T_0\right) \mathrm{d}t$$

has been fitted to each data set, where N is observed sample number, T_0 the mean time interval and t is expressed in unit of days. N and T_0 are taken from Tables I and III.

One can see in Figure 2 that the observed distribution is well represented by the exponential law at least for the time interval less than 900 days. Chi-squares test suggests that the deviation of crosses from the straight line is statistically insignificant. Nevertheless, a similar trend, that is, the excess over the exponential distribution for large time interval is seen for almost every data set, though the data points are more dispersed. If these deviations are real, the cause of them would be physical disruption of nuclei and/or

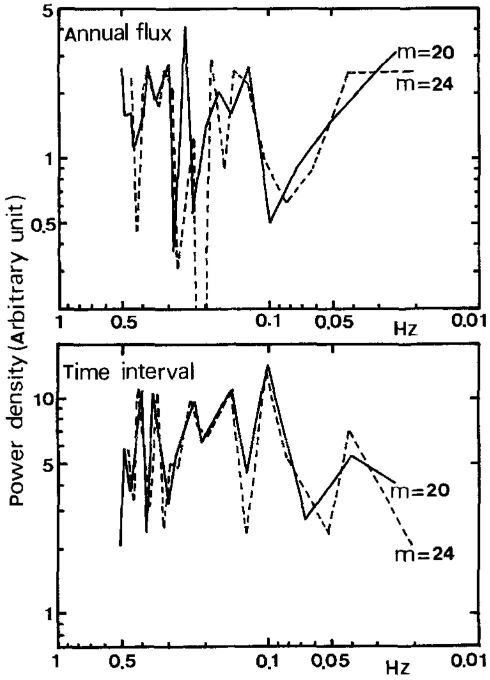

Fig. 3. Power spectra of annual flux and time interval for the data of 1850–1978, $50° \leqslant i < 130°$ and $q \leqslant 2$ AU. m is maximum lag number in Blackman–Tukey method.

observational selection. Let us assume tentatively that the original population of perihelion passages of long-period comets is expressed rigorously by the Poisson and exponential distributions. If the samples from these populations are affected by observational escape of count and/or physical loss of nuclei, time interval of perihelion passages will increase as a whole. Conversely, if the number of comets increase due to splitting of nuclei, the situation will reverse. As far as the above two effects take place *at random*, however, it can be proved that the resultant distributions are still the Poisson and exponential ones. Numerical simulation has also confirmed this result. Therefore, bend of the distribution at large time interval as shown in Figure 2, if real, may be caused by superposition of two populations with different mean time intervals.

Finally observed annual fluxes and time intervals of perihelion passages have been examined by a method of spectral analysis. Since perihelion passages of comets is not an intrinsically periodic phenomenon, we cannot of course expect to obtain any distinct periods from spectral analysis. In Figure 3 are shown the power spectra of annual flux and time interval for 1850–1978, $50° \leqslant i < 130°$ and $q \leqslant 2$ AU. We notice a dip in the upper figure and a peak in the lower one, both around 0.1 Hz. However, it will be premature to say whether these dip and peak are real or not. In this connection, analysis of synthesized data has occasionally shown similar dips and peaks around this frequency.

In conclusion, it is suggested that the intrinsic character of perihelion passages of long-period comets is almost completely of random nature, at least within the time scale of a few hundred years.

References

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