PROTOSTARS AND THE ORIGIN OF THE ANGULAR MOMENTUM OF THE SOLAR SYSTEM

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(Received 21 March, 1984)

Abstract. Protostars in a group exert gravitational tidal torques on an aspherical nebula located in the group. The net torque transfers angular momentum from the orbital motions of the stars to rotation of the nebula. A relation can be derived between the parameters describing the protostars and the final angular momentum of the nebula. While the parameters concerned are uncertain, a conservative choice results in a value for the angular momentum equal to about 1/3 of that of the present solar system. This suggests that if the Sun formed in a group, tidal interactions with other protostars may account for a significant part of the angular momentum of the solar system.

1. Introduction

In recent years it has become plausible that most – and perhaps all – stars form in groups; with single stars being escapers from such groups (see below). In a group of protostars, gravitational tidal interactions can have significant effects on any aspherical condensation located in the group. Especially, the protostars will exert a net torque on a nebula of the type destined to become a star with a planetary system. The main effect of this torque is to transfer angular momentum from the orbital motions of the stars to rotation of the nebula. In what follows, a simple model is used to derive a relation between the parameters describing the protostars and the final angular momentum of the nebula. This relation is then applied to the problem of the origin of the angular momentum of the solar system.

Observational work has revealed the existence of several regions wherein stars appear to be forming (see, e.g., Elias, 1978; Jaffe and Fazio, 1982; Myers, 1982). These regions typically contain about 20 stars in a volume a few parsecs across. In such a group, it is inevitable that tidal interactions have a signficant effect on any aspherical condensation located in the group. However, the role of such interactions is at present poorly understood. Theoretical work on star formation has traditionally been largely concerned with the collapse of a rotating cloud and the fate of the associated angular momentum (see De Jong and Maeder, 1977, and Bodenheimer, 1981, for reviews; and Boss, 1980, 1981, for references to recent work). It has commonly been assumed that the angular momentum of a star, and its associated planets if there are any, is inherited more or less directly from the rotation of the parent cloud. This assumption may be valid, but while some interstellar clouds are observed to have significant rotation, others do not (see Bodenheimer, 1981; p. 8 for a review). Also, where several protostars form in the same region, there must clearly be cases where the effects of tidal interactions cannot be neglected. In

Earth, Moon, and Planets **30** (1984) 275–280. 0167–9295/84/0303–0275$00.90.

particular, recent work has shown that tidal interactions can play a part in the formation of binary stars (Boss, 1981), and account for the origin and angular momentum of the solar nebula (Kobrick and Kaula, 1979). The fact that the Sun is a single star might at fist sight seem to imply that it formed in isolation, from the monogenic collapse of an interstellar cloud. But it is also plausible that the Sun formed as a member of a group, and that gravitational encounters with its neighbours led to its subsequent escape from the group (Kobrick and Kaula, 1979). The angular momentum of the present solar system is $3 \times 10^{50}\,\mathrm{g\,cm^2\,s^{-1}}$ approximately, and is the only concrete datum available on the angular momentum of a star with a planetary system. While it is *not* the intention to argue that this angular momentum originated solely from tidal interactions, it is felt that there is sufficient motivation for studying the major effects of such interactions in a protostellar group, and their implications for angular momentum especially.

It is useful to differentiate two aspects of tidal interactions which, while complementary, require different mathematical analyses for their elucidation: (i) In the case where there is a close encounter between a *single* protostar and a nebula, a net torque exists if the line from the protostar to the centre of the nebula and the semi-major axis of the nebula do not coincide. This torque can transfer angular momentum to the nebula during the encounter, and is analgous to the situation in the Earth/Moon system. The effects of this type of interaction have been studied in an important paper by Kobrick and Kaula (1979). (ii) In the case where the protostars are distributed randomly in a group which contains a nebula, a net torque exists due to the mean gravitational field of *all* the protostars. This torque transfers angular momentum to the nebula because the latter tries to orientate itself so as to reduce the torque it feels, and when the torque is cut off by the collapse of the nebula (see below) it is left with a net rotation. The effects of this type of interaction have been studied by Peebles (1969), Wesson (1982) and others.

The cases (i) and (ii) outlined above are not mutually exclusive, but (i) depends on a special event (a close encounter) while (ii) is present generally. Therefore, (ii) is concentrated on in what follows. It should be emphasized that the following account is preliminary: the aim is only to illustrate the major effects of this type of tidal interaction. A suggestion for a more sophisticated analysis is made in the last section.

2. A Simple Model

The effect of a group of protostars on a nebula can be investigated by applying a simple model of Peebles (1969; see also Wesson, 1982). Consider first the effect of one protostar of mass M_2 on a nebula of mass M_1. The latter is assumed to be spheroidal in shape, with semi-axes a, b and eccentricity $e \equiv (a^2 - b^2)^{1/2}/a$. A comment is made below on the origin of this initial asphericity. The angle between the line from the protostar to the centre of the nebula and the semi-minor axis of the nebula is θ. The distance from the protostar to the centre of the nebula is r, and it can be assumed that $r \gg a$ so the protostar can be treated as a point. Then the magnitude of the torque τ due to one protostar on the nebula is

$$\tau = \frac{kGM_1M_2e^2a^2 \sin 2\theta}{r^3}, \tag{1}$$

where k is a numerical constant ($k = 0.3$ for a homogeneous nebula according to the model of Peebles), and G is the gravitational constant.

The effect of many protostars on the nebula can be found by obtaining the root mean square (RMS) torque from (1). Let P be the probability of finding a protostar in a volume element $\mathrm{d}V$; and let $\langle\rangle$ denote the mean value of a quantity, averaged over the protostars. Then

$$\langle\tau^2\rangle = (kGM_1e^2a^2)^2\langle M_2^2\rangle\langle\sin^2 2\theta\rangle \int \frac{P\,\mathrm{d}V}{r^6}, \tag{2}$$

where the integral depends on how the protostars are distributed in the group. Let it be assumed that they are distributed randomly and homogeneously, with mean separation r_m. Then each star has associated with it a volume $4\pi d^3/3$, where $d = r_m/2$. Thus $P = (4\pi d^3/3)^{-1}$. Also, the lower limit of the integral in (2) is r_m, since, on the average, the nearest protostar will be at this distance from the nebula. The upper limit of the integral in (2), which can be denoted by r_0, will be of the order of the radius of the protostellar group. But it is not necessary to define this precisely, because $r_0 \gg r_m$ in any case, so by (2) its contribution to the integral is negligible. It is now straightforward to evaluate (2) and take its square root to get the RMS torque $\Gamma \equiv \langle\tau^2\rangle^{1/2}$. The latter is

$$\Gamma = \frac{2kGM_1M_*e^2a^2}{r_m^3}; \tag{3}$$

in which $M_* \equiv \langle M_2^2\rangle^{1/2}$ is the RMS protostar mass.

The RMS torque Γ of (3) does not act indefinitely. It will disappear if the nebula collapses, if the nebula dissipates, or if the protostellar group disintegrates. Since the first of these eventualities must anyhow precede the others, let it be assumed that Γ is effectively cut off after a time t_{eff} by the collapse of the nebula. For times $0 < t < t_{\mathrm{eff}}$, the torque has a value given by (3) with the instantaneous values of e and a. The angular momentum transferred to the nebula by the action of the torque (3) is given by

$$J = \int \Gamma\,\mathrm{d}t = \frac{2kGM_1M_*}{r_m^3}\int_0^{t_{\mathrm{eff}}} e^2a^2\,\mathrm{d}t. \tag{4}$$

This can unfortunately not be evaluated as it stands, because $e = e(t)$ and $a = a(t)$ are unknown. Numerical work indicates that e increases as a decreases, loosely speaking (Regev and Shaviv, 1981; Boss and Haber, 1982). But the behaviour of these parameters is not well enough known to allow (4) to be evaluated precisely. As an approximation, let $\bar{e}$ and $\bar{a}$ be the average values of these parameters, and write (4) as

$$J = \frac{2kGM_1M_*\bar{e}^2\bar{a}^2t_{\mathrm{eff}}}{r_m^3}. \tag{5}$$

In this expression, t_{eff} has to be chosen in conformity with the collapse dynamics of the nebula. Numerical work like that just quoted and that quoted in Section 1 shows that, under a wide variety of circumstances, a collapsing nebula evolves on a timescale of a few times the free-fall time, t_{ff}. In particular, numerical work on a model with parameters similar to those employed below shows that the shape of a collapsing nebula evolves on a timescale of just over $2t_{ff}$ (Boss, 1981, p. 872). In view of these results, a reasonable approximation is to put $t_{\mathrm{eff}} \simeq t_{ff} \simeq (\pi^2 \bar{a}^3/8GM_1)^{1/2}$ in (5), where $e \ll 1$ has been assumed. This gives

$$J = \frac{\pi k M_* \bar{e}^2}{r_m^3} \left(\frac{GM_1 \bar{a}^7}{2} \right)^{1/2} . \tag{6}$$

This is the angular momentum transferred to the nebula by tidal interactions with the surrounding protostars. The relation (6) makes use of several approximations, but they are reasonable ones, and it is expected to be accurate to at least order of magnitude.

For the purpose of illustration, the following plausible values may be substituted in (6): $M_* = M_1 = 1M_\odot$, $\bar{e} = 0.1$, $\bar{a} = r_m/10 = 0.1$ pc. These give $J \simeq 10^{50}\,\mathrm{g\,cm^2\,s^{-1}}$. The size of t_{eff} for this choice of parameters is $t_{\mathrm{eff}} \simeq t_{ff} \simeq 5 \times 10^5$ yr. The size of J for alternate choices of this parameter and the other parameters may easily be obtained from (5) and (6) by scaling. For the noted choices, J of (6) is about 1/3 of the angular momentum of the present solar system.

3. Conclusions

A relation (6) has been derived between the parameters describing a group of protostars and the angular momentum which a nebula located in the group acquires by tidal interactions with the protostars. While the parameters concerned are uncertain, a conservative choice results in a value for the angular momentum equal to about 1/3 of that of the present solar system.

It has been implicitly assumed above that the nebula concerned in the calculation ultimately forms one or more stars and perhaps planets as well. However, before applying (6) to real stellar systems and the solar system in particular, some cautionary comments are in order. (a) It should be recalled that the tidal mechanism for the acquisition of angular momentum outlined in Section 2 only works for a nebula located in a relatively dense group of protostars. To apply the result (6) to a single star like the Sun requires the additional assumption that it formed in such a group and subsequently escaped. This topic is not fully understood at the present time. The relation (6) in effect gives the angular momentum transferred from orbital motions of the protostars to rotation of the nebula, and it may be that this process itself significantly influences the dynamics of the group and the chances of escape. (b) The tidal mechanism of Section 2 presumes an initial asphericity for the nebula concerned. It is of course almost inevitable that a nebula is asperhical to some degree when it forms, and to this extent the transfer of an amount of angular momentum of the order of (6) is also inevitable. But it could be the case that part of the initial asphericity is due to prior rotation of the nebula, and if so the angular momentum associated with this has to be added to that of (6) in order to get the total.

This case is quite plausible, and implies that the observation of finite rotations for some interstellar clouds are not necessarily in conflict with the mechanism outlined in Section 2. However, in this case the calculation of the total angular momentum would be complicated, because the part associated with the prior rotation and the part transferred by tidal interactions would not in general agree in direction. (c) In order for the angular momentum given by any calculation to be compared to the angular momentum of a real stellar system, an assumption has to be made about conservation. In much numerical work on the collapse of nebulae, it is assumed that angular momentum is conserved (see, e.g., Bodenheimer, 1981; p. 9). However, while this assumption and the others made in Section 2 may be plausible, they are all open to some doubt. To test both the assumptions on which it rests and the applicability of (6) to real stellar systems, a more sophisticated analysis is required. In view of the complexity of the problem, this would presumably take the form of a numerical simulation. As mentioned in Section 1, the tidal interactions that have been studied by Kobrick and Kaula (1979) and those studied here are really complementary, and a simulation would have the added advantage of treating them as such.

Notwithstanding the comments of the preceding paragraph, some qualified conclusions can be drawn about the comparison of (6) with the solar system. If the nebula from which the Sun and the planets formed was at one time a member of a group of protostars, then it would have acquired of the order of 10^{50} g cm^2 s^{-1} in angular momentum by tidal interactions. While the collapse of the nebula has not been treated in detail, it is plausible that the accretion of material with finite angular momentum was responsible for the alignment of the solar rotation and the orbital rotations of the planets (Kobrick and Kaula, 1979). However, no comment can be made about the specific processes whereby the Sun became a discrete object, the planets aggregated and obtained their orbits and spins, and the initial angular momentum was redistributed to have the form observed in the present solar system. The processes responsible for these things must have been very different in nature from the tidal interactions treated above (see Giuli, 1968a, b, Wesson and Lermann, 1978, and Wesson, 1979, for references). But ignorance about these processes can be circumvented if conservation of angular momentum is assumed. In this case, it can be concluded that a significant part of the angular momentum of the present solar system (3×10^{50} g cm^2 s^{-1}) may have originated from tidal interactions with protostars.

Acknowledgements

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

References

Bodenheimer, P.: 1981, in Sugimoto, D., Lamb, D. Q., and Schramm, D. N. (eds.), 'Fundamental Problems in the Theory of Stellar Evolution', *IAU Symp.* **93,** D. Reidel Publ. Co., Dordrecht, Holland.

Boss, A. P.: 1980, *Astrophys. J.* **237**, 563.
Boss, A. P.: 1981, *Astrophys. J.* **246**, 866.
Boss, A. P. and Haber, J. G.: 1982, *Astrophys. J.* **255**, 240.
De Jong, T. and Maeder, A. (eds.): 1977, 'Star Formation', *IAU Symp.* **75**, D. Reidel Publ. Co., Dordrecht, Holland.
Elias, J. H.: 1978, *Astrophys. J.* **224**, 453.
Giuli, R. T.: 1968a, *Icarus* **8**, 301.
Giuli, R. T.: 1968b, *Icarus* **9**, 186.
Jaffe, D. T. and Fazio, G. G.: 1982, *Astrophys. J.* **257**, L77.
Kobrick, M. and Kaula, W. M.: 1979, *The Moon and the Planets* **20**, 61.
Myers, P. C.: 1982, *Astrophys. J.* **257**, 620.
Peebles, P. J. E.: 1969, *Astrophys. J.* **155**, 393.
Regev, O. and Shaviv, G.: 1981, *Astrophys. J.* **245**, 934.
Wesson, P. S.: 1979, *Astron. Astrophys.* **76**, 200.
Wesson, P. S.: 1982, *Vistas in Astron.* **25**, 411.
Wesson, P. S. and Lermann, A.: 1978, *Icarus* **33**, 74.