RAYLEIGH WAVES IN MAGNETO-THERMO-MICROELASTIC HALF-SPACE UNDER INITIAL STRESS

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Abstract. The aim of the present paper is to investigate the influence both of intitial stress and magnetic field on the propagation of Rayleigh waves in thermo-microelastic half-space subjected to certain boundary conditions. The wave velocity equation has been obtained. If the initial stress and the electromagnetic field are ignored, the frequency equation as obtained by Locket (1958).

Introduction

The problem of propagation of the electromagneto thermo-microelastic waves in electrically and thermally conducting solids are very important for the possibility of its extensive practical applications in various branches of science and technology, particularly in optics, acoustics, geophysics and plasma physics.

Many works on the subject are reviewed by Paria (1967). Recently, the works Tomita and Shindo (1979) investigated the influence of magnetic field upon the phase velocity of Rayleigh waves, Nowacki (1986) discussed the equations of thermo-electric-magneto-elasticity, Nowacki *et al.* (1969) investigated the generation of waves in an infinite micropolar elastic solid body.

In the present paper the wave propagation over the surface of a semi-infinite homogeneous, isotropic electro-magneto-thermo-microelastic solid with initial stress was considered. The object of this paper is to show the effect of the magnetic field and initial stress upon the phase velocity of Rayleigh waves. The frequency equation has been derived. The roots of this equation are in general complex and the imaginary part of an appropriate root measures the attenuation of the waves. It is noticed that the frequency of Rayleigh waves contains the term which involving the initial stress and electric conductivity. When the electromagnetic field is ignored, the frequency equation, for Rayleigh waves of a thermo-microelastic under initial stress case has formula, which is similar to that obtained by Elnaggar and Abd-Alla (1987).

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1. Formulation of the Problem

We consider a semi-infinite micro-elastic medium, occupying the half-space $x_2 \geqslant 0$ under initial stress P, embedded in a constant primary magnetic field H_3, acts in the positive direction of x_3-axis, and also disturbed from its initial state, subjected to certain boundary conditions.

The dynamic equations of motion in the absence of body forces can be written (cf. Nowacki, 1986) as

$$(\mu+\alpha)\nabla^2\mathbf{u}+(\lambda+\mu)\nabla(\nabla\cdot\mathbf{u})+2\alpha\,\mathrm{curl}\,\mathbf{u}-P\,\mathrm{curl}\,\boldsymbol{\omega}+\mu_0(\mathrm{curl}\,\mathbf{h}\times\mathbf{H})=$$
$$=\gamma\,\mathrm{grad}\,T+\rho\frac{\partial^2\mathbf{u}}{\partial t^2},$$
$$(r-\epsilon)\nabla^2\boldsymbol{\omega}-4\alpha\boldsymbol{\omega}-I\frac{\partial\boldsymbol{\omega}}{\partial t^2}+(\beta+F-\epsilon)\nabla(\nabla\cdot\boldsymbol{\omega})+2\alpha\,\mathrm{curl}\,\mathbf{u}=0\,, \tag{1}$$

equations of electro-dynamic have the form (cf. Nowacki, *op. cit.*) as

$$\mathrm{curl}\,\mathbf{E}=-\mu\frac{\partial\mathbf{h}}{\partial t},\qquad \mathrm{curl}\,\mathbf{h}=\mathbf{J},\quad \mathrm{div}\,\mathbf{h}=0\,,$$
$$\mathbf{J}=\lambda_0\left[\mathbf{E}+\mu_0\left(\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{H}\right)\right]-\nu_1\,\mathrm{grad}\,\theta\,, \tag{2}$$

and the heat conduction equation is

$$\left(\nabla^2-\frac{1}{k}\frac{\partial}{\partial t}\right)T-\eta\,\mathrm{div}\frac{\partial\mathbf{u}}{\partial t}-\frac{\Pi_0}{K}\mathrm{div}\,\mathbf{J}+\frac{Q}{K}=0\,, \tag{3}$$

where

$$\eta=\frac{\gamma T_0}{K},\qquad k=\frac{K}{\rho C_e},\qquad \gamma=(3\lambda+2\mu)\alpha_t\,;$$
$$\nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}\,;$$

$\mathbf{h}$, $\mathbf{E}$ denote vectors of perturbed intensities of magnetic and electric fields, and $\mathbf{J}$ stands for the vector of current density, $\mathbf{H}$ is the vector of the original constant magnetic field, $\mathbf{u}$ is the displacement vector, $\boldsymbol{\omega}$ is the rotation vector, Q is the intensity of heat source, μ_0 is the magnetic permeability, λ_0 is the electric conductivity, ν_1 is the coefficient linking the electric field with the temperature gradient, Π_0 is the coefficient relating velocity vectors with that of heat flow, $T=\theta-T_0$ is the temperature increment measured from the natural state, K is the heat conductivity, c_e is the specific heat and α, β, F, ϵ, μ, λ are the natural constants.

The components of stress under initial stress are given (cf. Boit, 1965) by

$$S_{11}=(\lambda+2\mu+P)\frac{\partial u_1}{\partial x_1}+(\lambda+P)\frac{\partial u_2}{\partial x_2}-\gamma T\,,$$

$$S_{22}=\lambda\frac{\partial u_1}{\partial x_1}+(\lambda+2\mu)\frac{\partial u_2}{\partial x_2}-\gamma T\,, \tag{4}$$

$$S_{12}=\mu\left(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}\right)-\alpha\left(\frac{\partial u_1}{\partial x_2}-\frac{\partial u_2}{\partial x_1}\right).$$

The components of couple stress are given (cf. Nowacki, 1969) as

$$\mu_{11}=2F\frac{\partial w_1}{\partial x_1}+\beta\tilde{k}\,,$$

$$\mu_{22}=2F\frac{\partial w_2}{\partial x_2}+\beta\tilde{k}\,, \tag{5}$$

$$\mu_{12}=F\left(\frac{\partial w_3}{\partial x_1}+\frac{\partial w_1}{\partial x_2}\right)+\epsilon\left(\frac{\partial w_1}{\partial x_2}-\frac{\partial w_3}{\partial x_1}\right);$$

where

$$\tilde{k}=\frac{\partial w_1}{\partial x_1}+\frac{\partial w_2}{\partial x_2},\qquad w_1=\frac{1}{2}\left(\frac{\partial u_3}{\partial x_2}\right),\qquad w_2=\frac{1}{2}\left(\frac{\partial u_3}{\partial x_1}\right),$$

$$w_3=\frac{1}{2}\left(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2}\right). \tag{6}$$

2. Solution of the Problem

Let us consider two-dimensional problem under the assumption that all the causes and the effects depend on the variables x_1, x_2 and t, and that primary magnetic field is parallel to x_3-axis – i.e., $\mathbf{H}=(0,0,H_3)$. Equations (1) can be separated into two independent sets of equations.

The first set takes the form

$$\Box_2 u_1+(\lambda+\mu+P-\alpha)\frac{\partial}{\partial x_1}e-2\alpha\frac{\partial w_3}{\partial x_2}-\mu_0 H_3\frac{\partial h_3}{\partial x_1}=\gamma\frac{\partial T}{\partial x_1}\,,$$

$$\Box_2 u_2+(\lambda+\mu-\alpha)\frac{\partial e}{\partial x_2}-2\alpha\frac{\partial w_3}{\partial x_1}-\mu_0 H_3\frac{\partial h_3}{\partial x_2}=\gamma\frac{\partial T}{\partial x_2}\,, \tag{7}$$

$$\Box_4 w_3+2\alpha\left(\frac{\partial u_1}{\partial x_1}-\frac{\partial u_1}{\partial x_2}\right)=0\,.$$

The second set becomes

$$\Box_4 w_1+(\beta+\lambda-\epsilon)\frac{\partial\tilde{k}}{\partial x_1}+2\alpha\frac{\partial u_3}{\partial x_2}=0\,,$$

$$\Box_4 w_2 + (\beta + \lambda - \epsilon)\frac{\partial \tilde{k}}{\partial x_2} - 2\alpha \frac{\partial u_3}{\partial x_1} = 0\,, \tag{8}$$

$$\Box_2 u_3 + 2\alpha\left(\frac{\partial w_2}{\partial x_1} - \frac{\partial w_1}{\partial x_2}\right) = 0\,;$$

where

$$\Box_2 = (\mu + \alpha)\nabla_1^2 - \rho\frac{\partial^2}{\partial t^2},$$

$$\Box_4 = (\lambda + \epsilon)\nabla_1^2 - 4\alpha - I\frac{\partial^2}{\partial t^2}, \tag{9}$$

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}, \qquad e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}.$$

In the second set of equations the magnitude u_3, w_1, w_2 are unperturbed by the fields of temperature and the electromagnetic field. Thus the first set of equations perturbed by both these fields should be taken into account. As h_1 and h_2 are respectively, equal to zero initially, by eliminating the vector **E** and **J** from Equation (2) we have

$$\left(\nu_H \nabla_1^2 - \frac{\partial}{\partial t}\right) h_3 = H_3 \frac{\partial}{\partial t} e\,. \tag{10}$$

Using Helmholtz theorem (cf. Morse and Feshbach, 1953) and introducing the potential ϕ and ψ by the equation

$$\mathbf{U} = (u_1, u_2, 0) = \operatorname{grad}\phi + \operatorname{curl}(0, 0, \psi)\,, \tag{11}$$

then from Equations (7) and (11) we get the following equations:

$$\left[(\lambda + 2\mu + P)\nabla_1^2 - \rho\frac{\partial^2}{\partial t^2}\right]\phi - \mu_0 H_3 h_3 = \gamma T\,, \tag{12}$$

$$\left[(\lambda + 2\mu)\nabla_1^2 - \rho\frac{\partial^2}{\partial t^2}\right]\phi - \mu_0 H_3 h_3 = \gamma T\,, \tag{13}$$

$$\left[\left(\mu + \frac{P}{2} + \alpha\right)\nabla_1^2 - \rho\frac{\partial^2}{\partial t^2}\right]\psi + 2\alpha w_3 = 0\,, \tag{14}$$

$$\left[\left(\mu - \frac{P}{2} + \alpha\right)\nabla_1^2 - \rho\frac{\partial^2}{\partial t^2}\right]\psi + 2\alpha w_3 = 0\,, \tag{15}$$

$$\left[(\lambda + \epsilon)\nabla_1^2 - 4\alpha - I\frac{\partial^2}{\partial t^2}\right] w_3 - 2\alpha\nabla_1^2\psi = 0\,. \tag{16}$$

In the absence of a heat source, Equations (10) and (3) can be written as

$$\left(\nu_H \nabla_1^2 - \frac{\partial}{\partial t}\right) h_3 = H_3 \frac{\partial}{\partial t}\nabla_1^2\phi\,, \tag{17}$$

$$\left[\nabla_1^2 - \frac{1}{K}\frac{\partial}{\partial t}\right] T = \eta \frac{\partial}{\partial t} \nabla_1^2 \phi \,. \tag{18}$$

We shall consider compressional and distortional waves along the x_1-axis only. These waves are represented by Equations (12), (15), and (16). From Equations (12), (17), (18) we obtain the wave equation:

$$\left[D_1 D_2 D_3 - \frac{\partial}{\partial t}\nabla_1^2\left(\eta m D_1 + \frac{R_H D_2}{\nu_H}\right)\right](\phi, h_3, T) = 0\,, \tag{19}$$

where the operators

$$D_1 = \nabla_1^2 - \frac{1}{\nu_H}\frac{\partial}{\partial t}, \quad D_2 = \nabla_1^2 - \frac{1}{K}\frac{\partial}{\partial t},$$

$$D_3 = \nabla_1^2 - \frac{\rho}{(\lambda + 2\mu + P)}\frac{\partial^2}{\partial t^2}; \quad m = \frac{\gamma}{\rho c_1^2},$$

$$R_H = \mu_0 H_3 / \rho c_1^2 \,.$$

For a plane harmonic wave propagation in the x_1-direction we write

$$\begin{aligned} \phi(x_1, x_2, t) &= \phi(x_2)\exp\left[iw\left(\frac{x_1}{c} - t\right)\right], \\ T(x_1, x_2, t) &= T(x_2)\exp\left[iw\left(\frac{x_1}{c} - t\right)\right], \\ h(x_1, x_2, t) &= h_3(x_2)\exp\left[iw\left(\frac{x_1}{c} - t\right)\right]. \end{aligned} \tag{20}$$

Substituting from Equations (20) into Equation (19) we obtain

$$\begin{aligned} &\frac{\mathrm{d}^6\phi(x_2)}{\mathrm{d}x_2^6} - \left(s_1 + \frac{w^2}{c^2} - \frac{iw}{K}\right)\frac{\mathrm{d}^4\phi(x_2)}{\mathrm{d}x_2^4} + \left(\frac{i\epsilon_T w y_1}{K_1} + \frac{w^2}{c^2}(s_1 + s_2) - \frac{iws_1}{K_1}\right) \times \\ &\quad \times \frac{\mathrm{d}^2\phi(x_2)}{\mathrm{d}x_2^2} - \left(\frac{w^4}{c^4}s_2 - \frac{iw^3}{c^2 K_1}(\epsilon_T y_1 - s_2)\right)\phi(x_2) = 0\,, \end{aligned} \tag{21}$$

where

$$y_1 = iw\frac{(K_1 - \nu_H)}{K_1 \nu_H},$$

$$s_1 = \frac{2w^2}{c^2} - \frac{\rho w^2}{(\lambda + 2\mu + P)} - \frac{iw}{\nu_H}(1 + R_H) - \frac{i\epsilon_T w}{K_1},$$

$$s_2 = \left(\frac{w^2}{c^2} - \frac{iw}{\nu_H}\right)\left(1 - \frac{\rho c^2}{(\lambda + 2\mu + P)} - \frac{iR_H w}{\nu_H} - \frac{i\epsilon_T w}{K_1}\right);$$

$\epsilon_T = \rho T_0\, \gamma^2 / w^2\, c_e(\lambda + 2\mu + P)$ and c is the phase velocity of Rayleigh waves in the thermo-magneto-microelastic medium under initial stress.

The solution of Equation (21) satisfying the condition that the corresponding stress vanish as $x_2 \to \infty$ is given by

$$\phi = A_1 \, e^{-\xi_1 x_2} + A_2 \, e^{-\xi_2 x_2} + A_3 \, e^{-\xi_3 x_2} , \tag{22}$$

where

$$\xi_j^2 = \frac{w^2}{c^2} - \xi_j^2, \quad R_e(\xi_j) \geq 0 \quad \text{and} \quad j = 1, 2, 3 ;$$

and ξ_1, ξ_2, and ξ_3 are the roots of the equation

$$\xi^6 - \left[\frac{\rho w^2}{(\lambda + 2\mu + P)} + \frac{iw}{\nu_H}(1 + R_H) + \frac{iw}{K_1}(1 + \epsilon_T)\right] \xi^4 + \\ + \left[\frac{iw^3}{\nu_H c_1^2} + \frac{iw^3}{K_1 c_1^2} + \frac{w^2}{K_1 \nu_H}(1 + \epsilon_T + R_H)\right] \xi^2 + \frac{\rho w^4}{K_1 \nu_h (\lambda + 2\mu + P)} = 0 . \tag{23}$$

Introducing in (23) the dimensionless quantities

$$x = \frac{w}{w^*}, \quad \epsilon_H = \rho \nu_H w^* / (\lambda + 2\mu + P), \quad c = v \quad \text{and} \quad c\xi / w = \rho$$

$$w^* = (\lambda + 2\mu + P) / \rho K.$$

Equation (23) takes the form

$$l^6 - \left\{\frac{\rho v^2}{(\lambda + 2\mu + P)} + \frac{i(1 + R_H)\rho v^2}{\epsilon_H x (\lambda + 2\mu + P)} + \frac{i(1 + \epsilon_T)}{x} \frac{\rho v^2}{(\lambda + 2\mu + P)}\right\} l^4 + \\ + \left\{\frac{i}{\epsilon_H x} + \frac{i}{x} - \frac{(i + \epsilon_T + R_H)}{\epsilon_H x^2}\right\} \frac{\rho^2 v^4}{(\lambda + 2\mu + P)^2} l^2 + \frac{\rho^3 v^2}{\epsilon_H (\lambda + 2\mu + P)^3} \frac{1}{x^2} \\ = 0 . \tag{24}$$

Similarly we consider the last two equations of (15) and (16), and eliminating w_3 from them. For harmonic wave propagation in the x_1-direction, one can write

$$\psi = \psi(x_2) \exp\left[iw\left(\frac{x_1}{c} - t\right)\right] . \tag{25}$$

Substituting (25) into Equations (15) and (16) after eliminating w_3 we obtain the solution of the form

$$\psi(x_2) = B_4 \, e^{-\xi_4 x_2} + B_5 \, e^{-\xi_5 x_2} , \tag{26}$$

where:

$$\xi_j^2 = \frac{w^2}{c^2} - \xi_j^2, \quad R_e(\xi_j) \geq 0, \quad j = 4, 5 ;$$

and ξ_4^2 and ξ_5^2 are the roots of the equation

$$\xi^4-\left\{\frac{\rho w^2}{\mu-(P/2)+\alpha}+\frac{w^2}{c_4^2}-\nu_1^2+\eta_1^2\right\}\xi^2+\frac{\rho w^2}{u-(P/2)+\alpha}\left(\frac{w^2}{c_4^2}-\nu_1^2\right)=0\,,$$

$$\nu_1^2=\frac{4\alpha}{\lambda+\epsilon},\quad \eta_1^2=\frac{4\alpha^2}{(\lambda+\alpha)(F+\epsilon)}, \tag{27}$$

$$c_4^2=\frac{F+\epsilon}{I}\,.$$

Then we can obtain the solution in the form

$$\phi=\sum_{j=1}^{3}A_j\,\mathrm{e}^{-\beta m_j x_2}\exp\left[iw\left(\frac{x_1}{c}-t\right)\right],$$

$$\psi=\sum_{j=4}^{j=1}B_j\,\mathrm{e}^{-\beta m_j x_2}\exp\left[iw\left(\frac{x_1}{c}-t\right)\right],$$

$$T=\frac{\rho c_1^2}{y_1\gamma}\left[\sum_{j=1}^{3}A_j[(\beta^2 m^4 j-s_1 m_j^2+s_2)\,\mathrm{e}^{-\beta m_j x_2}\right]\exp\left[iw\left(\frac{x_1}{c}-t\right)\right]\Bigg], \tag{28}$$

$$h_3=\frac{H_3}{y_1 R_H}\left[\sum_{j=1}^{3}A_j[\beta^2 m_j^4-(s_1+y_1)m_j^2+s_2\right.$$
$$\left.+\,y_1\left(1-\frac{\rho v^2}{\lambda+2\mu+p}\right)\right]\mathrm{e}^{-\beta m_j x_2}\exp\left[iw\left(\frac{x_1}{c}-t\right)\right]\Bigg],$$

where $m_j^2=1-L_j^2$, $j=1,2,3$ and L_j^2, $j=4,5$, are the roots of Equations (24) and (27), respectively.

In terms of the potential ϕ and ψ the stress components and couple stress are given by

$$S_{11}=(\lambda+P)\nabla^2\phi+2\mu\frac{\partial^2\phi}{\partial x_1^2}+2\mu\frac{\partial^2\psi}{\partial x_1\,\partial x_2}-\gamma T\,,$$

$$S_{12}=2\mu\frac{\partial^2\phi}{\partial x_1\,\partial x_2}+\mu\left(\frac{\partial^2\psi}{\partial x_2^2}-\frac{\partial^2\psi}{\partial x_1^2}\right)-\alpha\nabla^2\psi\,, \tag{29}$$

$$\mu_{12}=\frac{1}{2}(\epsilon-F)\left[\frac{\partial^3\psi}{\partial x_1^3}+\frac{\partial^3\psi}{\partial x_1^2\,\partial x_2^2}\right].$$

3. Frequency Equation

In this section frequency equation for the boundary conditions on the plane $x_2=0$ are

$$S_{11}=S_{12}=\mu_{12}=0$$

and

$$\frac{\partial T}{\partial x_1} = h_2 = 0 \begin{cases} \text{at } x_2 = 0\,, & (30) \\ \text{at } x_2 = 0\,. & (31) \end{cases}$$

The last two boundary conditions indicate that the medium is thermally insulated and maintains the primary magnetic field H_3 at all times.

If we eliminate the constants A_i, $(i = 1, 2, 3)$ and $B_j (j = 4, 5)$ by substituting Equations (28) in (29) and using the boundary conditions (30) and (31), the frequency equation is given in the form of the 5th-order determinant as

$$\begin{vmatrix} n_1 & n_2 & n_3 & n_4 & n_5 \\ n_6 & n_7 & n_8 & n_9 & n_{10} \\ 0 & 0 & 0 & n_{11} & n_{12} \\ n_{13} & n_{14} & n_{15} & 0 & 0 \\ n_{16} & n_{17} & n_{18} & 0 & 0 \end{vmatrix} = 0, \tag{32}$$

where

$$n_1 = (\lambda + P)\left(\beta^2 m_1^2 - \frac{w^2}{c^2}\right) - \frac{2\mu w^2}{c^2} - \frac{\rho^2 c_1^2 \beta^2}{y_1}(\beta^2 m_1^4 - s_1 m_1^2 + s_2)\,,$$

$$n_2 = (\lambda + P)\left(\beta^2 m_2^2 - \frac{w^2}{c^2}\right) - \frac{2\mu w^2}{c^2} - \frac{\rho^2 c_1^{22}}{y_1}(\beta^2 m_2^4 - s_1 m_2^2 + s_2)\,,$$

$$n_3 = (\lambda + P)\left(\beta^2 m_3^2 - \frac{w^2}{c^2}\right) - \frac{2\mu w^2}{c^2} - \frac{\rho^2 c_1^{22}}{y_1}(\beta^2 m_3^4 - s_1 m_3 + s_2)\,,$$

$$n_4 = \frac{2i\mu w}{c} m_4, \quad n_5 = \frac{2i\mu w}{c} \beta m_5, \quad n_6 = -iw\frac{\beta m_1}{c}\,,$$

$$n_7 = -iw\frac{\beta m_2}{c}, \quad n_8 = -iw\frac{\beta m_3}{c}\,,$$

$$n_9 = \left[(\mu - \alpha)\beta^2 m_4^2 + (\mu + \alpha) - \frac{\beta^2 w^2}{c^2}\right],$$

$$n_{10} = \left[(\mu - \alpha)\beta^2 m_5^2 + (\mu + \alpha)\frac{\beta^2 w^2}{c^2}\right],$$

$$n_{11} = \left[\frac{iw\beta^2 m_4^2}{c} - \frac{iw^3}{c^3}\right], \quad n_{12} = \left[\frac{iw\beta^2 m_5^2}{c} - \frac{iw^3}{c^3}\right],$$

$$n_{13} = [\beta^2 m_1^4 - s_1 m_1^2 + s_2], \quad n_{14} = [\beta^2 m_2^4 - s_1 m_2^2 + s_2]\,,$$

$$n_{15} = [\beta^2 m_3^4 - s_1 m_3^2 + s_2]\,,$$

$$n_{16} = \left[\beta^2 m_1^4 - (s_1 + y_1) m_1^2 + s_2 + y_1\left(1 - \frac{y^2}{c_1^2}\right)\right],$$

$$n_{17}=\left[\beta^2 m_2^4-(s_1+y_1)m_2^2+s_2+y_1\left(1-\frac{v^2}{c_1^2}\right)\right],$$

$$n_{18}=\left[\beta^3 m_3^4-(s_1+y_1)m_3^2+s_2+y_1\left(1-\frac{v^2}{c_1^2}\right)\right].$$

The transcendental equation (32), in the determinantal form, represents the required wave velocity equation of magneto thermo-microelastic medium under initial stress P. It is clear from this frequency equation (32) that the phase velocity c depends on initial stress. Also, the frequency equation changes with respect to initial stress. But in a thermo-microelastic case, with no electromagnetic effects, where $\lambda_0=0$, i.e. $\epsilon_H\to\infty$, one of the roots of Equation (24), say l_3^2 becomes zero and the other two roots satisfy the equation

$$l^4-\left\{\frac{v^2}{c_1^2}+\frac{i(1+\epsilon_T)}{x}\frac{v^2}{c_1^2}\right\}l^2+\frac{iv^4}{c_1^4 x}=0\,,$$

which is the frequency equation for Rayleigh waves under initial stress, similar to that which has been obtained by Elnaggar and Abd-Alla (1987). In addition, in absence of initial stress, the frequency equation of magneto-thermo-microelastic half-space has an expression similar to that which has been obtained by Tomita and Shimdo (1979). Letting $\alpha\to 0$ in Equation (27), we get $m_5^2=1$, $m_4^2=1-(v^2/c_2^{*2})$, $c_2^{*2}=\mu/\rho$ and the frequency equation (32) admits of the form

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15}\\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25}\\ 0 & 0 & 0 & A_{34} & A_{35}\\ A_{41} & A_{42} & A_{43} & 0' & 0\\ A_{51} & A_{52} & A_{53} & 0 & 0 \end{vmatrix}=0\,, \tag{33}$$

where

$$A_{11}=\lambda\left(\beta^2 m_1^2-\frac{w^2}{c^2}\right)-\frac{2\mu w^2}{c^2}-\frac{\rho^2 c_1^{*22}}{y_1}(\beta^2 m_1^4-s_1 m_1^2+s_2)\,,$$

$$A_{12}=\lambda\left(\beta^2 m_2^2-\frac{w^2}{c^2}\right)-\frac{2\mu w^2}{c^2}-\frac{\rho^2 c_1^{*22}}{y_1}(\beta^2 m_2^4-s_1 m_2^2+s_2)\,,$$

$$A_{13}=\lambda\left(\beta^2-\frac{w^2}{c^2}\right)-\frac{2\mu w^2}{c^2}-\frac{\rho^2 c_1^{*22}}{y_1}(\beta^2-s_1+s_2)\,,$$

$$A_{14}=\frac{2i\mu w}{c}\sqrt{1-\frac{v^2}{c_2^{*2}}},\quad A_{15}=\frac{2iw\beta}{c}\,,$$

$$A_{21}=-\frac{iw\beta m_1}{c},\quad A_{22}=-\frac{iw\beta m_2}{c},\quad A_{23}=-\frac{iw\beta}{c}\,,$$

$$A_{24}=\left[(\mu-\alpha)\beta^2\left(1-\frac{v^2}{c_2^{*2}}\right)+(\mu+\alpha)\frac{\beta^2 w^2}{c^2}\right],$$

$$A_{25} = \left[(\mu-\alpha)\beta^2 + (\mu+\alpha)\frac{\beta^2 w^2}{c^2}\right],$$

$$A_{34} = \left[\frac{iw\beta^2}{c}\left(1-\frac{v^2}{c_2^{*2}}\right) - \frac{iw^3}{c^3}\right], \quad A_{35} = \left[\frac{iw\beta^2}{c} - \frac{iw^3}{c^3}\right],$$

$$A_{41} = [\beta^2 m_1^4 - s_1 m_1^2 + s_2], \quad A_{42} = [\beta^2 m_2^4 - s_1 m_2^2 + s_2],$$

$$A_{43} = [\beta^2 - s_1 + s_2],$$

$$A_{51} = \left[\beta^2 m_1^4 - (s_1+y_1)m_1^2 + s_2 + y_1\left(1-\frac{v^2}{c_1^{*2}}\right)\right],$$

$$A_{52} = \left[\beta^2 m_2^4 - (s_1+y_1)m_2^2 + s_2 + y_1\left(1-\frac{v^2}{c_1^{*2}}\right)\right],$$

$$A_{53} = \left[\beta^2 - (s_1+y_1) + s_2 + y_1\left(1-\frac{v^2}{c_1^{*2}}\right)\right],$$

$$c_1^{*2} = \lambda + 2\mu/\rho,$$

coinciding with the frequency equation for the thermoelastic Rayleigh waves (cf. Locket, 1958). In addition, the setting $m_1^2 = 1-(v^2/c_1^{*2})$, $m_2 \to \infty$ for $\epsilon_T \to 0$, then the frequency equation coincides with the frequency equation for the surface wave in the micropolar medium given by Nowacki and Nowacki (1969) as

$$\begin{vmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} \\ 0 & 0 & 0 & D_{34} & D_{35} \\ D_{41} & D_{42} & D_{43} & 0 & 0 \\ D_{51} & D_{52} & D_{53} & 0 & 0 \end{vmatrix} = 0, \tag{34}$$

where

$$D_{11} = \lambda\left\{\beta^2\left(1-\frac{v^2}{c_1^{*2}}\right) - \frac{w^2}{c^2}\right\} - \frac{2\mu w^2}{c^2} - \frac{\rho^2 c_1^{*2}\beta^2}{y_1}\left\{\beta^2\left(1-\frac{v^2}{c_1^{*2}}\right)^2 - \right.$$
$$\left. - s_1\left(1-\frac{v^2}{c_1^{*2}}\right) + s_2\right\},$$

$$D_{12} = -\lambda\frac{w^2}{c^2} - \frac{2\mu w^2}{c^2} - \frac{\rho^2 c_1^{*2}\beta^2}{y_1}s_2,$$

$$D_{13} = \lambda\left(\beta^2 - \frac{w^2}{c^2}\right) - \frac{2\mu w^2}{c^2} - \frac{\rho^2 c_1^{*22}}{y_1}(\beta^2 - s_1 + s_2),$$

$$D_{14} = \frac{2i\mu w m_4}{c}, \quad D_{15} = \frac{2i\mu w}{c}\beta m_5,$$

$$D_{21} = -\frac{iw\beta}{c}\left(1-\frac{v^2}{c_1^{*2}}\right)^{1/2}, \quad D_{22} = 0, \quad D_{23} = -\frac{iw\beta}{c},$$

$$D_{24} = \left[(\mu - \alpha)\beta^2 m_4^2 + (\mu + \alpha)\frac{\beta^2 w^2}{c^2}\right],$$

$$D_{25} = \left[(\mu - \alpha)\beta^2 m_5^2 + (\mu + \alpha)\frac{\beta^2 w^2}{c^2}\right],$$

$$D_{34} = \left[\frac{iw\beta^2 m_4^2}{c} - \frac{iw^3}{c^3}\right], \quad D_{35} = \left[\frac{iw\beta^2 m_5^2}{c} - \frac{iw^3}{c^3}\right],$$

$$D_{41} = \left[\beta^2\left(1 - \frac{v^2}{c_1^{*2}}\right)^2 - s_1\left(1 - \frac{v^2}{c_1^{*2}}\right) + s_2\right] \quad]g4_2 = s_2\,,$$

$$D_{43} = [\beta^2 - s_1 + s_2]\,,$$

$$D_{51} = \left[\beta^2\left(1 - \frac{v^2}{c_1^{*2}}\right)^2 - (s_1 + y_1)\left(1 - \frac{v^2}{c_1^{*2}}\right) + s_2 + y_1\left(1 - \frac{v^2}{c_1^{*2}}\right)\right],$$

$$D_{52} = \left[s_2 + y_1\left(1 - \frac{v^2}{c_1^{*2}}\right)\right],$$

$$D_{53} = \left[\beta^2 - (s_1 + y_1) + s_2 + y_1\left(1 - \frac{v^2}{c_1^{*2}}\right)\right].$$

Since the computations base on the frequency-equation are too cumbersome and tedious to carry out, let us consider the case when the reduced frequency x is very small; so that its first and higher terms can be neglected in comparison with unity. It can be noted that the frequency equation (34) is dispersive.

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