CONTRACTION OF THE SOLAR NEBULA

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Abstract. The concept of Roche limit is applied to the Laplacian theory of the origin of the solar system to study the contraction of a spherical gas cloud (solar nebula). In the process of contraction of the solar nebula, it is assumed that the phenomenon of supersonic turbulent convection described by Prentice (1978) is operative and brings about the halt at various stages of contraction. It is found that the radius of the contracting solar nebula follows Titius–Bode law $R_p = R_\odot a^p$, where $R_\odot$ is the radius of the present Sun and $a = 1.442$. We call 'a' the Roche's constant. The consequences of the relation are also discussed. The aim, here, is an attempt to explain, on the basis of the concept of Roche limit, the distribution of planets in the solar system and try to understand the physics underlying it.

1. Introduction

Since the time Copernicus discovered that the planets revolve around the Sun, astronomers have been trying to understand the origin of the solar system. Numerous theories for the origin of the solar system have so far been advanced (ter Haar and Cameron, 1963); ter Haar (1967), Williams and Cremine (1968), Woolfson (1969), Alfvén and Arrhenius (1970a, b), Nieto (1972), Reeves (1978) and Prentice (1978).

Among all these theories of the formation of the solar system, Laplace's nebular hypothesis is favoured as it explains (1) the isotopic abundance of the elements, (2) the estimates of the ages of the Sun and the planets, (3) our understanding of transformation of a hot magnetic and rotating interstellar gas cloud into a star, (4) chemical and mineralogical composition of different objects in the solar system such as meteorites and (5) support from astronomical observations (Reeves, 1978). However, it faces the following problems: (1) the problem of explaining extraordinary character of the distribution of mass and angular momentum in the solar system, (2) the problem of explaining why the protosun shed a discrete system of rings and (3) the problem of explaining how the planets aggregated from each gaseous ring (for full details, see above mentioned references).

The difficulties faced by Laplacian hypothesis are considered by Prentice (1978). He presents an outline of the Laplacian theory, which he calls 'modern Laplacian theory' for the origin of the solar system. He considers the influence of a supersonic turbulent stress on the cloud and shows how this stress leads to the formation and detachment of a discrete system of gaseous rings, the ratio of the orbital radii R_p of successively disposed gaseous rings being a constant forming a geometric progression similar to the Titius–Bode law of planetary distances (ter Haar, 1950; Dermott, 1968; Nieto, 1972, and Rawal, 1978).

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In this work, the concept of Roche limit is applied to the contracting solar nebula. In the process of contraction of the solar nebula, it is assumed that the phenomenon of supersonic turbulent convection described by Prentice (1978) is operative. This brings about the halt at various stages of contraction. The aim, here, is an attempt to explain on the basis of the concept of Roche limit, the distribution of planets in the solar system and try to understand the physics underlying it.

2. Contracting Solar Nebula

It is assumed that there was a spinning spherical gas cloud of interstellar gas and dust with mass, M, slightly greater than $M_\odot$ ($M_\odot$ = Sun's mass) and certain radius, denoted by R_p. Under the influence of its self-gravitation, the cloud began to contract and because of the conservation of angular momentum, it began to spin faster and faster. A stage was reached at which the centrifugal force became equal to gravitational force at the equator giving rise to a rotational instability, as a result, a shell of matter evolved into a ring of matter at the equator. This whole process repeated itself until the solar nebula reached its present size in which form we call it the Sun. Here the contraction of the above solar nebula halting at various radii is described in a particular fashion given below. In this description, it is assumed that the halts at various radii are brought about by the phenomenon of supersonic turbulent convection. The supersonic turbulent convection does the following jobs: (1) It creates an additional source of pressure in a solar nebula called the radial turbulent stress which halts the free collapse of the solar nebula first at the dimensions of the planetary system, (2) it causes the interior of the solar nebula to rotate almost uniformly like a rigid body because of a large turbulent viscosity and drastically lowers the moment-of-inertia coefficient, f, of the protosun thereby allowing the protosun to give up its angular momentum to a very light planetary system and (3) it leads to the formation of a very dense ring of gas at the equator of the protosun, thereby causing the protosun to dispose of its excess angular momentum through the successive detachment of a discrete system of gaseous rings.

Assume that the above solar nebula shrank to a radius R_{p-1} such that R_p is the Roche limit of the cloud having radius R_{p-1}.

There are two versions of Roche limits. One for a fluid satellite and another for a rigid satellite (Smith and Jacobs, 1973). For a fluid satellite Roche limit, denoted by d_{Roche}' is given by

$$d_{\mathrm{Roche}} = 2.4554\,[\rho/\rho']^{1/3} \times R; \tag{2.1}$$

and, for a rigid satellite, it is given by

$$d_{\mathrm{Roche}} = 1.442\,[\rho/\rho']^{1/3} \times R, \tag{2.2}$$

where R is the radius of the primary (the central body), ρ is its density and ρ' is the density of the secondary.

As already mentioned, the supersonic turbulent convection makes the interior of the solar nebula to rotate almost uniformly like a rigid body and the conditions at the photosphere arise by which the heat can freely escape and the turbulent convection dies down. As soon as the turbulent convection dies down, so will the turbulent stress and the material above the photosphere becomes essentially non-turbulent. In order that the non-turbulent material above the photosphere withstand the huge turbulent stress of the highly turbulent material beneath it, the pressure equilibrium demands that the density of the material above the photosphere must exceed that of the photosphere by a very large factor of 100. This is achieved by the extrusion of material from the convective interior of the protostar to the equator giving rise to a very steep density inversion at the surface of the protostar. These considerations lead us, here, to adopt the second version of Roche limit.

In the case when $\rho = \rho'$, and we assume, here, that this is the case, the Roche limit, therefore, assumes the form

$$d_{\mathrm{Roche}} = 1.442R. \tag{2.3}$$

If we refer to $1.442 = a$, as the Roche constant,

$$d_{\mathrm{Roche}} = aR. \tag{2.4}$$

Therefore, the relation between R_p and R_{p-1} of the contracting spherical gas cloud can be written as

$$R_p = aR_{p-1}. \tag{2.5}$$

The shell of matter having width $R_p - R_{p-1}$ forms the Roche zone of the protosun having radius R_{p-1}. The matter in the shell having width $R_p - R_{p-1}$ settled down to form a ring at the equator of width $R_p - R_{p-1}$. The matter inside such a ring might have grown to planetesimals but naturally failing to form a full planet there, because the matter in this ring was still inside the Roche limit of the protosun having radius R_{p-1}. The matter inside such a ring had to wait for further contraction of the solar nebula to take place which could put it outside the Roche limit so that a full planet might form in it. For the process of planets formation, we refer the reader to the paper of Prentice (1978).

At the next stage of contraction, the cloud shrank to a radius R_{p-2} such that

$$R_{p-1} = aR_{p-2}. \tag{2.6}$$

Hence, we have

$$R_p = a^2R_{p-2}. \tag{2.7}$$

The annular ring (R_{p-2}, R_{p-1}) of width $R_{p-1} - R_{p-2}$ lay inside the Roche limit of the protosun now having radius R_{p-2}. At this stage, the previous ring (R_{p-1}, R_p) of matter came out of the Roche zone of the protosun having radius R_{p-2} and found the matter to grow to form a planet.

We assume that the contraction proceeded in this fashion until the solar nebula reached its present size in which form we call it the Sun, the halts at various radii are being

TABLE I

R_p, the radius of the contracting Solar nebula in units of AU	Annular ring (R_{p-1}, R_p)	Width $R_p - R_{p-1}$ of the annular ring (R_{p-1}, R_p) in units of AU	Mean radius of the annular ring (R_{p-1}, R_p) in units of AU	Known Object in the annular ring (R_{p-1}, R_p)	Observed mean distance of the known object in the annular ring (R_{p-1}, R_p) in units of AU
$R_\odot = 0.004652$					
	$(R_\odot, R_1)$	0.0024	0.006	–	–
$R_1 = 0.007$					
	(R_1, R_2)	0.003	0.008	–	–
$R_2 = 0.01$					
	(R_2, R_3)	0.004	0.012	–	–
$R_3 = 0.014$					
	(R_3, R_4)	0.006	0.017	–	–
$R_4 = 0.02$					
	(R_4, R_5)	0.01	0.025	–	–
$R_5 = 0.03$					
	(R_5, R_6)	0.012	0.036	–	–
$R_6 = 0.042$					
	(R_6, R_7)	0.018	0.051	–	–
$R_7 = 0.06$					
	(R_7, R_8)	0.027	0.073	–	–
$R_8 = 0.087$					
	(R_8, R_9)	0.043	0.11	–	–
$R_9 = 0.13$					
	(R_9, R_{10})	0.05	0.155	–	–
$R_{10} = 0.18$					
	(R_{10}, R_{11})	0.08	0.22	–	–
$R_{11} = 0.26$					
	(R_{11}, R_{12})	0.12	0.32	–	–
$R_{12} = 0.38$					
	(R_{12}, R_{13})	0.16	0.46	Mercury	0.4
$R_{13} = 0.54$					
	(R_{13}, R_{14})	0.26	0.67	Venus	0.7

TABLE I *Continued*

R_p, the radius of the contracting Solar nebula in units of AU	Annular ring (R_{p-1}, R_p)	Width $R_p - R_{p-1}$ of the annular ring (R_{p-1}, R_p) in units of AU	Mean radius of the annular ring (R_{p-1}, R_p) in units of AU	Known Object in the annular ring (R_{p-1}, R_p)	Observed mean distance of the known object in the annular ring (R_{p-1}, R_p) in units of AU
$R_{14} = 0.8$	(R_{14}, R_{15})	0.33	0.965	Earth	1
$R_{15} = 1.13$	(R_{15}, R_{16})	0.47	1.365	Mars	1.5
$R_{16} = 1.6$	(R_{16}, R_{17})	0.75	1.975	Asteroids	–
$R_{17} = 2.35$	(R_{17}, R_{18})	1.05	2.875	Asteroids	2.8
$R_{18} = 3.4$	(R_{18}, R_{19})	1.5	4.15	Asteroids	–
$R_{19} = 4.9$	(R_{19}, R_{20})	2.1	5.95	Jupiter	5.2
$R_{20} = 7$	(R_{20}, R_{21})	3.15	8.575	Saturn	9.5
$R_{21} = 10.15$	(R_{21}, R_{22})	4.5	12.4	Chiron	13.7
$R_{22} = 14.65$	(R_{22}, R_{23})	6.47	17.88	Uranus	19.2
$R_{23} = 21.12$	(R_{23}, R_{24})	9.34	25.79	Neptune	30.1
$R_{24} = 30.46$	(R_{24}, R_{25})	13.44	37.18	Pluto	39.4
$R_{25} = 43.9$					

brought about by the phenomenon of supersonic turbulent convection eventually leading us to the stage

$$R_1 = aR_\odot, \tag{2.8}$$

where $R_\odot$ is the radius of the present Sun. In terms of the radius of the present Sun, the sequence of the radii of the contracting solar nebula at various stages of the contraction can be expressed as

$$R_p = R_\odot a^p, \qquad p = 1, 2, 3, \ldots, k. \tag{2.9}$$

Table I shows various R_p. The known planet residing in the ring labelled (R_{p-1}, R_p), for various values of p are also mentioned.

From Table I, it can be seen that the planet Pluto lies in the ring (R_{24}, R_{25}), Neptune in the ring (R_{23}, R_{24}), Uranus in (R_{22}, R_{23}). Newly discovered Kowal's Object Chiron lies in the ring (R_{21}, R_{22}). Probably, this ring may be a ring of asteroids, Chiron being one of the members (Rawal 1978, 1981). Saturn lies in the ring (R_{20}, R_{21}), Jupiter in (R_{19}, R_{20}). Three rings (R_{18}, R_{19}), (R_{17}, R_{18}), and (R_{16}, R_{17}) represent Asteroid belt. Recently Low *et al.* (1984) have reported the findings of IRAS of three more asteroid belts at distances 2.2, 2.3, and 3.2 AU in addition to the already existing main asteroid belt at a mean distance 2.8 AU. All these asteroid belts correspond to the rings (R_{16}, R_{17}) and (R_{17}, R_{18}). It is felt that this zone of asteroids is even extended further out and our ring (R_{18}, R_{19}) corresponds to this outer part of asteroid zone (see, Table I). Astronomers believe (Prentice, 1978) that the asteroids just failed to aggregate in time before their gaseous ring disintegrated and hence could not form a planet. As a consequence the larger asteroids were left strewn around their mean orbit while the smaller ones were dragged away with the escaping gas. The planet Mars lies in the ring (R_{15}, R_{16}), Earth in (R_{14}, R_{15}), Venus in (R_{13}, R_{14}) and Mercury in (R_{12}, R_{13}). Until recently, no objects (rings/planets) were known to lie in any of the rings labelled (R_{12}, R_{11}) to $(R_\odot, R_1)$ and some astronomers like Prentice (see, Prentice 1978) got convinced that these rings were really vacant. At the same time, there were certain indications also to believe that a few of those rings might not be vacant. It was also expressed that if the material in these rings has survived both vaporization and drags through the age of the solar system, then it may well be there. In this context, it is interesting to note that Brecher *et al.* (1979) arrive at a consistent picture of the primordial ring of refractory material, possibly of graphite, allowed to reside around the Sun. According to them, it must lie at a distance $\gtrsim 4R_\odot$, its total mass $M \lesssim 6 \times 10^{25}$ gm, it could consist in that orbit of at most $N \simeq 10^6$ objects of minimum radius $A \simeq 10$ km. They are also hopeful of the observation of such a ring system around the Sun. Rawal (1978, 1981) has also arrived at similar conclusion on the basis of resonant structure in the solar system and on the basis of the modified Titius–Bode law in attempts to explain the ring structures around the planets and the Sun itself. All these tended to suggest that there might be a ring structure and one or two small planets yet undiscovered, going around the Sun within the orbit of Mercury. Interestingly enough, by balloon borne optical and infrared polarimeter, Isobe *et al.*

(1984) have observed from Jogjakarta a solar ring of refractory material during the total solar eclipse of June 11, 1983. They found the solar ring at a distance of $\gtrsim 4R_\odot$. In our work this corresponds to the ring labelled (R_3, R_4) (see Table I).

It is quite probable that some matter, may be in the form of a planet/planets/rings, lies in the region beyond the ring (R_{24}, R_{25}) which houses the planet Pluto. Therefore, there will be no surprise if some such objects are discovered beyond Pluto. The questions regarding comets have been discussed by several authors (for details see Kuiper, 1951).

On this scheme, Equation (2.9) is looked upon as giving outer and inner boundaries of various rings. The scenario, here, brings out that the ring-structure-feature is a common and natural feature of the heavenly bodies, in particular of the major members of the solar system (Rawal 1981, 1982). According to this scenario of the formation of the solar system, it is obvious that the distant planets were formed earlier.

On the basis of supersonic turbulent convection and the law of conservation of mass and angular momentum, Prentice, in his modern Laplacian theory, gets the ratio of the orbital radii R_p of successively disposed gaseous rings to be a constant, given by

$$R_p/R_{p+1} = \left(1 + \frac{m}{Mf}\right)^2 = \text{const.}, \tag{2.10}$$

where m is the mass of a disposed ring, M the remaining mass of the protosolar nebula and f the moment-of-inertia coefficient.

To reconcile our work with that of Prentice, we note that Prentice distributes the solar material $0.05\,M_\odot$ which has gone to form the planetary system (Urey, 1951; Kuiper, 1951; Hoyle, 1960; Hoyle and Wickramasinghe, 1968; Whipple, 1971, Prentice, 1978; and other cosmo-chemists) among twenty orbits that he gets between the present Sun and Neptune, ten between Mercury and the present size of the Sun and ten between Neptune and Mercury, by putting $m = 1000 M_\oplus$ ($M_\oplus$ = Earth's mass) and $f = 0.01$ in the Equation (2.10), and gets Bode's constant to be 1.69. As we are getting twenty five rings between Neptune and the present size of the Sun, each ring in our work gets $660 M_\oplus$ as its share. Hence, putting $m = 660 M_\oplus$ and $f = 0.01$, we find that

$$R_p/R_{p+1} = \left[1 + \frac{m}{Mf}\right]^2 = 1.442 = a, \tag{2.11}$$

showing the agreement between our work and that of Prentice. Several authors (ter Haar and Cameron, 1963; Dermott, 1968; Nieto, 1972; and Rawal, 1978) have arrived at different forms of Titius–Bode law in their attempts to explain planetary distances. So far, all were empirical relations. In comparison with those laws, Equation (2.9) giving outer and inner boundaries of various rings has a physical interpretation in the sense that it is based on the concept of Roche limit applied to contracting solar nebula, the halts at various radii are being brought about by the phenomenon of supersonic turbulent convection leading to the formation and detachment of a discrete system of gaseous rings.

The discussion, here, supports modern Laplacian theory of Prentice, and in turn, modern Laplacian theory provides an understanding between supersonic turbulent convection and Roche limit in that the rotational instability at the equator of the protosolar nebula arises at various stages of its quasistatic contraction precisely by the step of Roche's constant which is the same as Bode's constant of modern Laplacian theory leading to the formation and detachment of a discrete system of gaseous rings, the whole process being controlled by the phenomenon of supersonic turbulent convection.

The usefulness of this work is that once the radius of the primary is known, the relation can be set up very simply and uniquely. The present discussion could be considered as an alternative way of deriving the Titius–Bode law and trying to understand the physics underlying it.

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References

Alfvén, H. and Arrhenius, G.: 1970a, *Astrophys. Space Sci.* 8, 338.
Alfvén, H. and Arrhenius, G.: 1970b, *Astrophys. Space Sci.* 9, 3.
Brecher, K., Brecher, A., Morrison, P., and Wasserman, I.: 1979, *Nature* **282**, 50.
Dermott, S. F.: 1968, *Monthly Notices Roy. Astron. Soc.* **141**, 363.
ter Haar, D.: 1967, *Ann. Rev. Astron. Astrophys.* 5, 267.
ter Haar, D. and Cameron, A. G. W.: 1963, in *The Origin of the Solar System*, R. Jastrow and A. G. W. Cameron (eds.), Academic Press, New York, p. 1.
Hoyle, F.: 1960, *Quart. J. Roy. Astron. Soc.* 1, 28.
Hoyle, F. and Wickramasinghe, N. C.: 1968, *Nature* 217, 415.
Isobe, S., Tanabe, H., Maihara, T., Mizutani, H., and Koma, Y., 1984, 163rd AAS meeting in Las Vegas.
Kuiper, G. P.: 1951, in *Astrophysics*, J. A. Hynek (ed.), McGraw-Hill, New York, Chapter 8.
Low, F. J. *et al.*: 1984, *Astrophys. J.* 278, L19.
Nieto, M. M.: 1972, *The Titius–Bode Law of Planetary Distances, Its History and Theory*, Pergamon Press, Oxford.
Prentice, A. J. R.: 1978, in '*The Origin of the Solar System*', Dermott, S. F. (ed.), Wiley, London, p. 111.
Rawal, J. J.: 1978, *Bull. Astr. Soc. India* **6**, 92.
Rawal, J. J.: 1981, *The Moon and the Planets* **24**, 407.
Rawal, J. J.: 1982, *Indian J. of Radio and Space Phys.* **11**, 100.
Reeves, H.: 1978, in '*The Origin of the Solar System*', Dermott, S. F. (ed.), Wiley, London 1978, p. 1.
Smith, E. P. and Jacobs, K. C.: 1973, *Introductory Astronomy and Astrophysics*, W. B. Saunders Company, Philadelphia, p. 77.
Urey, H. C.: 1951, *Geochim. Cosmochim. Acta* **1**, 209.
Whipple, F. L.: 1971, *Earth, Moon and Planets*, Harvard Univ. Press, Cambridge, Mass.
Williams, I. P. and Cremin, A. W.: 1968, *Quart. J. Roy. Astron. Soc.* 9, 40.
Woolfson, M. M.: 1969, *Repts. Progr. Phys.* **32**, 135.