# LOW-FREQUENCY ELECTRIC FIELD FLUCTUATIONS EXCITED BY RING CURRENT PROTONS

S. V. SINGH
*Department of Physics, University of Durban-Westville, Durban 4000, South Africa*
G. S. LAKHINA*
*Indian Institute of Geomagnetism, Colaba, Bombay - 400 005, India*
R. BHARUTHRAM
*Department of Physics, University of Durban-Westvile, Durban 4000, South Africa*

(Received 14 November, 1996; Accepted 29 November, 1996)

**Abstract.** Energetic protons having ring type distributions are shown to generate low-frequency electrostatic waves, propagating nearly transverse to the geomagnetic field lines, in the ring current region by exciting Mode 1 and Mode 2 nonresonant instabilities and a resonant instability. Mode 1 nonresonant instability has frequencies around ~4 Hz with transverse wavelengths of ~(8–80) km, and it is likely to occur in the region $L$ = (7–8). Mode 2 nonresonant instability can generate frequencies ~(850–1450) Hz with transverse wavelengths ~(2–20) km. The typical frequencies and transverse wavelengths associated with the resonant instability are (950–1250) Hz and (30–65) km. Both the Mode 2 nonresonant instability and the resonant instability can occur in the ring current region with $L$ = (4–6). The low-frequency modes driven by energetic protons could attain maximum saturation electric field amplitude varying from 0.8 mV/m to 70 mV/m. It is suggested that the turbulence produced by the low-frequency modes may cause pitch angle scattering of ring current protons in the region outside the plasmapause resulting in the ring current decay.

## 1. Introduction

There are several observations indicating the presence of energetic proton and heavier ion distributions in various regions of the magnetosphere. These energetic protons have, generally, non-Maxwellian distributions which can drive several plasma instabilities. In fact, the wave particle interactions can play an important role in ring current, auroral, and plasma sheet dynamics. Particle data from Explorer 45, AMPTE/CCE, GEOS 1 and 2, and other spacecrafts clearly indicate the presence of hot nonthermal proton distributions in the ring current region (Fritz and Spjeldvik, 1979; Perraut et al., 1982; Kistler et al., 1989; Laakso et al., 1990; Lui et al., 1990). Energetic ion distributions have been observed on the auroral field lines, in the cusp/cleft region and in the plasma sheet boundary layer (Hultqvist et al., 1988; Lundin and Eliasson, 1991; Daglis et al., 1993; Ghielmetti et al., 1978; Parks et al., 1984; Kistler et al., 1990; Peterson et al., 1981). Several spacecraft namely OGO 3, IMP 6, Hawkeye 1, S3-3, GEOS 1 and 2, Viking etc. have observed the low-frequency fluctuations with frequencies ranging from essentially

*Address for correspondence:* Dr. G. S. Lakhina, Mail Stop 169–506, Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, CA 91109, USA.

*Earth, Moon and Planets* **74:** 231–242, 1996.

zero to a few kHz. Generally, the noise at the lowest frequencies is predominantly electromagnetic in nature, whereas the fluctuations at the higher frequencies are mostly electrostatic in nature (Russell et al., 1970; Anderson and Gurnett, 1973; Gurnett, 1976; Gurnett and Frank, 1977, 1978; Perraut et al., 1982; Laakso et al., 1990). It has been shown that various types of energetic proton distribution in the ring current region can excite ULF waves over a broad range of frequencies spanning from much below the proton cyclotron frequency to several of its harmonics (Cornwall et al., 1971; Gul'elmi et al., 1975; Curtis and Wu, 1979; Bhatia and Lakhina, 1980a,b; McClements and Dendy, 1993; McClements et al., 1994). Energetic ion distributions can also excite electrostatic waves in various regions of the magnetosphere. In the ring current region, a quasi-electrostatic instability can be driven by the loss-cone distribution of protons (Coroniti, 1972; Bernstein et al., 1974; Lakhina, 1976; Bhatia and Lakhina, 1980c). On the other hand, ion beams can excite various types of electrostatic instabilities leading to the generation of broad band electrostatic noise in the plasma sheet boundary layer (Grabbe and Eastman, 1984; Omidi, 1985; Lakhina, 1987), and generation of low-frequency electric field fluctuations in the cusp region (D'Angelo, 1977; Lakhina, 1987) and auroral acceleration region (Ashour-Abdalla and Schriver, 1989; Bergmann and Lotko, 1986; Bergmann et al., 1988; Lakhina, 1993). Low-frequency electric field fluctuations driven by energetic ion beams can attain quite large amplitudes and therefore can lead to significant scattering, heating and acceleration of the ions and electrons.

Recently, McClements et al. (1994) have studied the generation of obliquely propagating fast Alfvén waves driven by energetic proton distributions in the ring current as observed by GEOS 1 (Perraut et al., 1982). The energetic protons were found to have a ringlike distribution with finite spread of parallel ($v_{\|}$) and perpendicular ($v_{\perp}$) velocities. Energetic protons have energy of about 7.1 keV and number density of $N_H = 1.6\ \text{cm}^{-3}$. The bulk ions had energy of $\sim$1 eV and number density of $N_C \sim 65\ \text{cm}^{-3}$.

In this paper, we investigate the excitation of lower hybrid type waves driven by the observed energetic proton distributions (Perraut et al., 1982) in the ring current plasma. Our aim is to find out whether the observed ringlike proton distributions could, in addition to ULF waves, generate the intense low frequency electrostatic noise observed beyond the plasmapause (Anderson and Gurnett, 1973; Bernstein et al., 1974). We find that the observed energetic ringlike proton distributions can excite both the resonant and non-resonant type lower hybrid instabilities in the ring current region. Our model includes three species namely electrons, cold protons and energetic protons with ringlike distribution. We find that the instability can be excited over a wide range of wavenumber $k$ values in non-resonant and resonant cases. Growth rate is larger in the non-resonant case than the resonant case. In resonant case instability shows stabilizing effect for large $k$ values. Organization of the paper is as follows. In Section 2 we derive the dispersion relations and growth rates for the non-resonant and resonant instability. The results are discussed in Section 3.

## 2. Dispersion Relation

We model the ring current region by a plasma system consisting of electrons, cold protons and the energetic protons embedded in the geomagnetic field which we take as uniform, i.e. $\mathbf{B}_0 = B_0\hat{z}$. Let the number density and temperature of electron be $N_{0e}$ and $T_e$, and of cold protons be $N_{0c}$ and $T_c$. The electrons and the cold protons have Maxwellian distributions,

$$f_{0e,c} = \frac{N_{0e,c}}{\pi^{3/2} v_{te,c}^{3/2}} \exp\left[-\frac{v^2}{v_{te,c}^2}\right], \tag{1}$$

where $v_{te,c}$ are the thermal velocities of the electrons and cold protons. We consider the hot protons to have a number density of $N_{0H}$ with ringlike distribution function (McClements et al., 1994),

$$f_{0H} = \frac{N_{0H}}{2\pi^2 u \alpha_{\parallel H} \alpha_{\perp H} R} \exp\left[-\frac{v_{\parallel}^2}{\alpha_{\parallel H}^2} - \frac{(v_\perp - u)^2}{\alpha_{\perp H}^2}\right], \tag{2}$$

where $u$ is the mean perpendicular velocity of the ring, $\alpha_{\perp H}$ and $\alpha_{\parallel H}$ are the perpendicular and parallel thermal velocities of the hot protons respectively, and $R$ is the normalization factor given by,

$$R = \frac{1}{2\pi^{1/2}} \left[\sqrt{\pi}\left\{1 + \mathrm{erf}\left(\frac{u}{\alpha_{\perp H}}\right)\right\} + \frac{\alpha_{\perp H}}{u} \exp\left\{-\frac{u^2}{\alpha_{\perp H}^2}\right\}\right], \tag{3}$$

where erf$(u/\alpha_{\perp H})$ is the error function. We consider nearly transverse propagation of waves to the ambient magnetic field, i.e. $k_{\parallel}^2 \ll k_{\perp}^2$; $k_{\parallel}$ and $k_\perp$ are the parallel and perpendicular wavenumbers to the magnetic field $\mathbf{B}_0$, and the perturbation is assumed as $\exp[-i(\omega t - \mathbf{k}.\mathbf{r})]$, where $\omega$ is the wave frequency. We treat the background electrons and protons as cold. This requires that $\omega/k_{\parallel} v_{te,c} \gg 1$. Further, we consider the wave frequencies to lie in the range $\omega_{ci}^2 \ll \omega^2 \ll \omega_{ce}^2$, where $\omega_{ce}$, $\omega_{ci}$ are the electron and ion cyclotron frequencies respectively. Under these conditions the electrons are strongly affected by the magnetic field while the protons are unaffected by it. The response of electrons to the perturbation field is treated as electromagnetic and that of protons as electrostatic. Taking into account the above assumptions, the dispersion relation for the ring current instability can be written as (Bhatia and Lakhina, 1980c)

$$D(\omega, k) = P - Q\frac{k_{\parallel}^2 \omega_{pe}^2}{k^2 \omega^2} - \frac{\omega_{pc}^2}{\omega^2} + \chi_H = 0, \tag{4}$$

where

$$P = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\left[\frac{1 - I_0(\lambda_e)\exp(-\lambda_e)}{\lambda_e}\right] + \frac{\omega_{pe}^2}{c^2k^2}\frac{\omega_{pe}^2}{\omega_{ce}^2}\left[\frac{\{[I_0(\lambda_e) - I_1(\lambda_e)]\exp(-\lambda_e)\}^2}{1 + \beta_e \exp(-\lambda_e)[I_0(\lambda_e) - I_1(\lambda_e)]}\right], \tag{5}$$

$$Q = \frac{I_0(\lambda_e)\exp(-\lambda_e)}{\left[1 + \frac{\omega_{pe}^2}{c^2k^2} I_0(\lambda_e)\exp(-\lambda_e)\right]}, \tag{6}$$

and $\chi_H$ the susceptibility for the hot proton given by

$$\chi_H = \frac{\omega_{PH}^2}{k^2 N_{0H}} \int_0^{2\pi} \int_{-\infty}^{\infty} \int_0^{\infty} \frac{\mathbf{k}.\frac{\partial f_{0H}}{\partial \mathbf{v}}}{\omega - \mathbf{k.v}} \, d\theta \, dv_{\|} v_{\perp} \, dv_{\perp}. \tag{7}$$

In (5)–(7), $\omega_{pj} = (4\pi N_{0j} e^2/m_j)^{1/2}$ is the plasma frequency, and $\omega_{ce}$ is the cyclotron frequency of the $j$th species, where $j = e, c$ and $H$ for the electrons, cold protons, and the hot protons respectively, $\beta_e = (8\pi N_{0e} K T_e / B_0^2)$ is the plasma beta for electrons, and $I_0(\lambda_e)$ and $I_1(\lambda_e)$ are the modified Bessel functions with the argument $\lambda_e = (k_\perp^2 v_{te}^2 / 2\omega_{ce}^2)$. Now we shall solve the dispersion relation (4) analytically for some cases where $\chi_H$ reduces to some simple forms.

### 2.1. Non-resonant Instability

On substituting the distribution function $f_{0H}$ in the above equation, making use of the assumption $k_\| \ll k_\perp$, and doing the integration in Equation (7) over $\theta$ and $v_\|$, we obtain

$$\chi_H = -\frac{\omega_{PH}^2}{\pi^{1/2} u k_\perp \alpha_{\perp H} R} \frac{\partial}{\partial \omega}\left[\int_{-u/\alpha_{\perp H}}^{\infty} \frac{(u + \alpha_{\perp H} x)}{x - \xi} \exp(-x^2) dx\right], \tag{8}$$

where

$$x = (v_\perp - u)/\alpha_{\perp H}, \qquad \xi = (\omega - k_\perp u)/k_\perp \alpha_{\perp H}. \tag{9}$$

In the limit $\xi \gg x$, and neglecting the pole contribution, the expression for the hot proton susceptibility attains a simple form, namely,

$$\chi_H = -\frac{\omega_{PH}^2}{(\omega - k_\perp u)^2}, \tag{10}$$

Substituting the value of $\chi_H$ into Equation (4) and simplifying, we can reduce the dispersion relation to

$$(\omega^2 - \omega_a^2)[(\omega - k_\perp u)^2 - \omega_b^2] = \omega_a^2\omega_b^2, \tag{11}$$

where

$$\omega_a = \frac{\omega_{pc}}{\sqrt{P}}\left[1 + Q\frac{k_\parallel^2}{2}\frac{\omega_{pe}^2}{\omega_{pc}^2}\right]^{1/2} \tag{12}$$

$$\omega_b = \frac{\omega_{PH}}{\sqrt{P}} \tag{13}$$

Equation (11) yields two fluid type or nonresonant instabilities, with $\gamma \sim \omega_r$, where $\omega_r$ and $\gamma$ represent the real frequency and the growth rate defined by $\omega = \omega_r + i\gamma$, which we discuss below.

2.1.1. *Mode 1 Nonresonant Instability*

For $u \approx u^I = \omega_b/k_\perp$, we obtain an unstable root of Equation (11),

$$\omega = \left(\frac{\omega_a^2\omega_b}{2}\right)^{1/3}\frac{(1 + i\sqrt{3})}{2} \tag{14}$$

provided $\omega_a/\omega_b < 2$. The excitation of Mode 1 nonresonant instability demands $N_{0c}/N_{0H} \ll 1$, therefore this mode could be excited when the ring current is far away from the plasmapause. Figure 1 shows the variation of $\omega_r$ and $\gamma$ for this mode for some typical parameters of the ring current plasma in the region far beyond the plasmapause ($L \sim 7$ to 8). For the ring current parameters $N_{0H} = 1.6$ cm$^{-3}$ and $N_{0c} = 10^{-2}$ cm$^{-3}$, Figure 1 shows that growth rate $\gamma \sim 6.5\omega_{ci}$, where $\omega_{ci}$ is the ion cyclotron frequency (in this case proton cyclotron frequency). Growth rate remains almost constant for $\lambda_e = 10^{-3}$ to $10^{-1}$.

2.1.2. *Mode 2 Nonresonant Instability*

For $\omega \approx k_\perp u = k_\perp u^{II} + \eta$, where $u^{II} = \omega_a/k_\perp$ and $\eta \ll \omega_a$, the solution of Equation (11) gives an unstable root with

$$\begin{aligned} \omega_r &= \omega_a + \left(\frac{\omega_b^2\omega_a}{2}\right)^{1/3}, \\ \gamma &= \Im\eta = \frac{\sqrt{3}}{2}\left(\frac{\omega_b^2\omega_a}{2}\right)^{1/3}, \end{aligned} \tag{15}$$

provided $\omega_b^2/\omega_a^2 < 2$. This excitation of this mode requires $N_{0c}/N_{0H} \gg 1$, this means that presence of large number of cold protons would facilitate this generation

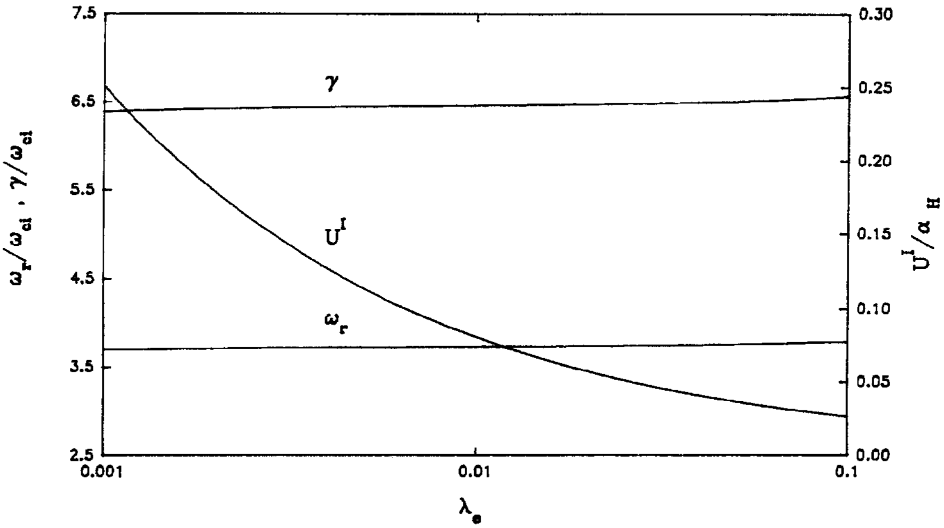


*Figure 1.* Variation of $\gamma/\omega_{ci}$, $\omega_r/\omega_{ci}$ and $u^I/\alpha_{\perp H}$ vs. $\lambda_e$ for the Mode 1 nonresonant instability in the ring current plasma with $N_{0H}$ = 1.6 cm$^{-3}$, $N_{0c} = 10^{-2}$ cm$^{-3}$, and $k_\parallel/k = 0.002$.

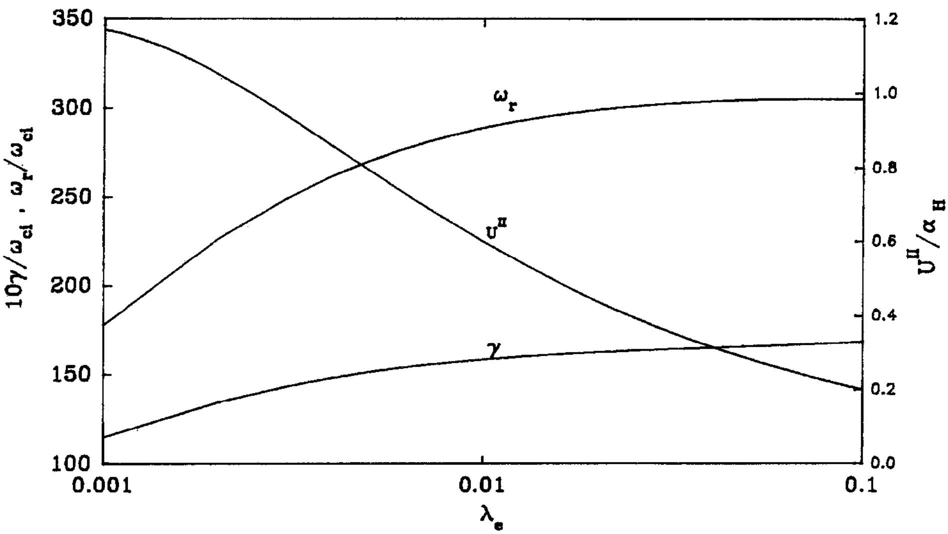


*Figure 2.* Variation of $\gamma/\omega_{ci}$, $\omega_r/\omega_{ci}$ and $u^{II}/\alpha_{\perp H}$ vs. $\lambda_e$ for the Mode 2 nonresonant instability in the ring current plasma with $N_{0/c}$ = 65.0 cm$^{-3}$, $N_{0H}$ = 1.6 cm$^{-3}$, and $\theta = \cos^{-1}(k_\parallel/k) = 80°$.

of this mode. Figure 2 shows the real frequencies, growth rate and $u^{II}$ for this mode for the observed ring current parameters. The real frequency varies from $\omega_r \approx 175\omega_{ci}$ to $300\omega_{ci}$, and growth rate varies from $\gamma \approx 11\omega_{ci}$ to $17\omega_{ci}$ as $\lambda_e$ is increased from 0.001 to 0.1.

## 2.2. RESONANT INSTABILITY

While discusssing the fluid type instabilities, we neglected the contribution from the pole at $x = \xi$ in Equation (7) or (8) for $\chi_H$. For the resonant instabilities this contribution is crucial. Taking into account the effect of singularity, the susceptibility for the hot protons, in the limit $x \gg \xi$, can be written as

$$\chi_H = \Re_{\chi H} + i\Im_{\chi H}, \tag{16}$$

$$\begin{aligned} \Re_{\chi H} = &\frac{2\omega_{PH}^2}{\pi^{1/2}k^2 u\alpha_{\perp H}R} \\ &\times\left[\frac{1}{2}\exp\left(-\frac{u^2}{\alpha_{\perp H}^2}\right) + \frac{\omega}{k_\perp \alpha_{\alpha H}}\left\{\frac{\sqrt{\pi}}{2}\left[1+\mathrm{erf}\left(\frac{u}{\alpha_{\perp H}}\right)\right]\right.\right. \\ &\left.\left. + \frac{\omega - k_\perp u}{k_\perp \alpha_{\perp H}}\frac{1}{2}E_1\left(\frac{u^2}{\alpha_{\perp H}^2}\right) - 2\pi^{1/2}R\left(\frac{\omega - k_\perp u}{k_\perp \alpha_\perp H}\right)^2\right\}\right] \end{aligned} \tag{17}$$

$$\Im_{\chi H} = \frac{2\pi^{1/2}\omega(\omega - k_\perp u)\omega_{PH}^2}{k_\perp^2 k^2 u \alpha_{\perp H}^3 R}\exp\left[-\left(\frac{\omega - k_\perp u}{k_\perp \alpha_{\perp H}}\right)^2\right] \tag{18}$$

where $E_1$ is an exponential integral function (Abramowitz and Stegun, 1964). Then, substituting for $\chi_H$ in (4), the dispersion relation can be written as

$$D_r(\omega, k) + iD_I(\omega, k) = 0, \tag{19}$$

where

$$D_r(\omega, k) = P - Q\frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} + \Re_{\chi H}, \tag{20}$$

$$D_I(\omega, k) = \Im_{\chi H} \tag{21}$$

The contribution from the term $\Re_{\chi H}$ is usually much smaller than the rest of the terms in (20). Therefore, this term can be ignored while calculating the real frequency from $D_r(\omega, k) = 0$. For the case of $\gamma/\omega_r \ll 1$, the real frequency and growth rate are given by,

$$\omega_r \approx \frac{\omega_{pc}}{\sqrt{P}}\left[1 + Q\frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega_{pc}^2}\right]^{1/2} = \omega_a, \tag{22}$$

$$\gamma \approx -\frac{\sqrt{\pi}\omega_r^2(\omega_r - k_\perp u)\omega_{PH}^2}{k_\perp^2 k^2 u \alpha_{\perp H}^3 RP}\exp\left[-\left(\frac{\omega_r - k_\perp u}{k_\perp \alpha_{\perp H}}\right)^2\right]. \tag{23}$$

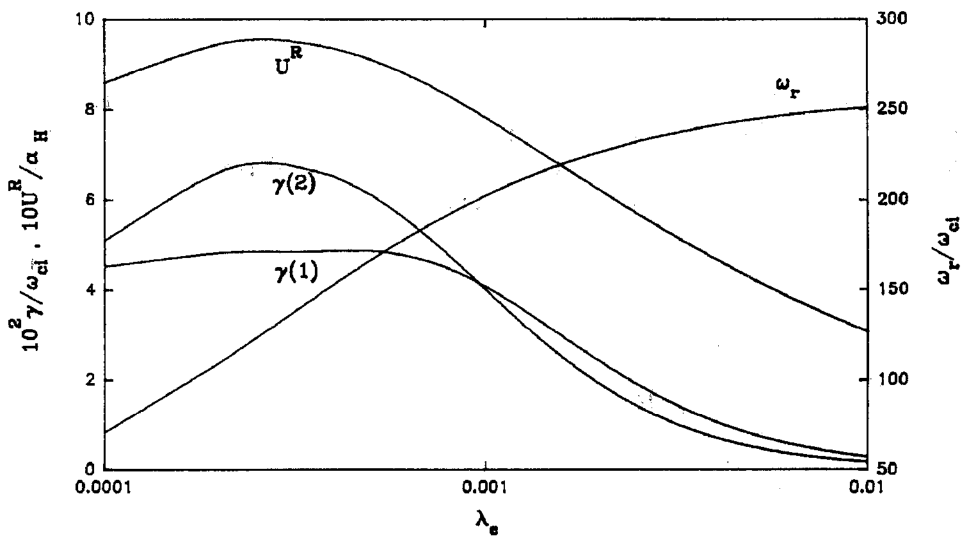


*Figure 3.* Variation of $\gamma/\omega_{ci}$ and $\omega_r/\omega_{ci}$ and $u^R/\alpha_{\perp H}$ vs $\lambda_e$ for the resonant instability in the ring current plasma for $N_{0c}$ = 65.0 cm$^{-3}$, $N_{0H}$ = 1.6 cm$^{-3}$, $\theta$ = 82° and $u/\alpha_{\perp H}$ = 1.2 and 1.6 for the growth rate curves 1 and 2 respectively. The real frequency is not affected by the change in the parameter $u/\alpha_{\perp H}$.

The instability will occur provided $u > U^R$, where $U^R = \omega_a/k_\perp$ is the phase velocity perpendicular to the magnetic field. Figure 3 shows the real frequency and the growth rate for the resonant instability as calculated from (22) and (23) respectively for some typical ring current parameters. From Figure 3 it is seen that instability growth rate increases by an increase in the ring speed $u$ (cf. curves 1 and 2 for $\gamma$), and it is excited at propagation angle of $\theta = \cos^{-1}(k_\parallel/k) = 82°$. The real frequencies and the maximum growth rates associated with this instability are respectively $\omega_r \approx (200–250)\omega_{ci}$, and $\gamma_{\max} \approx (0.05–0.07)\omega_{ci}$ for the parameters of Figure 3. It is seen that instability requires ring speeds $u \sim (1.2–1.6)\ \alpha_{\perp H}$ in order to have significant growth rates $\gamma_{\max} \sim 0.05\omega_{ci}$.

## 3. Results and Discussion

The model for the ring current instabilities described above is quite general. However, here we shall apply our model to a very special class of the ring current proton distributions observed by Perraut et al. (1982). Such energetic proton distributions have been used by McClement et al. (1994) in their analysis pertaining to the excitation of ULF waves in the ring current region.

Our analysis shows that two types of nonresonant instabilities and a resonant instability can be excited in the ring current region by the energetic protons having ringlike distribution functions. Figure 1 indicates that Mode 1 nonresonant instability can be excited when the cold proton number density is very small. This condition is satisfied when the ring current is at $L \sim$ 7–8 or beyond. From Figure 1, it is clear that nominal ring speeds of $u \approx U^I \leq 0.25\alpha_{\perp H}$ are required to excite Mode 1 nonresonant instability. The instability is observed only for almost perpendicular propagation of the wave i.e. for $\cos\theta = k_{\|}/k \sim 0.002$ or so, where $\theta$ is the angle between the wave vector and static magnetic field. Considering the ring current parameters in the region $L$ = 7–8, as $N_{0H} = 1.6$ cm$^{-3}$, $N_{0c} = 10^{-2}$ cm$^{-3}$, $\alpha_{\perp H} = 195.8$ km s$^{-1}$ corresponding to hot-proton temperature of 0.2 keV, and typical average value of $B_0 = 80$ nT (i.e. $\omega_{ci} = 1.2$ Hz), we find from Figure 1 that the Mode 1 nonresonant instability would generate modes with real frequencies $\omega_r \approx 4.25$ Hz and growth rates $\gamma \approx 7.5$ Hz for ring speeds of $u \sim$ (10–50) km s$^{-1}$. Taking electron temperature $T_e$ = 0.2 keV, the electron Larmor radius $\rho_e = (v_{te}/\sqrt{2}\omega_{ce})$ comes out to be about 420 m. Then, the unstable modes would have transverse wavelengths, $\lambda_{\perp} = 2\pi/k_{\perp}$, in the range of $\lambda_{\perp} \sim$ (8–80) km. Our choice of hot–proton temperature of $T_{\perp H}$ = 0.2 keV relates more or less to the parameter $v_{r\perp}/u = 0.15$ as considered by McClements et al. (1994), where the hot proton's ring speed $u = 1166$ km s$^{-1}$ corresponds to the energy of 7.1 keV (Perraut et al., 1982). Since the condition $u \approx U^I \leq 0.25\alpha_{\perp H}$ must be satisfied, Mode 1 nonresonant instability would be excited only when either the ring speed $u$ decreases to as low values as (10–50) km s$^{-1}$ (corresponding to energies of ~0.05 keV or less) when $T_{\perp H}$ = 0.2 keV, or the hot proton temperature gets increased to very high values ~28 keV when the ring speed is fixed at $u$ = 1166 km s$^{-1}$. The latter possibility appears to be unlikely. Furthermore, the assumption $\xi \gg x$ made in Section 2.1, demands that the Doppler shifted phase velocities for these modes be much greater than the perpendicular thermal velocities of the hot-protons. This condition is easily fulfilled for hot-proton temperature of $T_{\perp H}$ = 0.2 keV or less.

The assumptions of $\omega_b < \omega < \omega_a$, and $\omega_b^2/\omega_a^2 < 2$, made during analysis of Mode 2 non-resonant instability, are satisfied for the ring current parameters: $N_{0e}$ = 65.0 cm$^{-3}$, $N_{0H}$ = 1.6 cm$^{-3}$, $\theta = 80°$. This parametric region is expected to exist for $L \sim$ 4–6. It is seen from Figure 2 that ring speeds of $u \approx U^{II}$ = (0.2–1.2)$\alpha_{\perp H}$ are needed for the excitation of Mode 2 nonresonant instability. In the ring current region with $L$ = 4–6, an average value of $B_0$ = 320 nT and electron temperature of $T_e$ = 0.2 keV would yield $\omega_{ci}$ = 4.9 Hz as an average value of the proton cyclotron frequency, and electron Larmor radius of $\rho_e$ = 105 m, Once again considering $\alpha_{\perp H}$ = 195.8 km s$^{-1}$ corresponding to hot-proton temperature of 0.2 keV, Figure 2 predicts the real frequencies of $\omega_r$ = (850–1450) Hz and growth rates of $\gamma$ = (50–85) Hz for the Mode 2 nonresonant instability provided the ring speeds are in the range $u$ = (40–235) km s$^{-1}$ (corresponding to hot-proton ring energies of ~(0.04–0.24) keV). It should be noted that for higher hot-proton temperature, larger ring speeds will be required. For example, for hot-proton temperature of $T_{\perp H}$= 5.0 keV, hot-proton ring energies of the order of (1.0–6.0) keV will be required to excite Mode 2 nonresonant instability (cf. Figure 2). The transverse wavelengths associated with these unstable modes are expected to be in the range of $\lambda_{\perp} \sim$ (2–20) km. Further, for the parameters considered here, the Doppler shifted phase velocities, in the perpendicular direction, associated with Mode 2 nonresonant instabilities are much greater than the perpendicular thermal velocities of the hot-protons, thus justifying the assumption of $\xi \gg x$

It should be noted that the above conclusions apply to the special subpopulation of the ring current protons having ring type distributions with energies $\sim$7 keV and not to the bulk of the energetic part of the ring current population which may have much higher energies $\sim$ a few hundred keV. However, once the excited modes attain a finite level, they could scatter the energetic protons and electrons of the ring current.

The resonant instability is excited for the ring current parameters which are similar to Mode 2 nonresonant instability, i.e. $N_{0c} = 65.0$ cm$^{-3}$, $N_{0H} = 1.6$ cm$^{-3}$, but for the ring speeds greater than the critical value $U^R \approx (0.5\text{–}1.0)\alpha_{\perp H}$. The most favored region for this instability would also be $L \sim$ 4–6. Considering an average value of $B_0 = 320$ nT and $T_e = 0.2$ keV, and $\alpha_{\perp H} = 195.8$ km s$^{-1}$ as above for $L \sim$ 4–6, Figure 3 predicts real frequencies and growth rates of $\omega_r$ = (950–1250) Hz and $\gamma_{\max}$ = (0.25–0.35) Hz respectively for the resonant instability provided $u \geq$ (230–310) km s$^{-1}$. This means that the hot-proton ring energies should exceed (0.24–0.32) keV for the hot-proton temperature of $T_{\perp H} = 0.2$ keV. On the other hand for $T_{\perp H} = 5.0$ keV, the ring energies must exceed (6.0–8.0) keV to excite the resonant mode instabilities. The transverse wavelengths corresponding to the maximum growth rates are expected to be in the range of $\lambda_{\perp} \sim$ (30–65) km. Note that for the resonant modes the perpendicular Doppler shifted phase velocities are of the same order as the perpendicular thermal velocities of the hot-protons.

We must emphasize that our model for the generation of low-frequency electrostatic modes in the ring region would work provided the hot-protons have ring distributions which last for times longer than a few growth times of the instabilities discussed here. As discussed above, the typical growth times for the Mode 1 nonresonant instability, Mode 2 nonresonant instability, and the resonant instability are of the order of 0.15 s, 0.01–0.02 s, and 3–4 s respectively. The hot-proton ring distributions observed by Perraut et al. (1982) last for 3 minutes or longer. Hence, there is sufficient time for the low frequency instabilities to be excited by the ring distributions and for the unstable modes to attain saturation.

The above estimates for real frequencies and the growth rates for resonant and nonresonant instabilities are based on a single value of the angle of propagation in each case. Each of the three instabilities discussed above occurs over a certain range of $k_{\|}/k$ values. Therefore the frequency range of the excited modes will be broader than the estimates given here. An estimate of the saturation electric field of the waves can be made by equating the wave energy density to the available free energy density for the instability (Liewer and Davidson, 1977; Bhatia and Lakhina, 1980c). This gives the level of saturated electric field $E_s$ for the nonresonant modes as

$$E_s = \alpha[2\pi m_p N_{0H} U^{i2}/P]^{1/2}, \tag{24}$$

where the superscript $i$ = I and II for the Mode 1 nonresonant instability and Mode 2 nonresonant instability respectively, and $\alpha$ is a parameter such that $0 < \alpha \leq 1$. For the resonant modes an upper limit for the level of saturated electric field $E_s$ is given by,

$$E_s = \alpha[2\pi m_p N_{0H}(u - U^R)^2/P]^{1/2}. \tag{25}$$

Considering the nominal values of $\alpha = 0.23$ (Papadopoulos *et al.*, 1971), and for the parameters of Figures 1–3, we get $E_s$ = (0.8–4.0) mV/m, (10.0–70.0) mV/m and (20.0–35.0) mV/m for the Mode 1 nonresonant instability, Mode 2 nonresonant instability, and for the resonant instability respectively. Thus, the saturation level of the electric field is the largest for the Mode 2 nonresonant modes for the ring current parameters considered here.

The saturated electric field amplitudes for Mode 2 nonresonant modes, and for resonant modes are of the same order as those for the loss-cone modes discussed by Coroniti et al. (1972). The electrostatic turbulence produced due to low-frequency modes discussed here may be responsible for the pitch angle scattering and loss of ring current protons as well as electrons from the region near but outside the plasmapause (i.e. $L \sim$ 4–6). Thus these modes may be relevant to the ring current decay as well as in the formation of stable auroral red (SAR) arcs.

## 4. Acknowledgments

Dr. S. V. Singh wishes to express his gratitude for the hospitality provided by the Physics Department, University of Durban-Westville, Durban, South Africa. This work was supported by the Foundation of Research and Development, South Africa. G. S. L. thanks the Department of Science and Technology, Government of India, for a research grant under the SERC scheme.

## References

Abramowitz, M. and Stegun, I. A.: 1964, *Handbook of Mathematical Functions*, National Bureau of Standards, Appl. Math. Series 55, U.S. Government Printing Office, Washington DC.
Anderson, R. R. and Gurnett, D. A.: 1973, *J. Geophys. Res.* **78**, 4756–4764.
Ashour-Abdalla, M. and Schriver, D.: 1989, *Geophys Res. Lett.* **16**, 21–24.
Bergmann, R. and Lotko, W.: 1986, *J. Geophys. Res.* **91**, 7033–7045.
Bergmann, R., Roth, I. and Hudson, M. K.: 1988, *J. Geophys. Res.* **93**, 4005–4020.
Bernstein, W., Hultqvist, B. and Borg, H.: 1974, *Planet. Space Sci.* **22**, 767–776.
Bhatia, K. G. and Lakhina, G. S.: 1980a, *Astrophys. Space Sci.* **68**, 175–182.
Bhatia, K. G. and Lakhina, G. S.: 1980b, *Astrophys. Space Sci.* **70**, 467–481.
Bhatia, K. G. and Lakhina, G. S.: 1980c, *Proc. Indian Acad. Sci. (Earth and Planet. Sci.)* **89**, 99–108.
Cornwall, J. M., Coroniti, F. V. and Thorne, R. M.: 1971, *J. Geophys. Res.* **76**, 4428–4445.

Coroniti, F. V., Fredricks, R. W. and White, R.: 1972, *J. Geophys. Res.* **77**, 6243–6248.
Curtis, S. A. and Wu, C. S.: 1979, *J. Geophys. Res.* **84**, 2597–2607.
D'Angelo, N.: 1977, *Rev. Geophys. Space Phys.* **25**, 299–307.
Daglis, I. A., Sarris, E. T. and Wilken, B.: 1993, *Ann. Geophys.* **11**, 685–696.
Fritz, T. A. and Spjeldvik, W. N.: 1979, *J. Geophys. Res.* **84**, 2608–2618.
Ghielmetti, A. G., Johnson, R. G., Sharp, R. D. and Shelley, E. G.: 1978, *Geophys. Res. Lett.* **5**, 59–62.
Grabbe, C. L. and Eastman, T. E.: 1984, *J. Geophys. Res.* **89**, 3865–3872.
Gul'elmi, A. V., Klaine, B. I. and Potapov, A. S.: 1975, *Planet. Space Sci.* **23**, 279–286.
Gurnett, D. A.: 1976, *J. Geophys. Res.* **81**, 2765–2770.
Gurnett, D. A. and Frank, L. A.: 1977, *J. Geophys. Res.* **82**, 1031–1050.
Gurnett, D. A. and Frank, L. A.: 1978, *J. Geophys. Res.* **83**, 1447–1462.
Hultqvist, B., Lundin, R., Stasiewicz, K., Block, L., Lindqvist, P. A., Gustafsson, G., Koskinen, H., Bahnen, A., Potemra, T. A. and Zanetti, L. J.: 1988, *J. Geophys. Res.* **93**, 9765–9776.
Kistler, L. M., Ipavich, F. M., Hamilton, D. C., Gloeckler, G., Wilken, B., Kremser, G. and Stüdemann, W.: 1989, *J. Geophys. Res.* **94**, 3579–3599.
Kistler, L. M., Möbius, E., Klecker, B., Gloeckler, G., Ipavich, F. M. and Hamilton, D. C.: 1990, *J. Geophys. Res.* **95**, 18,871–18,885.
Laakso, H., Junginger, H., Roux, A., Schmidt, R. and de Villedary, C.: 1990, *J. Geophys. Res.* **95**, 10,609–10,621.
Lakhina, G. S.: 1976, *Planet. Space Sci.* **24**, 609–611.
Lakhina, G. S.: 1987, *J. Geophys. Res.* **92**, 12,161–12,170.
Lakhina, G. S.: 1993, *Ann. Geophysicae* **11**, 705–710.
Liewer, P. C. and Davidson, R. C.: 1977, *Nucl. Fusion* **17**, 85–100.
Lundin, R. and Eliasson, L.: 1991, *Ann. Geophysicae* **9**, 202–223.
Lui, A. T. Y., McEntire, R. W., Sibeck, D. G. and Krimigis, S. M.: 1990, *J. Geophys. Res.* **95**, 20,839–20,851.
McClements, K. G. and Dendy, R. O.: 1993, *J. Geophys. Res.* **98**, 11,689–11,700.
McClements, K. G., Dendy, R. O. and Lashmore-Davies, C. N.: 1994, *J. Geophys. Res.* **99**, 23,685–23,693.
Omidi, N.: 1985, *J. Geophys. Res.* **90**, 12,330–12,334.
Papadopoulos, K., Davidson, R. C., Dawson, J. M., Haber, I., Hammer, D. A., Krall, D. A. and Shanny, R.: 1971, *Phys. Fluids* **14**, 849–857.
Parks, G. K., McCarthy, M., Fitzenreiter, R. J., Etcheto, J., Anderson, K. A., Eastman, T. E., Frank, L. A., Gurnett, D. A., Huang, C., Lin, R. P., Lui, A. T. Y., Ogilvie, K. W., Pedersen, A., Reme, H. and Williams, D. J.: 1984, *J. Geophys. Res.* **89**, 8885–8906.
Perraut, S., Roux, A., Robert, P., Gendrin, R., Sauvaud, J. A., Bosqued, J.-M., Kremser, G. and Korth, A.: 1982, *J. Geophys. Res.* **87**, 6219–6236.
Peterson, W. K., Sharp, R. D., Shelley, E. G., Johnson, R. G. and Balsiger, H.: 1981, *J. Geophys. Res.* **86**, 761–767.
Russell, C. T., Holzer, R. E. and Smith, E. J.: 1970, *J. Geophys. Res.* **75**, 755–768.