THE BOUNDARY TO THE SOLAR SYSTEM AS SET BY A HYPOTHETICAL SOLAR COMPANION

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Abstract. If a binary companion to the Sun exists as proposed by Davis *et al.* and Whitmire and Jackson, then one can consider a planet/comet–Sun–solar companion system and use King–Innanen's formula to calculate the limiting direct and retrograde orbits around the Sun. The limiting retrograde orbit could be considered as the boundary to the Solar System. We study the problem for the companion having a mass in the range $0.005M_{\odot}$–$0.3M_{\odot}$ and find the corresponding boundary to the Solar System.

1. Introduction

Recent evidence has indicated that the impact of a comet or an asteroid may have been responsible for the mass extinction at the ends of both the Cretaceous (Alvarez *et al.*, 1980) and the Eocene (Ganapathy, 1982; Alvarez *et al.*, 1982) periods. Quantitative analysis by Raup and Sepkoski (1984) showed that the mass extinctions occur with a 26 Myr period, similar to the period seen in qualitative pelagic records by Fischer and Arthur (1977). To account for the possibility of periodic comet showers, Davis *et al.* (1984) and Whitmire and Jackson (1984) proposed that such showers of comets from the Oort comet cloud could be triggered by an unseen solar companion as it passes through perihelion. To test a prediction implicit in this model Alvarez and Muller (1984) examined records of large impact craters on the Earth. They reported that most of the craters occur in a 28.4 Myr cycle. Within measurement errors, this period and its phase are the same as those found in the fossil mass extinctions. The probability that such agreement is accidental is 1 in 10^3.

If an aforesaid solar companion exists, then one can consider a planet/comet–Sun–Solar companion system and use King–Innanen's formula (King, 1962; Innanen, 1979) to calculate the limiting direct and retrograde orbits around the Sun. The limiting retrograde orbit around the Sun could be considered as the boundary to the Solar System.

Smoluchowski and Torbett (1984) have considered a system consisting of a comet and the Sun rotating around the galactic center and periodically traversing the nearly harmonic field of the galactic plane and have described the results of a three-dimensional stability study. They have also determined the shape of the boundary of the Solar System defined as the surface within which the gravitational attraction of the Sun, rather than that of the rest of the Galaxy, controls the orbital motion of bodies such as planets and comets. A two-dimensional model of greatly simplified

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version of this problem has been studied previously by Chebotarev (1965, 1966) and the corresponding three-dimensional Hill surface has been discussed by Antonov and Latyshev (1972). We calculate here the boundary to the Solar System set by the solar companion proposed by Davis *et al.* (1984) and Whitmire and Jackson (1984) using King–Innanen's formula. We study the problem for the companion star having its a mass in the range $0.005M_\odot$–$0.3M_\odot$ and find the corresponding boundary to the Solar System.

2. Discussion and Results

First, we determine the semi-major axis, a, of the elliptical orbit described by the solar companion corresponding to some selected values of its mass, M, in the range $0.005M_\odot$–$0.3M_\odot$ and the orbital period 26 Myr using Kepler's third law: $(m + M)P^2 = a^3$ where m and M are the masses of the binary components in solar units and a is in AU (Table I).

2.1. Work of King and Innanen

King (1962) in his study of clusters estimates the tidal limit of a cluster by defining it to be a point on the line connecting the center of the cluster with the galactic center at which a star can remain on the line of centers with an acceleration along that line that is zero with respect to the center of the cluster. That is, at the moment of perigalactic passage, that star is pulled neither toward nor away from the cluster. He obtains the expression for limiting tidal radius, $r_{\rm lim}$, of the cluster to be

$$r_{\rm lim} = \left[\frac{Gm}{\Omega^2 - {\rm d}^2V/{\rm d}R^2}\right]^{1/3}, \tag{1}$$

where r is the radial distance of a star from the center of the cluster; m, the mass of the cluster; Ω, the angular velocity of the cluster around the galactic center; V, the gravitational potential energy of the galaxy and R, the radial distance of the cluster from the galactic center. If we represent the force field of the galaxy by an inverse-square law due to a mass M, then

$${\rm d}^2V/{\rm d}R^2 = -2GM/R^3; \tag{2}$$

and, hence,

$$r_{\rm lim} = \left[\frac{m}{3M}\right]^{1/3} R. \tag{3}$$

If the cluster's orbit about the galactic center is an ellipse, then the angular velocity at any point is given by

$$\Omega^2 = GMa(1 - e^2)/R^4, \tag{4}$$

where e is the eccentricity of the ellipse. At the perigalactic point, R takes the value

$$R_p = a(1 - e), \tag{5}$$

and Equation (1) in this case becomes

$$r_{\lim} = \left[\frac{m}{(3+e)M}\right]^{1/3} \times R_p.$$

Innanen (1979) applies King's formula for a system of a star–cluster–galaxy to systems of a moon–planet–Sun and a star–dwarf galaxy–galaxy. A moon revolving about a planet that, in turn, is revolving in the same sense about the Sun will, at some limiting distance from the planet become unstable because of the action of the Sun's tidal force. At greater limiting distance from the planet, this retrograde moon would eventually succumb to the Sun's tidal force. This limiting retrograde radius should properly define the true gravitational sphere of influence of a planet. Innanen (1979) uses the equation for acceleration in a rotating coordinate frame with an additional Coriolis term of magnitude $2\Omega v_r$, where v_r is the velocity of the Moon relative to the planet. The familiar right-hand rule immediately shows that the Coriolis term is always directed radially between the Moon and the planet. It counteracts the planet's gravity for direct motion of the Moon. but effectively supplements the planet's gravity for retrograde motion. For the limiting direct and retrograde radii of a moon around a planet, r_d and r_r, respectively, he gets

$$r_r/r_d = 3^{2/3} \tag{7}$$

and

$$r_d = \left[\frac{m}{3^2 M}\right]^{1/3} R. \tag{8}$$

For the case where the planet's orbit has eccentricity e, and pericentric distance $R = R_p$, we have

$$r_d = \left\{\frac{m}{[f(e)]^2 M}\right\}^{1/3} \times R_p \tag{9}$$

where

$$f(e) = \left[\frac{5 + e + 2(4+e)^{1/2}}{3+e}\right]$$

and

$$r_r/r_d = [f(e)]^{2/3}. \tag{10}$$

For a general two-body problem, we have

$$r_d = \left\{\frac{1}{[f(e)]^2}\,\frac{m}{M}\,\frac{1}{[1+(m/M)]}\right\}^{1/3} \times R_p. \tag{11}$$

2.2. Boundary to the Solar System set by a solar companion

Innanen (1979) considers a moon–planet–Sun system and finds the limiting direct and retrograde orbits for all planets, thus setting the boundaries to all satellite

TABLE I

Mass M of the solar companion in solar units	Semi-major axis, a, of the companion in AU	Pericentric distance, R_p, in AU	Limiting direct radius, r_d, in AU	Limiting retrograde radius, r_r, in AU
0.005	87880	26364	13530	26320
0.05	89190	26757	13520	26310
0.1	90570	27161	13530	26310
0.2	93240	27972	13530	26320
0.3	95740	28722	13530	26310

systems. In his work the Sun is a perturber. Here, we consider a planet/comet–Sun–solar companion system, wherein the solar companion acts as a perturber, and calculate the limiting direct and retrograde orbits, r_d and r_r respectively, for a planet/comet around the Sun. They are set out in Table I. In these calculations, we have assumed the value of e to be 0.7 (Davis *et al.*, 1984). r_r could be considered as the boundary to the Solar System set by the solar companion.

2.3. Boundary to the Solar System and the Oort comet cloud

On the basis of his studies of motion of long-period comets, Oort (1950) postulated the existence of a vast cloud of comets surrounding the Solar System extending up to a distance as large as 150000 AU or more from the Sun. The question now is whether or not the entire Oort comet cloud is bounded to the Sun? If it is entirely bounded to the Solar System, then the discussion mentioned above and the results obtained (Table I) implies that the extension of the Oort cloud cannot be up to 150000 AU or more but is limited by the boundary of the Solar System determined by the proposed solar companion (Table I). However, seeing the span of the Oort cloud cited in the literature, it appears that it may have extended up to a distance as large as 150000 AU or more. If this is so, then it extends beyond the boundary of the Solar System set by the proposed companion of the Sun, and in this case, the inner part of the Oort cloud is bounded to the Sun but not the outer part. The comets residing in the outer part of the Oort cloud move independently of the Sun with a gaussian velocity distribution characterized by the mean circular velocity and that the capture takes place if a comet enters the gravitational sphere of influence of the Sun at less than the local escape velocity (Clube and Napier, 1982). Whenever a near encounter between the Sun and its companion or a nearby star or a giant molecular cloud takes place, it perturbs the Oort cloud and sends a large number of comets towards the Sun from the outer part of the Oort cloud.

If it is true that the entire Oort cloud up to 150000 AU or more is bounded to the Sun, it follows that in this case the solar companion is not bounding the Solar System. Its status is like that of a planet – a member of the Solar System. In this case, the boundary to the Solar System is truly determined by the Galaxy. This problem has been discussed by us elsewhere.

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References

Alvarez, L. W., Alvarez, W., Asaro, F., and Michel, H. V.: 1980, *Science* **208**, 1095.
Alvarez, W., Asaro, F., Michel, H. V., and Alvarez, L. W.: 1982, *Science* **216**, 886.
Alvarez, W. and Muller, R. A.: 1984, *Nature* **308**, 718.
Antonov, V. A. and Latyshev, I. N.: 1972, in G. A. Chebotarev, Kazimirchakpolonskaya and B. G. Marsden (eds.), *The motion, Evolution of Orbits, and Origin of Comets,* D. Reidel Publ. Co., Dordrecht, Holland, p. 341.
Chebotarev, G. A.: 1965, *Soviet Astron. J.* **8**, 787.
Chebotarev, G. A.: 1966, *Soviet Astron. J.* **10**, 341.
Clube, S. V. M. and Napier, W. M.: 1982, *Quart. J. Roy. Astron. Soc.* **23**, 45.
Davis, M., Hut, P., and Muller, R. A.: 1984, *Nature* **308**, 715.
Fischer, A. G. and Arthur, M. A.: 1977, in H. P. Cook and P. Enos (eds.), *Deep-Water Carbonate Environments*, Society of Economic Paleontologists and Mineralogists, Tulsa, Oklahoma, Spec. Publ. 25, p. 19.
Ganapathy, R.: 1982, *Science* **216**, 885.
Innanen, K. A.: 1979, *Astron. J.* **84**, 960.
King, I.: 1962, *Astron. J.* **67**, 471.
Oort, J. H.: 1950, *Bull. Astron. Inst. Neth.* **11**, 91.
Raup, D. M. and Sepkoski, J. J.: 1984, *Proc. Nat. Acad. Sci. U.S.A.* **81**, 801.
Smoluchowski, R. and Torbett, M.: 1984, *Nature* **311**, 38.
Whitmire, D. and Jackson, A.: 1984, *Nature* **308**, 713.