THE ESTIMATE OF THE DECELERATION IN THE EARTH'S ROTATION DUE TO THE SUN*

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Abstract. The tidal long-term decrease in the angular velocity of the Earth's rotation has been estimated on the basis of the angular momentum tidal balance in the Earth-Moon-Sun system. The observed (LLR) tidal long-term decrease in the Moon's mean motion, the apparent secular acceleration in the mean longitude of the Sun and the long-term decrease in the 2nd degree zonal geopotential parameter were used.

1. Introduction

The long-term variation $d\omega/dt$ in the angular velocity ω of the Earth's rotation is the significant global geodynamic phenomenon proved by observations. It is due to the tidal evolution of the Earth-Moon-Sun system because of tidal friction, as well as, to the non-tidal forces giving rise to the longterm increase in ω.

The problem is complicated because of the facts as follows: (a) not only the tidal forces are responsible for the phenomenon; (b) no observation of the *tidal* decrease $dn_\odot/dt$ in the Sun's mean motion $n_\odot$ and/or of the tidal increase $da_\oplus/dt$ in the semi-major axis $a_\oplus$ of the Earth's heliocentric orbit is possible practically. However, the tidal part in $d\omega/dt$, corresponding to $dn_\odot/dt$, may be estimated on the basis of the angular momentum balance of the system.

2. Data Obtained from Observations

The observed data on the basis of which the problem is to be solved, are as follows: (1) The tidal long-term decrease $dn_{☾}/dt$ in the Moon's mean motion $n_{☾}$ from LLR (Newhall *et al.*, 1990)

$$dn_{☾}/dt = -(24.9 \pm 1.0) \text{ arcsec cy}^{-2} . \tag{1}$$

(2) The apparent secular acceleration $\nu_\odot$ in the mean longitude of the Sun observed in UT during ~2000 y on the basis of the solar eclipses (de Sitter, 1927)

$$\nu_\odot = (1.80 \pm 0.16) \text{ arcsec cy}^{-2} . \tag{2}$$

The most recent value is by Lyttleton and Fitch (1978) as

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$$\nu_\odot = (1.47 \pm 0.8) \text{ arcsec } cy^{-2} = \\ = (7.16 \pm 0.39) \times 10^{-25} \text{ rad s}^{-2}\,; \qquad (2')$$

the last will be used in the solution.

First of all, the secular variation $\mathrm{d}n_\odot/\mathrm{d}t$ and/or $\mathrm{d}a_\oplus/\mathrm{d}t$ is small, its estimate is as

$$\mathrm{d}n_\odot/\mathrm{d}t \sim -4.2 \times 10^{-7} \text{ arcsec cy}^{-2}, \qquad (3)$$

and/or

$$\mathrm{d}a_\oplus/\mathrm{d}t \sim 3.2 \times 10^{-4} \text{ m cy}^{-1}\,; \qquad (3')$$

however, its contribution to $\mathrm{d}\omega/\mathrm{d}t$ is significant, about 1/5 of the total value. Estimates (3), (3′) are based on the supposition not verified that the product $(k\epsilon)$ of the Love number k and the phase lag angle ϵ is identical for both Earth-Moon and Earth-Sun systems. That is why we face the problem of determining $(\mathrm{d}\omega/\mathrm{d}t)_\odot$, i.e., the long-term tidal decrease in ω due to the Sun.

Compared with (2′), the value (3) is small and we may consider (2′) as free of the tidal contribution due to the Sun. In that case,

$$\mathrm{d}\omega/\mathrm{d}t = -2\nu_\odot\omega/n_\odot = -(5.24 \pm 0.29) \times 10^{-22} \text{ rad s}^{-2}. \qquad (4)$$

3. Solution of the Problem

The theory of the tidal evolution of the Earth-Moon-Sun system suggests, that value (4) is, at least partly, of the tidal origin. The tidal contribution to it $(\mathrm{d}\omega/\mathrm{d}t)_☾$, due to the Moon only, can be computed on the basis of observed value (1) as (Burša, 1991)

$$(\mathrm{d}\omega/\mathrm{d}t)_☾ = -(5.36 \pm 0.20) \times 10^{-22} \text{ rad s}^{-2}. \qquad (5)$$

Let us compare (5) and (4). We learn that, there is "no space" in (4) for $(\mathrm{d}\omega/\mathrm{d}t)_\odot$, because values (5) and (4) are nearly equal. However, if (5), it should hold that $(\mathrm{d}\omega/\mathrm{d}t)_\odot \neq 0$.

The only explanation for this formal ("seen") contradiction is that, there should be a nontidal positive long-term increase $(\mathrm{d}\omega/\mathrm{d}t)_{\text{nontidal}}$ in (4). The phenomenon was predicted by Pariiskii in 1954, however, the value he predicted was too large: 3×10^{-22} rad s^{-2}. Supposing $(k\epsilon)_\odot = (k\epsilon)_☾$ the value was pre-computed (Burša, 1984) as $(1 \pm 0.35) \times 10^{-22}$ rad s^{-2}.

However, there is a global geodynamic phenomenon actually being observed, which is believed to give rise to $(\mathrm{d}\omega/\mathrm{d}t)_{\text{nontidal}}$, under supposition of zero long-term variation in the sum of the Earth's principal moments of inertia $\delta(A + B + C) = 0$, $C > B > A$. This phenomenon is the long-term decrease in the second zonal geopotential parameter J_2 detected from LAGEOS orbit dynamics (IERS Standards, 1989)

$$dJ_2/dt = -(2.8 \pm 0.3) \times 10^{-9}\,\mathrm{cy}^{-1}\,. \tag{6}$$

Under the conditions stated above one gets

$$(d\omega/dt)_{\mathrm{nontidal}} = +(1.147 \pm 0.084) \times 10^{-22}\,\mathrm{rad\,s^{-2}}\,. \tag{7}$$

No other geodynamic phenomenon has been observed which gives rise to the positive long-term acceleration in the Earth's rotation.

If (7) is a part of (4) and if no other geodynamic phenomenon exists which gives rise to the contribution to the positive long-term increase in the angular velocity of the Earth's rotation, then $(d\omega/dt)_\odot$ can be expressed as

$$\left(\frac{d\omega}{dt}\right)_\odot = \frac{d\omega}{dt} - \left(\frac{d\omega}{dt}\right)_{☾} - \left(\frac{d\omega}{dt}\right)_{\mathrm{nontidal}} =$$
$$= -(1.03 \pm 0.30) \times 10^{-22}\,\mathrm{rad\,s^{-2}}. \tag{8}$$

It is the pre-computed tidal contribution due to the Sun to the long-term decrease in the angular velocity of the Earth's rotation.

We now face the problem to estimate the actual accuracy of (8). Let us try to do it with the expression of the corresponding tidal torque $\overline{\delta N_\odot}$ due to the Sun exerted on Earth. It should hold that

$$C\left(\frac{d\omega}{dt}\right)_\odot = \overline{\delta N_\odot}\,; \tag{9}$$

$C = 8.0358 \times 10^{37}\,\mathrm{kg\,m^2}$ is the Earth's maximum principal moment of inertia. Tidal torque $\delta N_\odot$ is the component of the total tidal torque, along the Earth's rotation axis. It was derived as a part of the total solutions (cf. Burša, 1987)

$$\delta N_\odot = -3(k\epsilon)_\odot \frac{GM_\odot^2}{AU^6} R^5 \cos^2 \delta_\odot\,; \tag{10}$$

$AU = 1.495 \quad 9787 \times 10^{-11}\,\mathrm{m}$, $GM_\odot = 13\,271\,244 \times 10^{13}\,\mathrm{m^2\,s^{-2}}$, $G = 6.672\,59 \times 10^{-11}\,\mathrm{m^3\,s^{-2}\,kg^{-1}}$, $R = 6\,371\,000\,\mathrm{m}$. The geocentric declination of the Sun, $\delta_\odot$ is a periodic function of time; however, it has a permanent (zero frequency) part, the integral mean value being

$$\overline{\cos^2 \delta_\odot} = \tfrac{1}{2}(1 + \cos^2 \epsilon_0)\,,$$
$$\overline{\sin^2 \delta_\odot} = \tfrac{1}{2}\sin^2 \epsilon_0\,; \tag{11}$$

$\epsilon_0 = 23°26'21.412''$ and numerically $\frac{1}{2}(1 + \cos^2 \epsilon_0) \doteq \cos \epsilon_0 \doteq \overline{\cos^2 \delta_\odot}$. The permanent (zero frequency) tidal torque $\overline{\delta N_\odot}$ is

$$\overline{\delta N_\odot} = -3(k\epsilon)_\odot \frac{GM_\odot^2}{AU^6} R^5 \cos \epsilon_0 =$$
$$= -6.80(k\epsilon)_\odot \times 10^{17}\,\mathrm{kg\,m^2\,s^{-2}}\,. \tag{12}$$

Inserting (12) into (9) we can express the representative "effective global" (i.e., for both ocean and continental tides) value of the product $(k\epsilon)_\odot$ of the Love number and the tidal phase lag angle due to the Sun to be:

$$(k\epsilon)_\odot = 0.0122 \pm 0.0036 \,. \tag{13}$$

It represents the effect of both the continental, as well as, the oceanic tides. Analogously (again, $\cos^2 \delta_{\mathbb{C}} \doteq \cos \epsilon_0$)

$$\delta N_{\mathbb{C}} = -3.23(k\epsilon)_{\mathbb{C}} \times 10^{18}\,\mathrm{kg\,m^2\,s^{-2}} \tag{14}$$

and with the use of (5)

$$(k\epsilon)_{\mathbb{C}} = 0.0134 \pm 0.005 \,. \tag{15}$$

4. Conclusions

(1) The long-term decrease in J_2 (6) fits well the tidal angular momentum balance in the Earth-Moon-Sun system. (2) The long-term decrease in ω due to the Sun, $(\mathrm{d}\omega/\mathrm{d}t)_\odot$ computed on the basis of observed value (2), (4) and (6) may be considered as realistic.

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