

# LATITUDE DEPENDENCE OF AURORAL FREQUENCY IN RELATION TO SOLAR - TERRESTRIAL AND INTERPLANETARY PARAMETERS

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**Abstract.** The auroral frequency of occurrences ( $A$ ) for the 20th solar cycle and for the geomagnetic latitudes  $54^\circ$ – $63^\circ$  N has been investigated in relation to sunspot numbers ( $R_z$ ), number of flares ( $F$ ), the solar wind streams derived from the coronal holes ( $H$ ) and the geomagnetic index ( $A_p$ ). The relationship between  $A$  and the other indices were found to be strongly latitude dependent. At around  $57^\circ$ – $58^\circ$  N, a drastic change in this relationship occurs, and an attempt is made qualitatively to evaluate this latitudinal variation.

## 1. Introduction

The long known link between magnetic phenomena and the aurora implies an auroral-sunspot connection, but the auroral statistics were so incomplete that this could be doubted as late as 1868. This relation has been illustrated however by the systematic records of the aurora made at the Yerkes observatory by Meinel *et al.* (1954). Lassen (1967) in addition has also discussed the relation between polar Cap aurora,  $A_p$  and Sunspot activity at  $77.5^\circ$  N geomagnetic latitude.

For this reason in the present study the frequency of occurrence of aurorae borealis ( $A$ ) is investigated through other solar-terrestrial parameters, that is, the relative index of Zürich referred to the number of sunspots ( $R_z$ ), the number of flares ( $F$ ), of importance  $\geq 1$  of the solar winds stream ( $H$ ) of the coronal holes and the geomagnetic index  $A_p$ .

This study is referred to the period 1903–1982 though a complete account is given for the 20th solar cycle period. The correlation between ( $A$ ) and the other indices will be discussed in relation to the geomagnetic latitude effect, the solar magnetic field reversals (SMFR) the 11-yr solar cycle and the interplanetary magnetic field ( $B_z$ ).

The impact of the SMFR on various solar phenomena was mentioned initially by Jokipii *et al.* (1977) and the subsequent efforts to reconfirm such an effect was made by Shea and Smart (1981) and Chirkov (1979) for the aa geomagnetic index for the period 1954/80 and by Xanthakis *et al.* (1981) for the cosmic ray intensity. The auroral region has been defined in the international auroral Atlas as the region from  $60$ – $70^\circ$  latitude. The distribution of  $A$  varies over the solar cycle but the maximum does not shift, though the curve is skewed more to higher latitudes at sunspot minimum (Stringer and Belon, 1967).

## 2. Selection of Data

The number of annual auroral frequency were taken from the Yerkes observatory (Meinel *et al.*, 1954) and cover the period 1897/1951. Figure 1. The auroral frequency numbers for the later period 1952/1982 were taken from the Balfour/Stewart (1952/1975) all-sky camera and British Astronomical Association group (1975/1982), (Figure 2), recording also the occurrence of the aurora at various geomagnetic latitudes in Great Britain from  $54^{\circ}$ – $63^{\circ}$  N.

The numbers of flares ( $F$ ) with  $\text{Imp} \geq 1$ , were taken from Smith and Smith (1963) and Dodson *et al.* (1977). These are ground based observations. The number of  $R_z$  and  $A_p$  were obtained from the Solar-Geophysical data series and the number of solar wind streams from flares ( $f$ ) and coronal holes ( $H$ ) from Lindblad and Lundstedt (1981). The latter are measured from space probes and earth-orbiting spacecraft. According to Lindblad and Lundstedt, a high-speed plasma stream (HSPS) includes both ( $H$ ) and ( $f$ ) and this is recorded whenever the velocity difference  $\Delta v_0$  (the difference between the smallest 3-hr velocity value in a given day and the largest 3-hr value the following day) is greater to  $80 \text{ km s}^{-1}$ . The flare ( $f$ ) associated recognition was adopted from tables and diagrams published by Hundhausen (1972) and Iucci *et al.* (1979). We have found that the correlation coefficient ( $r$ ) between ( $f$ ) and flares ( $F$ ) of Importance  $\geq 3$  is 0.735, for flares of  $\text{Imp} \geq 2$  is 0.82 and for flares ( $F$ ) of  $\text{Imp} \geq 1$  is 0.87.

These coefficients are very significant at the probability level  $P \leq 0.01$ . For these reasons we have used the ( $F$ ) index throughout this study. Studies on SMFR have been made by Makarov *et al.* (1981), Haward (1974, 1972) and Hundhausen (1979).

## 3. Correlation Study between $A$ and $R_z$

It has been supported that there is no doubt of the existence of an 11-yr cycle in the frequency of the overall occurrence of aurora (Jones, 1974). Nevertheless we have observed a 1-yr lag with respect to the sunspot number curve for the period 1897/1951. Meinel *et al.* (1954) and Pokorný (1973) have also noted that ( $A$ ) lags ( $R_z$ ) by 2 y. Slater and Smith (1981) have found that the occurrence rates of stable auroral red arc follows the solar activity cycle though with a phase lag of 2-3 y for the 19th and 20th solar cycles in Battelle Observatory ( $46.4^{\circ}$  N and  $240.4^{\circ}$  E). For the above reasons it was examined and extended the correlation between ( $A$ ) and ( $R_z$ ) with various time-lags for the time period 1903–1982 in Liritzis and Petropoulos (1986, Table I).

From that study it was seen that the time lag varied from 0-3 yr for ( $A$ ) and ( $R_z$ ). If we consider the time series of the auroral frequency and solar activity indices for each solar cycle as accidental statistical samples of the corresponded populations, we can estimate true correlation for the populations of the series involved (1903-82) at a confidence level 0.05. This could be done by applying Fisher's Z-transformation

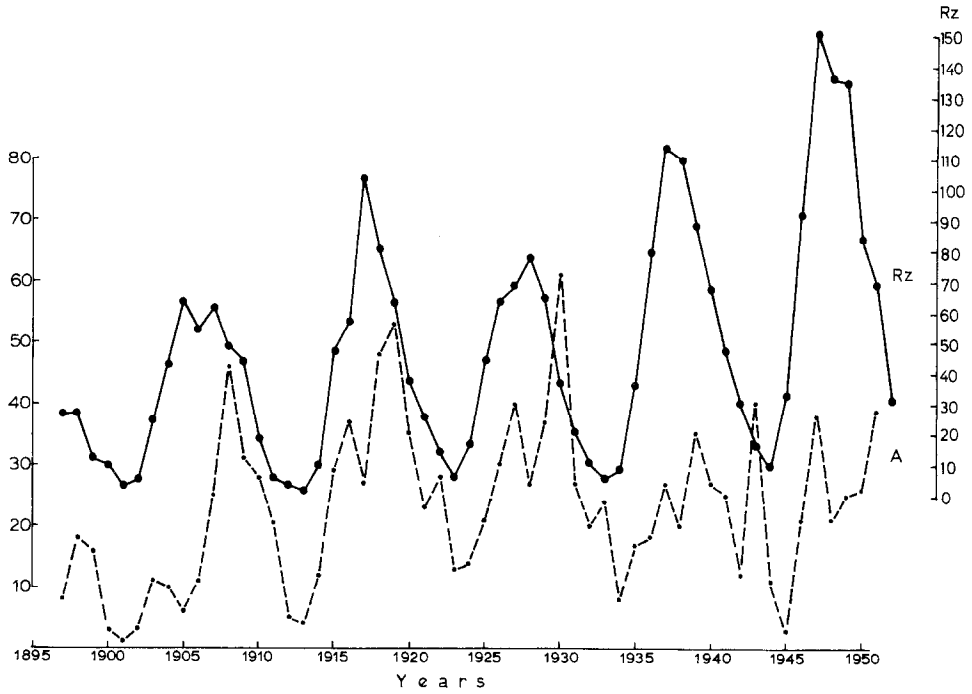


Fig. 1. Variations of  $A$  and  $R_z$  for the period 1895–1953.

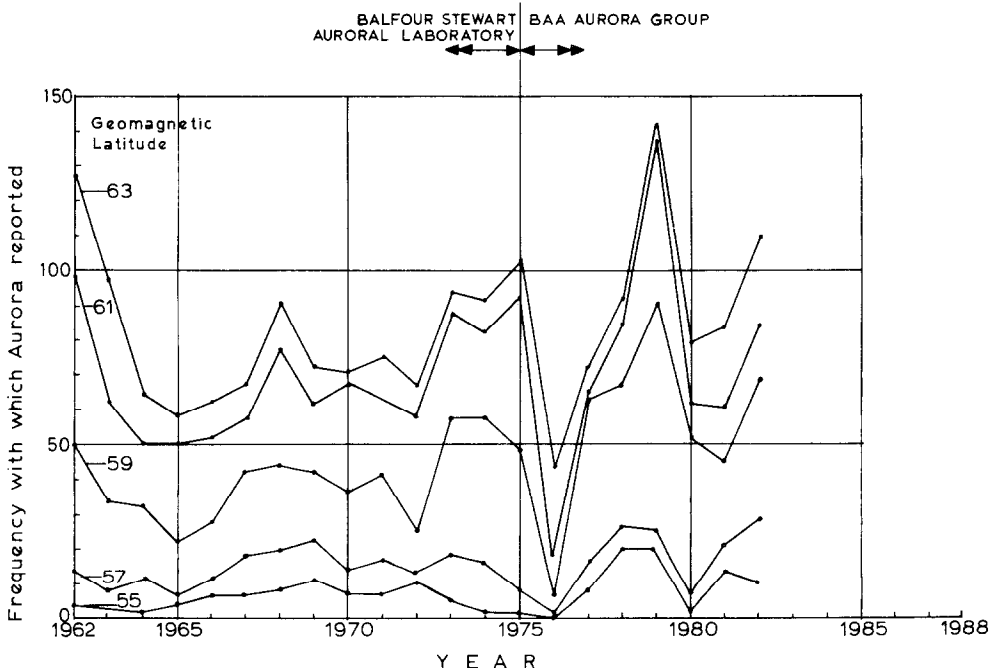


Fig. 2. Variation of aurorae occurrences for various northern latitudes for the period 1962–1982 (data kindly supplied by Dr Livesey of the British Astronomical Association).

according to the equation

$$Z = 1/2 \ln[(1 + r)/(1 - r)],$$

where ( $r$ ) is the correlation coefficient of the above-mentioned samples and  $Z$  a statistic following approximately a normal distribution with mean  $\mu_z$  and standard deviation  $S_z$ . The statistics  $Z$ ,  $\mu_z$  and  $S_z$  help to define the criterion

$$F' = \frac{Z - \mu_z}{S_z},$$

which also follows an approximate normal distribution. If we apply a one – tail test on the normal distribution of  $F'$ , we can define the lower value of the correlation coefficient or true correlation of the populations corresponded to  $F'$  for a certain confidence level (Fisher, 1958). The true ( $r$ ) appears to reach the highest value of 0.66 (15th solar cycle) while the whole time series from 1903–1983 present an  $r = 0.48$ . Overall the true ( $r$ ) is quite low at 0.05 confidence level. This low ( $r$ ) was increased in combining all four indices of  $H$ ,  $R_z$ ,  $A_p$ , and  $F$ .

The periodic, however, character of  $A$  values examined previously by spectrum analysis revealed the existence of two periods of 3–4 and 8–10 y (Liritzis and Petropoulos, 1986). In fact Link (1978), suggests a mean period of solar cycles of nearly  $10.5 \pm 0.6$  y obtained from auroral minima. Further, for the  $A - R_z$  intercomparison we should recall that the correlation coefficient ( $r$ ) for certain time lag depends on the solar cycle as well as on the geomagnetic latitude ( $L$ ). In particular for the 19th solar cycle the time lag is 0 to + 1 y as Slater and Smith (1981) had reported for red arc aurora.

For the 20th solar cycle the lag is 1-yr for  $L = 59^\circ$  N and it is zero for  $L = 55^\circ$  N in contrast to Slater and Smith who report 2–3 yr time lag (Table IIa).

Regarding the correlation coefficient between ( $A$ ) and ( $R_z$ ) it seems to vary for even-odd solar cycles (peculiarities of solar cycles) and a secular variation of this ( $r$ ) shows-up in Figure 3.

#### 4. Effects of Solar Magnetic Field Reversal on $A$ and $A - R_z$ Relationships

Solar Magnetic field reversals (SMFR) have been studied by Makarov *et al.* (1981) Babcock (1955), Howard (1972, 1974) for the period 1904–1981 at heliolatitudes  $50^\circ$  to  $90^\circ$  north and south. The first found that the years of polarity reversals shift towards the pole with different time intervals (or solar cycles). For every solar cycle the periods of polarity reversals are shown in Figure 4.

For SMFR periods the respective  $A - R_z$  correlation give:  $r = 0.756$  for 1958/68 and  $r = 0.443$  for 1968/79 for two successive maxima; while  $r = 0.860$  for 1954/64 and  $r = 0.137$  for 1964/76 for two successive minima (Figure 4). At periods of optimum flare occurrence the ( $r$ ) between  $A$  and  $R_z$  is disturbed (Liritzis and Petropoulos, 1986). Of particular importance is the observation that the 2nd auroral peak coincides with the period of SMFR for the period 1902–1982. (Figure 4a, b, c).

TABLE I

(A, X) correlations for ascending (A) and descending (D) parts of solar cycles in relation to SMFR. (Note the non-significant correlation between (A, SMF) during an interval of SMFR.)

Indices	Time	$r$	$L^\circ$	Constant term	Slope	Ascending (A) Descending (D) branch	SMFR
$A-F$	1964/68	0.46	54	1.97	0.003	A	$X$
„	1969/74	0.49	„	3.03	0.0049	D	
„	1964/68	0.71	63	56.76	0.0365	A	
„	1969/74	0.53	„	87.02	-0.027	D	
$A-H$	1964/68	0.47	54	5.03	-0.11	A	$X$
„	1969/74	0.69	„	11.106	-0.257	D	
„	1964/68	0.57	63	50.73	0.926	A	
„	1969/74	0.95	„	32.80	1.808	D	
$A-A_p$	1964/68	0.30	54	0.254	0.255	A	$X$
	1969/74	0.80	54	12.0	-0.528	D	
	1964/68	0.88	63	13.13	5.07	A	
	1969/74	0.88	63	36.10	3.01	D	
$A-A_p$	1975/79	0.47	63	-35.26	8.9	A	
$A-R_z$	1964/68	0.41	54	2.05	0.017	A	$X$
„	1969/74	0.68	„	1.09	0.051	D	
„	1964/68	0.76	63	55.5	0.22	A	
„	1969/74	0.77	63	99.0	-0.30	D	
$A-R_z$	1975/79	0.86	63	55.5	0.53	A	
$A-R_z$ (other solar cycles)	1954/57	0.80	59.0	47.23	0.30	A	$X$
	1958/64	0.91	59.0	30.69	0.45	D	
	1944/47	0.93	52.6	2.48	0.219	A	
	1948/51	0.81	52.6	47.24	-0.18	D	
	1933/37	0.54	52.6			A	$X$
	1937/43	0.12	52.6			D	
	1923/28	0.86	52.6	10.19	0.302	A	
	1929/33	0.53	52.6	23.65	0.366	D	$X$
$A-SMF$	1934/44	0.33(0) 0.55(+1) 0.44(+2)		16.9 12.5 14.7	0.017 0.0031 0.0025		$X$

**5. Dependence of Auroral Occurrence from Other Solar-Terrestrial Parameters**

The occurrence of visual discrete aurorae depends not only from the solar activity  $R_z$  but on other phenomena of the solar corona that emit corpuscular radiation accelerated by the interplanetary magnetic field ( $B_z$ ), as well as on terrestrial ones. (Flares, sunspots, coronal holes, and geomagnetic field). Figure 5 shows the variation

TABLE IIa  
Correlation coefficients of  $(A, X)$  for different latitudes

$L$ ( $^{\circ}$ N)	$(A, R_z)$	$(A, A_p)$	$(A, F)$	$(A, H)$	$T(A, R_z)$
54 $^{\circ}$	0.560	-0.182	0.470	-0.286	0
55 $^{\circ}$	0.722+	-0.365	0.625 +	-0.340	0
56 $^{\circ}$	0.921**+	0.314	0.733 +	0.253	-1
57 $^{\circ}$	0.761**+	0.449	0.509	0.483	-1
58 $^{\circ}$	0.410	0.886**+	0.206	0.771*	-1
59 $^{\circ}$	0.340	0.853**+	0.116	0.730+	-1
60 $^{\circ}$	0.321	0.817**+	0.040	0.848*	-1
61 $^{\circ}$	0.410	0.904**+	0.121	0.821*	-1
62 $^{\circ}$	0.409	0.899**+	0.111	0.822**+	-1
63 $^{\circ}$	0.396	0.882**+	0.0872	0.797**+	-1
$r(X, L)$	-0.67+	0.899**+	-0.878**+	0.881**+	

\* Significant at 99% ( $P < 0.01$ ).

+ Significant at 95% ( $P < 0.05$ ).

$T$  = Time-lag for  $R_z$ .

TABLE IIb  
Constant of  $(A, X)$  relationships for different latitudes ( $C_L$ )

$L^{\circ}$ N	$(A, R)$	$(A, A_p)$	$(A, F)$	$(A, H)$
54 $^{\circ}$	1.77	5.42	2.48	5.74
55 $^{\circ}$	3.09	10.54	4.2	9.43
56 $^{\circ}$	5.31	7.82	6.99	7.84
57 $^{\circ}$	9.34	7.62	11.79	8.87
58 $^{\circ}$	21.25	-7.44	22.27	4.57
59 $^{\circ}$	34.5	1.35	36.86	14.15
60 $^{\circ}$	40.7	10.7	46.44	18.12
61 $^{\circ}$	55.0	20.14	61.93	33.34
62 $^{\circ}$	58.4	17.97	66.43	33.14
63 $^{\circ}$	64.63	30.34	71.73	43.42

TABLE IIc  
Slope of  $(A, X)$  relationships for different latitudes ( $B_L$ )

$L^{\circ}$ N	$(A, R)$	$(A, A_p)$	$(A, F)$	$(A, H)$
54 $^{\circ}$	0.034	-0.121	0.0815	-0.0836
55 $^{\circ}$	0.056	-0.312	0.0077	-0.0127
56 $^{\circ}$	0.077	0.196	0.0098	0.103
57 $^{\circ}$	0.087	0.569	0.0093	0.267
58 $^{\circ}$	0.120	2.90	0.0118	1.102
59 $^{\circ}$	0.108	2.99	0.059	1.121
60 $^{\circ}$	0.103	2.92	0.002	1.322
61 $^{\circ}$	0.140	3.51	0.0063	1.391
62 $^{\circ}$	0.166	4.069	0.0073	1.624
63 $^{\circ}$	0.139	3.45	0.005	1.362

TABLE II d

Standard error ( $\sigma$ ) and accuracy  $A(\%)$  between ( $A$ ,  $X$ ) correlations in different latitudes

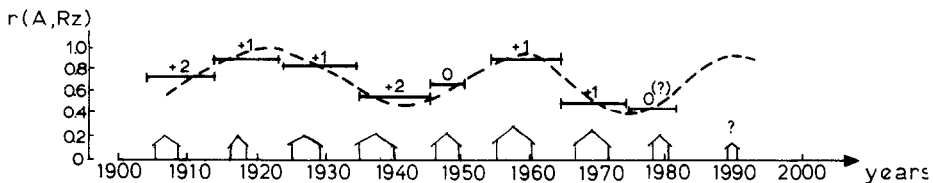
$L^\circ N$	$\sigma^A A_p$	$\sigma^A R_z$	$\sigma^A H$	$\sigma^A I$	$\sigma^A_{tot}$
54°	2.17 (44) <sup>b</sup>	1.83 (53)	2.14 (45)	1.96 (49.7)	1.66 (57.3)
55°	4.16 (37)	1.87 (72)	2.55 (61.3)	2.20 (66.6)	1.56 (77)
56°	3.04 (70)	1.17 (88)	3.05 (69.8)	2.17 (78.5)	0.89 (91)
57°	3.70 (75.2)	2.82 (81)	3.71 (75)	3.76 (74.7)	2.61 (82)
58°	4.98 (82.6)	9.86 (65)	6.94 (75.8)	19.28 (32.8)	3.60 (87.5)
59°	6.10 (84.4)	10.90 (69)	8.06 (79)	11.73 (69.6)	5.67 (83)
60°	6.95 (85)	11.13 (76)	6.22 (86.7)	11.93 (74.6)	4.95 (89.5)
61°	5.56 (91)	13.91 (78)	8.92 (86)	13.13 (79.4)	4.17 (93.5)
62°	6.58 (90)	13.60 (80)	8.73 (87)	15.00 (78)	4.87 (993)
63°	6.11 (91.6)	11.80 (84)	7.82 (89.4)	12.86(82.5)	4.35 (94)

$$^a \sigma = \sqrt{\frac{\sum (A^{obs} - A^{cal})^2}{N - 1}}, \quad N = 10$$

$$^b A = \left(1 - \frac{\sigma}{\bar{A}_{obs}}\right) \%,$$

$A_{obs}$  = mean number of aurorae 1964/74 per each latitude.  
 Accuracies are given in parenthesis.

of  $B_z$  in relation to  $R_z$ ,  $H$ , and  $A$  for  $L = 59^\circ N$  and the 20th solar cycle and a discussion will follow linking the above. We note that in 1972 the aurora shows drastic drop that is accompanied by a respective inflection (shoulder) for the secondary peaks of  $B_z$  and  $R_z$  (Figure 5). That a conspicuous change took place in the location of activity on the solar surface in 1972 has been mentioned by Dodson and Hedeman (1975). Indeed significant centers of activity developed in the two heliolongitude zones of  $\sim 0^\circ$  and  $\sim 180^\circ$  that for three years had been relatively deficient in such phenomena. Flare-rich McMath plage 11748 crossed the central



↑ SMFR  
 +2 → r(A,Rz) for respective Solar Cycle  
 0,+1,+2 = time Lag.

Fig. 3. Variation of the correlation coefficient ( $r$ ) between ( $A$ ,  $R_z$ ) as a function of time and most intense SMFR since 1905. Note the expected high ( $r$ ) for the next predicted SMFR.

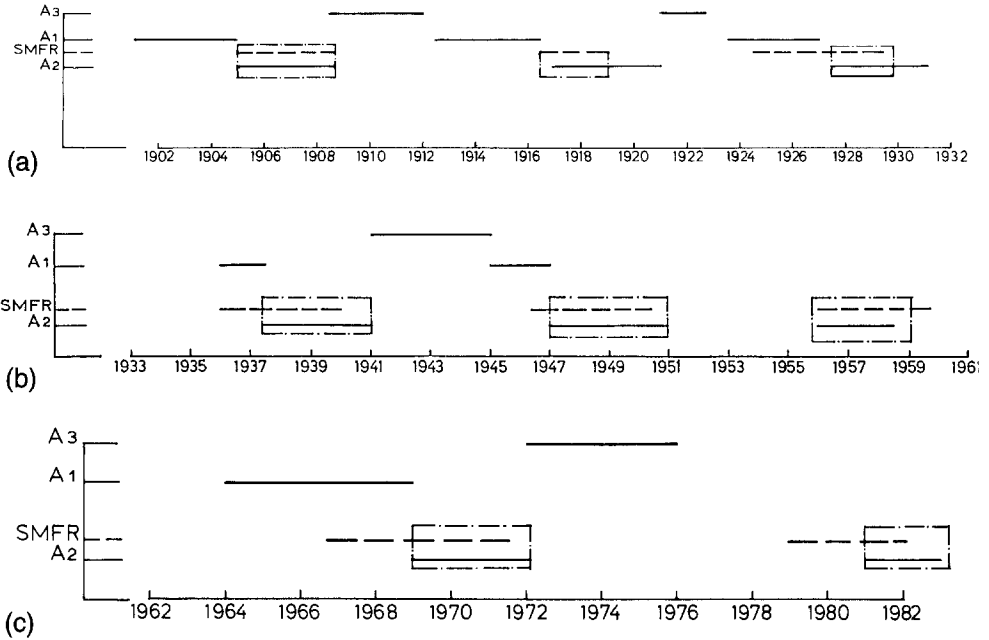


Fig. 4a, b, c. Aurorae peak (A1, A2, A3) position in time intervals associated with intervals of SMFR for the period 1902–1982.

meridian and these 32 flares and subflares were associated with sudden ionospheric disturbances the largest in number. The number of occurrence of red auroral also has a maximum in 1972 (Slater *et al.*, 1981). For total visual discrete aurorae the maximum took place also in 1972.

In addition, Warwick and Hansen (1959) have statistically found an excellent correlation between high values of magnetic disturbance indices and flares of importance  $\geq 3$  at least for the maxima of the sunspot cycle. Intense low latitude displays of aurora can usually be associated with solar flares occurring up to 1 to 4 days earlier.

These coronal holes, at any rate, have been identified with high speed solar wind streams (i.e. recurrent corpuscular streams) that is one-parameter influencing geomagnetic disturbances (the other is sporadic corpuscular streams well related with the flare activity) cf. Chirkov (1979).

Sheeley and Harvey (1978) found that modest auroral expansions occur during the main body of high-speed streams from coronal holes and that great expansions ( $L < 53^\circ$ ) occur only during intervals of intense interplanetary magnetic field, such as may occur at the leading edge of a high speed stream or at a flare produced interplanetary shock.

For all the above reasons we have made a correlation study between ( $A$ ) and the following parameters:  $R_z$ ,  $H$ ,  $F$  ( $\text{Imp} \geq 1$ ) and  $A_p$ . Initially a correlation study was made between ( $A$ ) and each of  $R_z$ ,  $H$ ,  $F$ , and  $A_p$  separately (Table IIa).



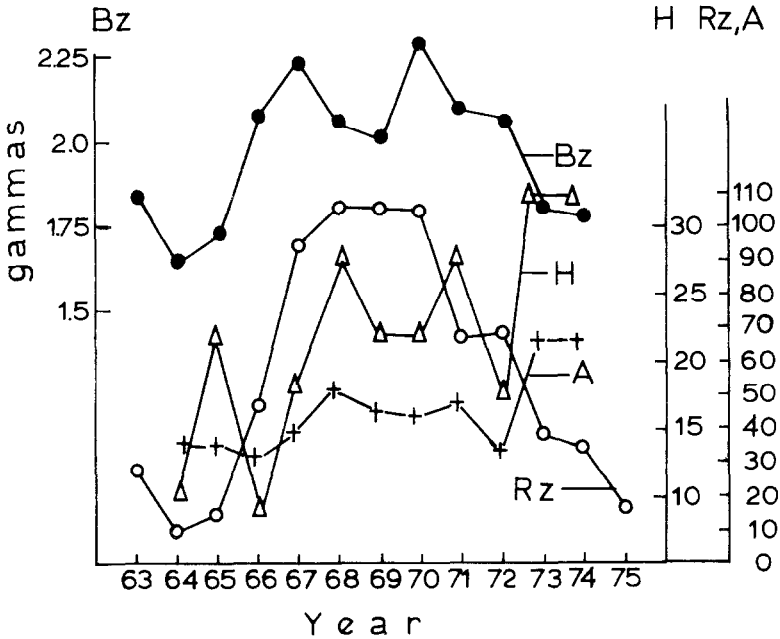


Fig. 5. Simultaneous variations of  $A$ ,  $R_z$ ,  $H$ ,  $B_z$  indices for the 20th solar cycle.

If we accept that the aurora frequency can be correlated separately with each of these indices ( $X$ ) for every geomagnetic latitude  $L$  the slopes and the constant terms are given by the following relation  $A_L = B_L X_i + C_L$ . The values for  $B_L$  and  $C_L$  are given in Tables IIb, c.

We have computed the standard error ( $\sigma$ ) for the above relation, which together with the accuracies are given in Table IId. It is found that the  $A_p$  and  $H$  indices have shown higher ( $r$ ) with  $A$  for  $L \geq 58^\circ$ , whereas the  $R_{(t-1)}$  and  $F$  indices have low ( $r$ ).

In contrast for  $L \leq 57^\circ$  the  $R_{t-1}$  and  $F$  appear to correlate reasonably well with  $A$ , and the  $A_p$ ,  $H$  have low ( $r$ ) values Figure 6. As the number of coronal holes appear more significant in the minima of the solar cycle, that is why, we examined the influence of ascending and descending parts of each of  $R_z$ ,  $A_p$ ,  $H$ ,  $F$  to  $A$  for the 20th solar cycle for  $L = 54^\circ$  and  $63^\circ$  N Table I.

It is noted that the ( $r$ ) increases considerably in the ascending part for higher latitudes, but not for the descending part of  $A$  vs  $F$ .

For a uniform distribution the calculated and observed aurora are equal. For this case the number of freedom  $\nu = N - 1$ , where  $N$  is the number of data points, that is,  $\nu = 10$ .

For five latitudes, namely  $55^\circ$ ,  $56^\circ$ ,  $61^\circ$ ,  $62^\circ$ ,  $63^\circ$  N the observed variation for the calculated aurorae is explained in terms of random fluctuations and the calculated aurorae match quite well the observed ones at a significance level better than 95%. For the other latitudes this significance level is getting worst down to  $\sim 58\%$  for  $L = 60^\circ$  N. The order of worst significance by latitude is as follows:  $56^\circ$ ,  $63^\circ$ ,  $61^\circ$ ,

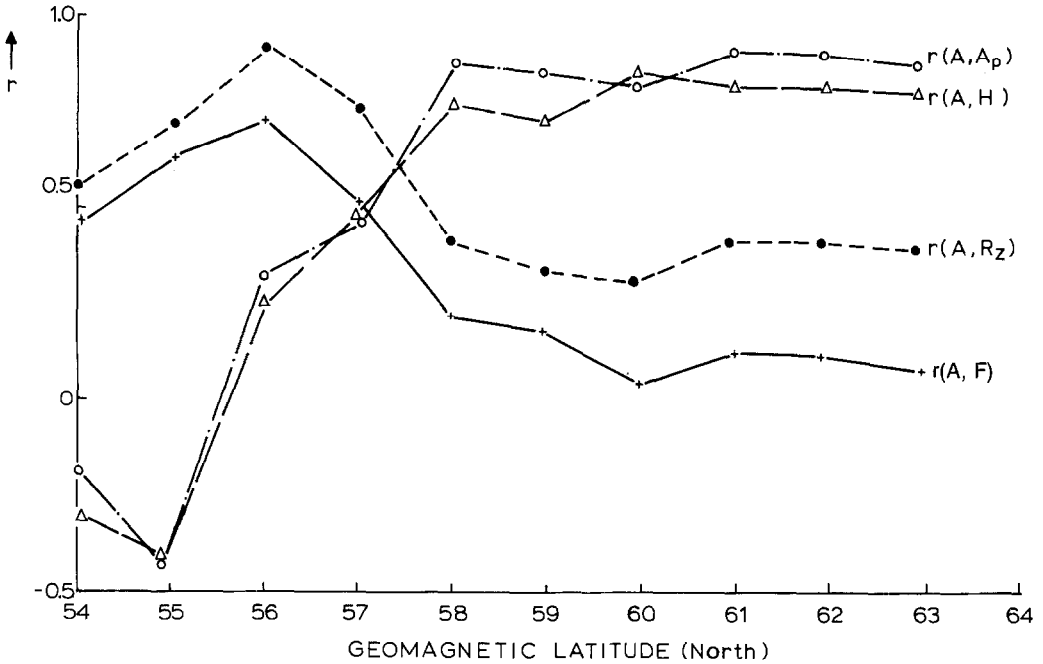


Fig. 6. Variation of  $r(A, X)$  where  $X$  is one of the indices of  $R_z$ ,  $A_p$ ,  $H$ ,  $F$ , with latitude. Note the 'break' around the  $57^\circ\text{N}$ .

$62^\circ$ ,  $55^\circ$ ,  $57^\circ$ ,  $58^\circ$ ,  $54^\circ$ ,  $59^\circ$ ,  $60^\circ$  while for the computed accuracy,  $A = (1 - \sigma/\bar{A})\%$  where  $\sigma$  = standard error and  $A$  = mean auroral number for 1964/74 per each latitude, in Liritzis and Petropoulos, 1986) the worst accuracy per latitude is  $63^\circ$ ,  $61^\circ$ ,  $62^\circ$ ,  $56^\circ$ ,  $58^\circ$ ,  $60^\circ$ ,  $59^\circ$ ,  $57^\circ$ ,  $55^\circ$ ,  $54^\circ$ . We note that the best agreement occurs for the low and higher latitudes. In the intermediate, that is,  $57^\circ$ ,  $58^\circ$ ,  $59^\circ$ ,  $60^\circ\text{N}$  the relationship weakens. This is not quite true for the accuracy although the general pattern is so for the  $54^\circ$  data we are reserved due to the very low auroral data and thus poorer statistics. That a drastic change in the correlation between  $A$  and the other indices occurs, in the transition zone  $56^\circ - 59^\circ$ , has been noted and qualitatively explained below (Section 7d) that might explain the poor relationship too.

For  $A$  versus  $H$  the ( $r$ ) of ascending parts increases little towards higher latitudes but considerably for the descending parts. Thus the influence of  $F$  and  $H$  to  $A$  as a function of  $L$  is inversely proportional. More significant is the impact of  $A_p$  on the ascending part of  $A$  for high  $L$ 's.

The ( $r$ ) between  $A$  and SMF for the period 1934/1944 is best for time lag = + 1. For SMF, the annual sums were taken which varied from 400–5500. The generalised relationship of  $A$  with all the solar-terrestrial indices for  $L = 54^\circ$  to  $63^\circ\text{N}$  has been found earlier employing the theory of residuals and it was shown the apparent higher accuracy when  $A$  is combined with all the four indices rather than with a selected few

TABLE III

Significance levels from  $\chi^2$  statistics for the relationship between calculated and observed aurorae, per latitude. The data pertain to the 20th solar cycle (see text)

Latitude ° N	54	55	56	57	58	59	60	61	62	63
$\chi^2$ ( $V = 10$ ) at various significance levels										
$\geq 95\%$ ( $\chi^2_{crit} = 3.94$ )		3.93	0.666					2.37	2.8	2.33
~ 93%				4.38						
~ 78%					6.34					
~ 73%	7.06									
~ 61%						8.26				
~ 58%							9.38			

TABLE IV

Correlation coefficient of slope Versus latitude for ( $A$ ,  $X$ ) relationships

Indices ( $A$ , $X$ )	Correlation coefficient for coefficients of slope vs latitude from:	
	Table IIc	Generalized equation (Table I, Liritzis <i>et al.</i> , 1986)
(Constant term)	0.568	—
$H$	0.826	0.935
$A_p$	0.863	0.926
$R_z$	0.793	0.930
$F$	0.806	0.07(?)

(see also Table IId, where

$$A(t) = f(R_z, H, F, A_p). \tag{1}$$

We have applied also the  $\chi^2$ -test to the observed and calculated by Equation (1) auroral values in order to statistically check the agreement to a certain significance level. As is well known,

$$X^2 = \sum \frac{(A_0 - A_c)^2}{A_c} \tag{2}$$

where  $A_0$  observed and  $A_c$  calculated aurorae.

The number of parameters is 11; so that the degrees of freedom  $\nu = 10$ . From tables of  $X^2$  the reduced chi-square  $X^2_\nu = X^2/\nu$ , corresponding to the probability of exceeding  $X^2$  vs  $\nu$  is readily given.

If the  $X^2_{calcul.} < X^2_{critical}$  at a 95% significance level then the calculated values are statistically significant. The results are given in Table III.

TABLE V

Electron density maximum  $h_m$ , maximum of electron concentration  $n_m$ , electron temperature  $T_e$ , electron concentration  $n_e$  and oxygen concentration  $n(O^+)$  variations as a function of latitude (Deminov *et al.*, 1980)

Latitude	$h_m$	$n_m$	$n_e$	$T_e$	$n(O^+)$
$L < 55^\circ$	constant to ~ 320 km	low, ~ $0.5 \times 10^5$ $\text{cm}^{-3}$	rise	rise	constant fluctuating
$55^\circ < L < 59^\circ$	drastic drop from 320 to 120 km	rise to $1.5 \times 10^5$ $\text{cm}^{-3}$	small drop and trough	slight drop	rise
$59^\circ < L < 68^\circ$	Very low plateau at ~ 100 km	rise, fluctuating, at $3 \times 10^5$ $\text{cm}^{-3}$	rise and dip at ~ $68^\circ$	slight increase fluctuating	trough

### 6. The Latitude Dependence of the Regression Coefficients from $A = f(A_p, H, R_z, F)$ Relationships

The coefficients of sloper (response function) between  $A$  and each one of  $H$ ,  $A_{pp}$ ,  $R_z$ , and  $F$  separately, as well as between  $A$  and the combined indices, both, as a function of latitude, have generally high ( $r$ ) (Table IV). For the purpose of interrelations between  $A$  and the other indices, the ( $r$ ) derived from the coefficients of the generalized relationship (Table II d) is more representative for such latitude dependence and it is expected to further refine it with the inclusion of more northern latitudes. (Note the poor ( $r$ ) for  $F$ ).

It would be interesting also to examine any longitude effect (e.g. due to pitch angle distribution of the incident particles and dipole tilt) on such relationships as both latitude and longitude characterize (together with the altitude) the morphology and occurrence of auroral displays.

### 7. Discussion

Auroral spectra in this region are dominated by the nitrogen and oxygen atomic and molecular bands; while visual auroral are produced by electrons of a few keV.

We shall try to qualify the above relationships comparing them with other physical mechanisms that take place in the same altitude and latitude as aurora.

We shall, thus, consider variations with latitude of:

- (a) The  $N_2^+$  3914 Å emission line (which requires 19 eV for excitation).
- (b) The trapped and precipitated particles.
- (c) The electron density profile is mainly related to  $O^{2+}$  and  $NO^+$  profiles (Friedman, 1964). Table V presents the latitudinal situation of the  $n_m$  as well as the

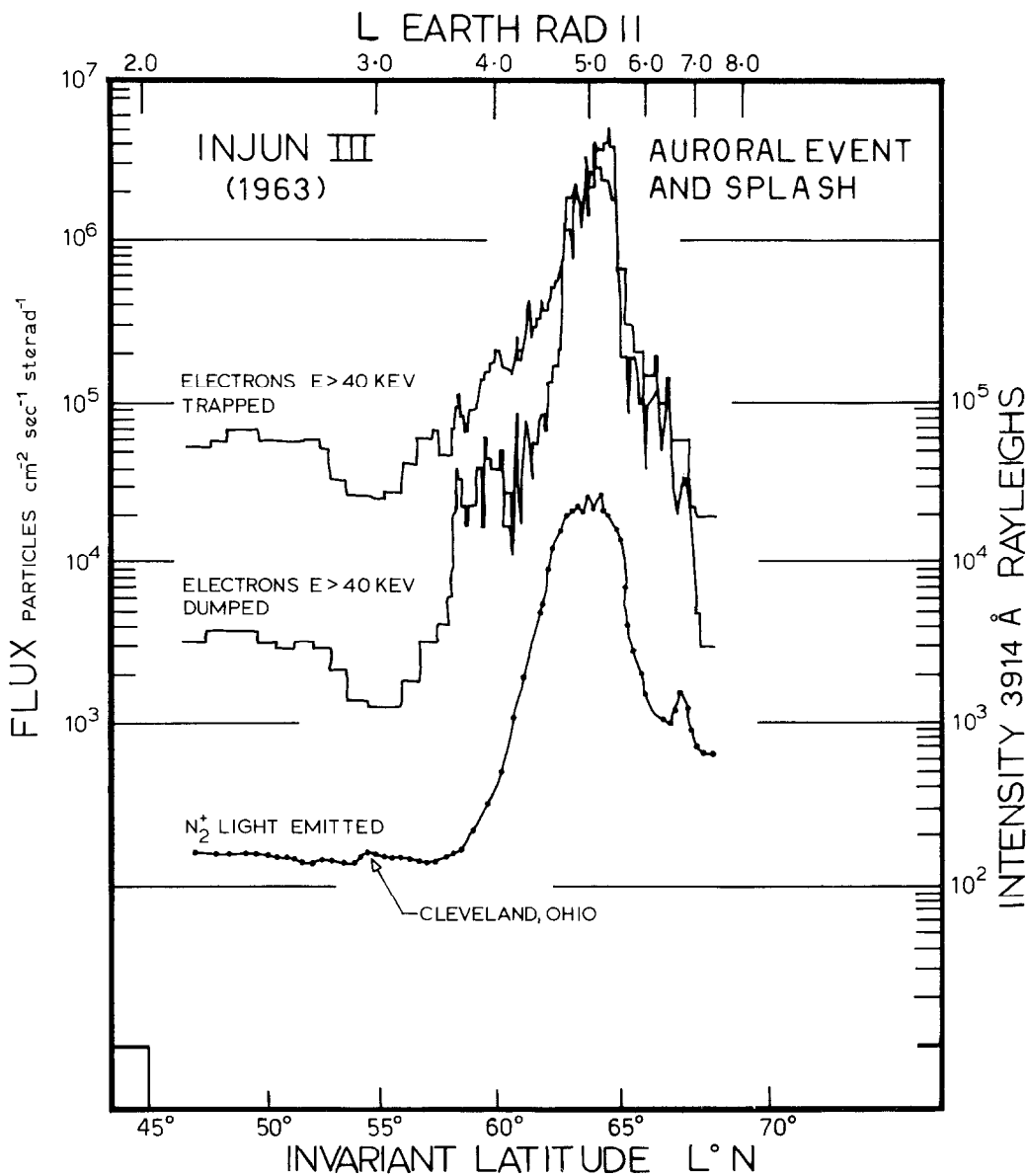


Fig. 7. Variation of trapped and precipitated electrons and N<sup>2+</sup> light as a function of latitude L°N (from O'Brien, 1964).

other parameters. The maximum concentration of electrons varies with an increase for  $L > 55^\circ$  and fluctuates for  $L > 59^\circ$ . At the same time, the  $n_e$  rises for  $L < 55^\circ$  and drops a little at  $55^\circ < L < 59^\circ$  (troughs) with a rise and dip at  $L = 68^\circ$ .

The  $T_e$  rises sharply at  $L = 54^\circ$  and drops at  $L = 55^\circ$ , subsequently drops slightly and then fluctuates increasing a little at  $L > 68^\circ$ .

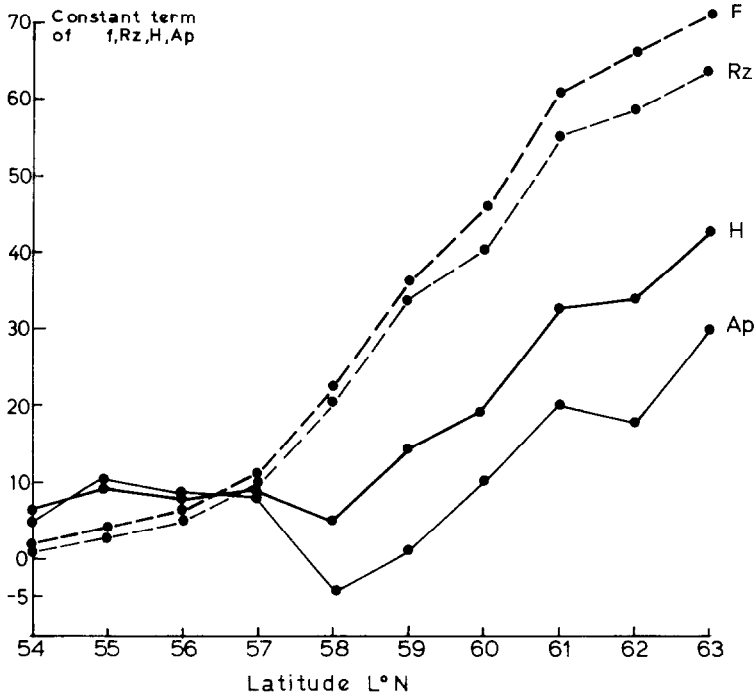


Fig. 8. Variation of the constant term coefficient between  $(A, X)$ , where  $X = \text{Index}$ , as a function of latitude.

At altitudes of auroral display which correspond to the E-region of the earth's ionosphere (90–140 km) the  $T_e$  variations correlate linearly with the  $R_z$  variations with response coefficient (slope) from  $3-5 \times 10^{-3}$  (altitude and seasonal effect). This value has apparently a latitude effect since for  $L = 20.48-51.31^\circ N$  is 0.00355 and for  $L = 59.58^\circ N$  is 0.00412 (which is the greatest value), Schwentek *et al.* (1981). Muggleton (1971) has presented also, the solar cycle control of E-region's peak electron density  $N_m(E)$  which is proportional to  $(f_oE)^2$  for the period 1949/59, which implies that solar activity gives rise to strong electron density which, consequently, influence auroral displays. In the altitude region  $\sim 150$  to  $\sim 250$  km there is also a drastic increase in this electron density profile (similarly for the electron temperature profile) (Bauer, 1973; Rees and Walker, 1968).

(d) The  $n(O^+)$  variations are shown in Table 5. It is worth noticing the trough at  $L = 54^\circ$  with subsequent increase peaking at  $L = 58-59^\circ$  and then follows a drop.

(e) Regarding  $(h_m)$  there is a drastic drop at  $L = 58^\circ$  to  $L = 59^\circ$  with subsequent constant but low  $h_m$  values to  $L = 69^\circ$ . These anomalously low heights  $(h_m)$  are thought to be largely associated with an increase in the concentration of the molecular components of the neutral atmosphere and in ion temperature because of joule heating and particle precipitation (see also (a), above).

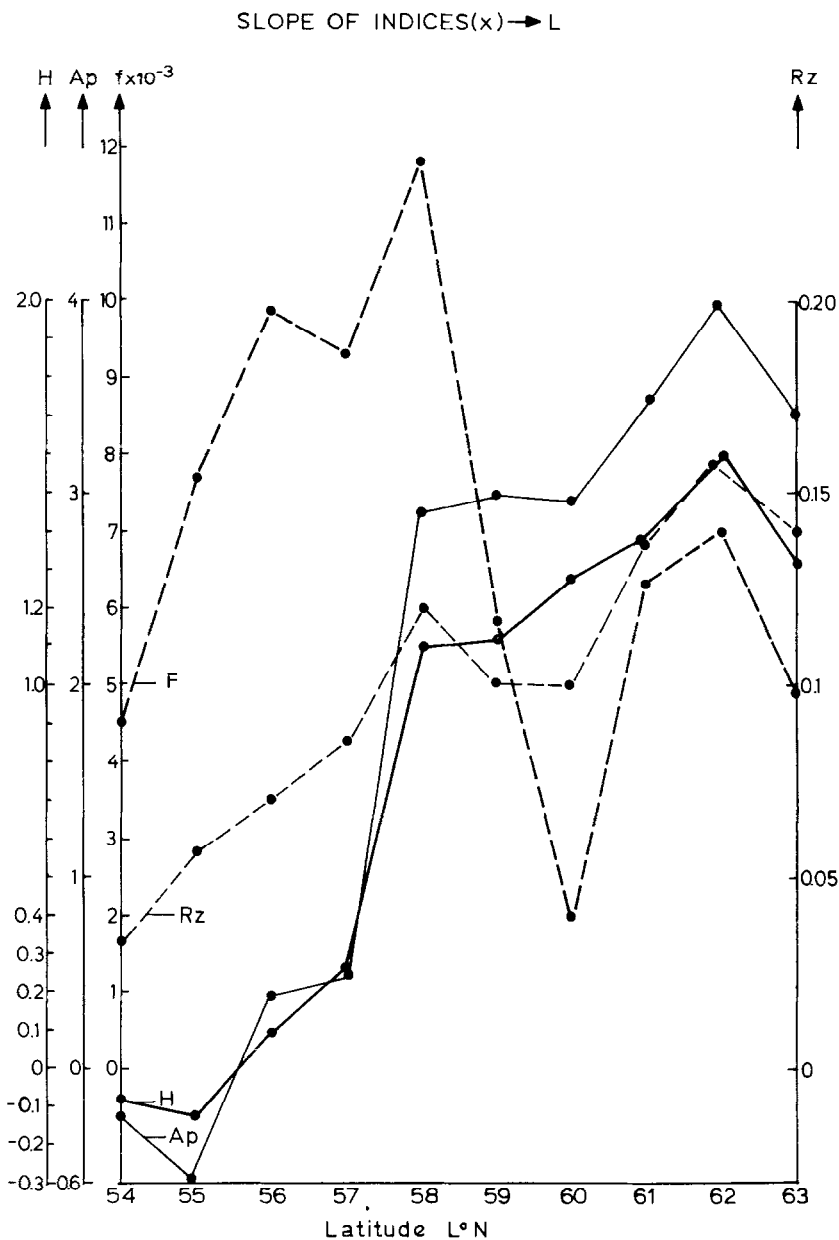


Fig. 9. Variation of the slopes between (A, X), where X = index, as a function of latitude.

This low  $h_m$  leads to the said decrease in the  $n(O^+)$  (see (d) above) and it is due to either the diffusion or complete disappearance of  $h_m$  at heights higher than 200 km. (Deminov *et al.*, 1980).

The  $n_e(h)$  (electron density by height) distribution shows anomalously low heights ( $h_m$ ) of the main electron density maximum at above 100–150 km.

The above considerations bring together the auroral production mechanisms and other physical parameters that appear in the same latitudes and latitudes.

These, then, lead one to examine the dependence of coefficients of the generalized equation as a function of latitudes, grouping them in three zones, that is (i) for  $(A, A_p)$  and  $(A, H)$  relationships:  $54-55^\circ$ ,  $55-59^\circ$  and  $59-63^\circ$ ; and (ii) for  $(A, R_z)$  and  $(A, F)$ :  $54-56^\circ$ ,  $56-59^\circ$ , and  $59^\circ-63^\circ$ . Therefore the above could explain the fact that between  $54-56^\circ\text{N}$  the  $(r)$  between  $A$  and  $R_z$  and  $A$  and  $F$  is much higher from the respective  $(A, A_p)$  and  $A, H$ . This indicates that in this zone the auroral occurrence is controlled mainly from sunspot and flare activities (Figure 6).

In the following transition zone of  $56-59^\circ$  a drastic change in the correlation occurs, while above  $59^\circ$  to  $63^\circ\text{N}$  the trend of variation smooths-out. It should be noted a latitudinal shift for the change of  $(A, A_p)$ ,  $(A, H)$  and  $(A, R_z)$ ,  $(A, F)$  relationships of  $\sim 1^\circ$  (occurs at  $54^\circ$  for the former and at  $55^\circ$  for the latter). This division in three zones is further reinforced from the plot of linear regression coefficients of  $A = f(F, R_z, H, A_p)$  as a function of latitude (Figures 8 and 9).

Furthermore, in the Figure 8 we note that the constant terms of  $F$  and  $R_z$  up to  $\sim 56^\circ\text{N}$  are smaller from those of  $H$  and  $A_p$ , while from  $L \geq 56^\circ\text{N}$  onwards this behaviour is reversed. For the latter, when the coefficients of  $A_p$ ,  $R_z$ ,  $H$ , and  $F$  of the generalized equation are plotted as a function of  $L$ , the constant term of this equation shows a minimum in  $L = 58^\circ$ .

Overall, a parallel picture is observed between the five above mentioned physical factors and the interrelationships between  $A, A_p, R_z, H, F$  indices as a function of latitude, and altitude.

Although the relationship between  $A$  and the other indices holds for at least 5 latitude, where a high significance is attached, the conspicuous change that took place in 1972 and recorded in cosmic ray intensity, the interplanetary magnetic field, flares and, radioemissions from quasar, resulted in an apparently disturbed solar-terrestrial-interplanetary system which subsequently is bound to perturb their (if any) relationship.

Emphasis should be placed on the apparent changes in coefficients of solar-terrestrial correlations at  $54^\circ$  to  $55^\circ$ ,  $56^\circ$  and  $58^\circ-59^\circ\text{N}$ .

In conclusion, it can be said that the auroral forms and displays depend on solar-terrestrial parameters which in turn are latitude-dependent. Such correlation studies are helpful for assessing the solar activity impact on certain geophysical parameters such as the secular variation of the geomagnetic field, climate and cosmic-ray intensity variations.

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