PLANETARY DISTANCE LAW

(*Letter to the Editor*)

J. J. RAWAL

Nehru Planetarium, Nehru Centre, Worli, Bombay, India

(Received 16 January, 1989)

Abstract. A general form of the planetary distance law has been proposed in this paper.

1. Introduction

On the basis of the 'modern Laplacian theory' of formation of the Solar System discussed by Prentice (1978a, b) together with the concept of Roche limit, Rawal (1984, 1986, 1989) has arrived at the distance relation in the Solar System and in the satellite systems of planets in the form

$$R_p = R_* a^p, \tag{1.1}$$

where R_p is the distance of pth secondary from its respective primary (the Sun or a planet). R_* is the present radius of the primary and 'a' the Roche constant. In the case of the Solar System the value of 'a' is 1.442 and in the cases of satellite systems 'a' has the value 1.26. This form of the distance relation is on the assumption that the density of the primary equals that of the secondary. This is a simple and ideal assumption, and, in general, may not be strictly true. In this paper, it is proposed to set up the most general form of the planetary distance law.

2. General Form of the Planetary Distance Law

Ter Haar (personal communication), suggests that instead of taking 'a', which has the value $=1.442$ in the Solar System and $=1.26$ in the satellite systems on the assumption that the density of the primary ρ_p equals that of the secondary ρ_s as the Roche constant, one should take $h = ab'$ ($b' = b^{1/3}$, $b = \rho_p/\rho_s$ is of order unity) as the Roche constant. This gives us

$$R_p = R_* a^p (b'_1 \times b'_2 \times \cdots \times b'_{p-1} \times b'_p) \tag{2.1}$$

$b'_i = (b_i)^{1/3}$, $i = 1, 2, \ldots, p$, b_i is the ratio of the density of the primary to that of the secondary at the i-th stage of the contraction and is of order unity. This requires us to know b_i ($i = 1, 2, \ldots, p$) at every stage of contraction. Let $G = (b'_2 \times b'_2 \times \cdots \times b'_p)^{1/p}$. G is the geometric mean of $b'_1, b'_2, \ldots, b'_p$. Clearly, G depends upon the ratio of the density of the primary to that of the secondary at each stage of contraction of the solar nebula or subsolar nebulae. We may, then, write

$$R_p = R_* a^p G^p \,, \tag{2.2}$$

Taking, a $G = C$, we have

$$R_p = R_* C^p, \quad p = 1, 2, \ldots, k \,. \tag{2.3}$$

The orbital period relation (Kepler's third law) then assumes the form

$$T_p = T_*(C^{3/2})^p \,. \tag{2.4}$$

The resonance relation among the pairs of successive orbit is given by

$$n_{p+1}/n_p = C^{-3/2} = F^{-1} \tag{2.5}$$

and the resonance among a triad of successive orbits is given by

$$n_p - (1 + F)n_{p+1} + Fn_{p+2} = 0 \,. \tag{2.6}$$

Equations (2.3)–(2.6) in C are uncertain as 'C' contains G, an uncertain factor. G can be determined by computing $b'_1, b'_2, \ldots, b'_p$ and taking their geometric mean. As we do not know ρ_p and ρ_s at every stage, we do not know $b'_i = (\rho_p/\rho_s)^{1/3}$ $(i = 1, 2, \ldots, p)$ at every stage. However, we may determine b_i $(i = 1, 2, \ldots, p)$ taking ratio of the actual mean distance of the secondary to the mean distance of the ring containing the secondary. The mean distance of the ring containing the secondary is the mean distances of its outer and the inner edges calculated assuming $b'_i = 1$.

Equation (2.3) is the most general form of the planetary distance relation in the Solar System as well as in the satellite systems of planets. Equations (2.4)–(2.6) are the general forms of the associated relations.

Acknowledgements

Thanks are due to Professor J. V. Narlikar and Professor S. M. Chitre of the Tata Institute of Fundamental Research, Bombay and Professor S. Ramadurai of the Indian Institute of Science, Bangalore for helpful discussions and useful suggestions.

References

Prentice, A. J. R.: 1978a, in S. F. Dermott (ed.), *The Origin of the Solar System*, Wilen, London p. 111.

Prentice, A. J. R.: 1978b, *The Moon and the Planets* **19**, 341.

Rawal, J. J.: 1984, *Earth, Moon and Planets* **31**, 175.

Rawal, J.J.: 1986, *Earth, Moon and Planets* **34**, 93.

Rawal, J. J.: 1989, *Earth, Moon and Planets* **44**: 265–274, (this issue).