

# CONTRACTIONS OF SUBSOLAR NEBULAE

J. J. RAWAL

*Nehru Planetarium, Nehru Centre, Worli, Bombay, India*

(Received 16 January, 1989)

**Abstract.** The present paper attempts to study the contractions of subsolar nebulae along the lines with contraction of the solar nebula.

## 1. Introduction

Prentice (1978a, b) has established 'the modern Laplacian theory' of the formation of the Solar System. Rawal (1984) studied the collapse of the solar nebula. On the basis of the concept of Roche Limit and the work of Prentice, he arrives at the relation:  $R_p = R_\odot a^p$  ( $R_\odot =$  Sun's present radius,  $a = 1.422$ , the Roche constant) for the various radii at which the collapse of the solar nebula was halted time to time. In this, Kepler's Third Law assumes the form:  $T_p = T_\odot (a^{3/2})^p$  where  $T_\odot$  is the rotational period of the Sun at the time when it attained the present radius (Rawal, 1986). Successive triads of  $T_p$  satisfy the Laplace's resonance relation (Rawal, 1986).

As the satellite systems mimic the Solar System in almost all respects, it implies that they might have followed similar pattern for their formation, the conclusion also reached by Alfvén and Arrhenius (1976). It is on this assumption that the present paper attempts to study the contractions of the subsolar nebulae.

## 2. Different Types of Roche Limits

When we consider the Solar System, the Sun is a primary body and the planets are secondary ones. The Roche limit is the boundary of a region around a primary body within which a secondary body is subject to tidal disruptive forces of the primary and disintegrates. Alternatively, the primordial matter of the solar nebula within this distance would not get condensed into a satellite. There are three types of Roche limits:

### (i) ROCHE LIMIT FOR A FLUID SATELLITE

In this, particles remain as a stable gravitational configuration as long as their mutual attractive force is greater than or equal to the difference of forces on each by the primary. This leads to a Roche limit given by

$$d_{\text{Roche}} \approx 2.4554 [\rho_p / \rho_s]^{1/3} \times R \quad (2.1)$$

where  $\rho_p$  is the density of the primary,  $\rho_s$ , that of secondary and  $R$ , the radius of

the primary. If we assume that  $\rho_p \approx \rho_s$ , then, in this particular situation, Roche limit assumes the form

$$d_{\text{Roche}} \approx 2.4554R. \quad (2.2)$$

(ii) ROCHE LIMIT FOR A RIGID SATELLITE

Here we consider the combined effect of the differential gravitational acceleration and the differential centripetal acceleration between the centre of a satellite and its outer edge due to the primary at the time of rotational instability on the disruption of the satellite, these two accelerations are balanced by the satellite's self-gravitational acceleration. The corresponding Roche limit is given by

$$d_{\text{Roche}} \approx 1.442[\rho_p/\rho_s]^{1/3} \times R. \quad (2.3)$$

In the case,  $\rho_p \approx \rho_s$ , we have

$$d_{\text{Roche}} \approx 1.442 \times R. \quad (2.4)$$

(iii) ROCHE LIMIT IN WHICH DIFFERENTIAL CENTRIPETAL FORCE IS DISREGARDED

In the situation where the radius of the secondary is negligible compared to the distance from the primary, we may disregard the centripetal acceleration between the centre of the satellite and its outer edge due to the primary and get the Roche limit to be

$$d_{\text{Roche}} \approx 1.26[\rho_p/\rho_s]^{1/3} \times R. \quad (2.5)$$

In the case,  $\rho_p \approx \rho_s$ , we have

$$d_{\text{Roche}} \approx 1.26R. \quad (2.6)$$

### 3. Contractions of the Sub-Solar Nebulae

Urey (1951), Kuiper (1951), Hoyle (1960), Whipple (1971), Prentice (1978a, b) and other cosmochemists have estimated the amount of material of the solar nebula that has gone to form the planetary system. Their work indicates that  $0.05 M_{\odot}$  ( $M_{\odot}$  = Sun's mass) material has gone to form the planetary system. Rawal (1984) shows that the Sun has shed 25 rings before attaining the present radius. Distributing  $0.05 M_{\odot}$  material among 25 rings equally, we see that each ring gets  $\sim 660 M_{\oplus}$  ( $M_{\oplus}$  = Earth's mass) as its share. Therefore, our spinning subsolar nebula has its initial mass  $M$  round about  $660 M_{\oplus}$ .

Let  $R_p$  be initial radius of the subsolar nebula. Under the influence of its self-gravitation, the subsolar nebula began to contract and because of the conservation of the angular momentum, it began to spin ever faster. A stage was reached at which the centrifugal force became equal to the gravitational

force at the equator giving rise to a rotational instability, as a result, a shell of matter evolved into a ring of matter at the equator which got detached. This whole process repeated itself until the subsolar nebula reached its present size of a planet. Here the contraction of the above subsolar nebula halting at various radii are brought about by the phenomenon of supersonic turbulent convection (Prentice, 1978a, b; Rawal 1984). The supersonic turbulent convection does the following jobs: (1) It creates an additional source of pressure in a subsolar nebula called the radial turbulent stress which for the first time halts the free collapse of the subsolar nebula from its initial dimension calculated by Innanen (1979) to the dimension of the satellite system, (2) it causes the interior of the subsolar nebula to rotate almost uniformly like a rigid body because of a large turbulent viscosity and drastically lowers the moment of inertia-coefficient,  $f$ , of the protoplanet thereby allowing the protoplanet to give up its angular momentum to a very light satellite system and (3) it leads to the formation of a ring of gas at the equator of the protoplanet, thereby causing the protoplanet to dispose off its excess angular momentum through the successive detachemnt of a discrete system of gaseous rings.

Assume, then, that the above subsolar nebula shrank to a radius  $R_{p-1}$  such that  $R_p$  is the Roche limit of the cloud which has now radius  $R_{p-1}$ .

As discussed above there are three versions of Roche limits. One for a fluid satellite, the second for a rigid satellite and the third is the situation in which we disregard the differential centripetal force.

As we are discussing here the subsolar nebula for the formation of the protoplanet and the satellite system, we assume that the third kind of Roche limit is applicable. In the case when  $\rho_p \approx \rho_s$ , and we assume that this is the case here, the Roche limit, therefore, assumes the form given by Equation (2.6).

If we write  $1.26 = a$ , referred to here as the Roche constant, then

$$d_{\text{Roche}} = aR . \quad (3.1)$$

Therefore, the relation between  $R_p$  and  $R_{p-1}$  of the contracting subsolar nebula can be written as

$$R_p = aR_{p-1} . \quad (3.2)$$

The shell of matter having width  $R_p - R_{p-1}$  forms the Roche zone of the protoplanet which has now radius  $R_{p-1}$ . The matter in the shell having width  $R_p - R_{p-1}$  settled down to form a ring at the equator of width  $R_p - R_{p-1}$ . The matter inside such a ring might have grown to satellitesimals but naturally failing to form a full satellite there, because the matter in the ring was still inside the Roche limit of the protoplanet which has radius  $R_{p-1}$ . The matter inside such a ring had to wait for further contraction of the subsolar nebula to take place which could put it outside the Roche limit so that a full satellite might form in it.

At the next stage of contraction, the subsolar nebula shrank to a radius  $R_{p-2}$

such that

$$R_{p-1} = aR_{p-2}. \quad (3.3)$$

Hence, we have

$$R_p = a^2 R_{p-2}. \quad (3.4)$$

The annular ring ( $R_{p-2}, R_{p-1}$ ) of width  $R_{p-1} - R_{p-2}$  lay inside the Roche limit of the protoplanet which has now radius  $R_{p-2}$ . At this stage, the previous ring ( $R_{p-1}, R_p$ ) of matter came out of the Roche zone of the protoplanet which has radius  $R_{p-2}$  and found the matter to grow to form a satellite.

We assume that the contraction proceeded in this fashion until subsolar nebula reached the present size of a planet, the halts at various radii were brought about by the phenomenon of supersonic turbulent convection eventually leading to the stage

$$R_1 = aR_o \quad (3.5)$$

where  $R_o$  is the present radius of a planet. In terms of the present radius of a planet, the sequence of the radii of the contracting subsolar nebula at various stages of the contraction can be expressed as

$$R_p = R_o a^p (p = 1, 2, 3, \dots, k). \quad (3.6)$$

Tables I-V show various  $R_p$  for the planets Jupiter, Saturn, Uranus, Neptune, and Mars. The known satellite residing in the ring labelled ( $R_{p-1}, R_p$ ) for, various values of  $p$  are also mentioned.

The thermal stirring in the proto-planetary cloud in the vicinity of the planet may be responsible for the smaller masses of some of the inner satellites as well as for the formation of the rocky rings of a planet like Uranus. The icy rings of a planet like Saturn are suggested to be the product of condensation processes in a continuous gaseous disc within the Roche radius of the planet.

The densities and structures of planets suggest that the planets consist predominantly of an envelope of H (hydrogen) and He (helium) surrounding a small rock/ice core.

The existence of such central cores of roughly comparable mass is in accord with the view that the first stage in the formation of the planets was the aggregation of such a mass of rock/ice satellitesimals in gaseous rings each of mass about  $660 M_{\oplus}$  which were shed by the contracting protosun at the orbits of these planets. Once these dense planetary cores have formed, they can act as a gravitational sink for the residual gases in each of the ring.

After a gaseous ring has been shed, the various condensates in the gas, appropriate to the prevailing density and temperature, settle onto the central circular Keplerian orbit of radius  $R_p$  with angular velocity  $\omega = [GM/R_p^3]^{1/3}$  to form a concentrated stream of satellitesimals. The circular orbiting stream of satellitesimals may then subsequently aggregate under the action of its own

TABLE I  
Jupiter System

$R_p$ , the radius of the contracting subsolar nebula in units of 1000 km	Annual ring ( $R_{p-1}, R_p$ )	Known object in the annular ring ( $R_{p-1}, R_p$ )	Observed mean distance of the known object in the annular ring ( $R_{p-1}, R_p$ ) in units of 1000 km
$R_0 = 72.00$	( $R_1, R_0$ )	-	-
$R_1 = 90.72$	( $R_2, R_1$ )	-	-
$R_2 = 114.30$	( $R_3, R_2$ )	Ring, 1979J <sub>1</sub> , 1979J <sub>3</sub>	131
$R_3 = 144.10$	( $R_4, R_3$ )	V	181
$R_4 = 181.50$	( $R_5, R_4$ )	1979J <sub>2</sub>	221
$R_5 = 228.80$	( $R_6, R_5$ )	-	-
$R_6 = 288.20$	( $R_7, R_6$ )	-	-
$R_7 = 363.20$	( $R_8, R_7$ )	I(Io)	422
$R_8 = 457.60$	( $R_9, R_8$ )	-	-
$R_9 = 576.60$	( $R_{10}, R_9$ )	II (Europa)	671
$R_{10} = 726.60$	( $R_{11}, R_{10}$ )	-	-
$R_{11} = 915.60$	( $R_{12}, R_{11}$ )	III (Ganymede)	1070
$R_{12} = 1153.00$	( $R_{13}, R_{12}$ )	-	-
$R_{13} = 1454.00$	( $R_{14}, R_{13}$ )	-	-
$R_{14} = 1832.00$	( $R_{15}, R_{14}$ )	IV (Callisto)	1880
$R_{15} = 2309.00$	( $R_{16}, R_{15}$ )	-	-
$R_{16} = 2909.00$	( $R_{17}, R_{16}$ )	-	-
$R_{17} = 3665.00$	( $R_{18}, R_{17}$ )	-	-
$R_{18} = 4618.00$	( $R_{19}, R_{18}$ )	-	-
$R_{19} = 5820.00$	( $R_{20}, R_{19}$ )	-	-
$R_{20} = 7333.00$	( $R_{21}, R_{20}$ )	-	-
$R_{21} = 9241.00$	( $R_{22}, R_{21}$ )	XIII, VI	10170, 11470
$R_{22} = 11640.00$	( $R_{23}, R_{22}$ )	X, VII	11710, 11740
$R_{23} = 14680.00$	( $R_{24}, R_{23}$ )	-	-
$R_{24} = 18490.00$	$R_{25}, R_{24}$ )	XII, VI, VIII	20700, 22350, 23300
$R_{25} = 23300.00$			

gravity into a single satellite mass. The latter process is expected to take place only as long as the gaseous ring remains intact and can act as a sink for the excess motions of the aggregating satellitesimals (Hourigan, 1977).

Prentice and ter Haar have studied the formation of satellites and find that the agreement between predicted and observed satellite compositions and masses is very good. Io and Europa have densities of  $\sim 3.5 \text{ g cm}^{-3}$  consistent with a rocky composition and this agrees with the condensation temperatures at those distances from Jupiter. Similar is true for Ganymede and Callisto. This is generally true for all Jovian satellites not only for Galilean satellites. This scenario remains valid for other satellite systems as well.

The mass of a satellite depends on how much of the condensing material can segregate onto the central Keplerian orbit of the gaseous ring before the ring evaporates or is dispersed. The segregation time depends on the size of the condensing particles, being longest for the small dusty particles. Most of the rock

TABLE II  
Saturn System

$R_p$ , the radius of the contracting subsolar nebula in units of 1000 km	Annular ring ( $R_{p-1}, R_p$ )	Known object in the annular ring ( $R_{p-1}, R_p$ )	Observed mean distance of the known object in the annular ring ( $R_{p-1}, R_p$ ) in units of 1000 km
$R_0 = 60.00$	$(R_1, R_0)$	Ring	–
$R_1 = 75.61$	$(R_2, R_1)$	Ring	–
$R_2 = 95.28$	$(R_3, R_2)$	Ring	–
$R_3 = 120.00$	$(R_4, R_3)$	Ring	–
$R_4 = 151.00$	$(R_5, R_4)$	Mimas	186
$R_5 = 190.60$	$(R_6, R_5)$	Enceladus	238
$R_6 = 240.30$	$(R_7, R_6)$	Tethys	295
$R_7 = 302.70$	$(R_8, R_7)$	Dione	378
$R_8 = 381.50$	$(R_9, R_8)$	–	–
$R_9 = 480.70$	$(R_{10}, R_9)$	Rhea	527
$R_{10} = 605.60$	$(R_{11}, R_{10})$	–	–
$R_{11} = 763.20$	$(R_{12}, R_{11})$	–	–
$R_{12} = 961.60$	$(R_{13}, R_{12})$	Titan	1222
$R_{13} = 1212.00$	$(R_{14}, R_{13})$	Hyperion	1481
$R_{14} = 1527.00$	$(R_{15}, R_{14})$	–	–
$R_{15} = 1924.00$	$(R_{16}, R_{15})$	–	–
$R_{16} = 2425.00$	$(R_{17}, R_{16})$	–	–
$R_{17} = 3055.00$	$(R_{18}, R_{17})$	Iapetus	3560
$R_{18} = 3850.00$	$(R_{19}, R_{18})$	–	–
$R_{19} = 4851.00$	$(R_{20}, R_{19})$	–	–
$R_{20} = 6112.00$	$(R_{21}, R_{20})$	–	–
$R_{21} = 7702.00$	$(R_{22}, R_{21})$	–	–
$R_{22} = 9705.00$	$(R_{23}, R_{22})$	–	–
$R_{23} = 12230.00$	$(R_{24}, R_{23})$	Phoebe	12945.5
$R_{24} = 15410.00$	$(R_{25}, R_{24})$	–	–
$R_{25} = 19420.00$			

condensate may, therefore, have remained suspended in the gas. It is also likely that there have been considerable stirring of the inner gaseous rings due to the intense heat bath of the contracting proto planet which could have frustrated the settling out of the finer grains (Cameron, 1978).

We can observe that the ice-like members of the satellite systems of the planets almost contain their full share of icy-material. In contrast, the masses of the rocky ones fall short of the expected values. Similar situation is also observed in the distribution of planetary masses. There, we find that Uranus and Neptune each contains about 10 to 15  $M_{\oplus}$  of ices consistent with full condensation from gaseous rings each of mass 660  $M_{\oplus}$  shed by the contracting protosolar cloud, while the terrestrial planets contain only a fraction of the available mass  $\sim 4 M_{\oplus}$  of rocky condensate that was available in their respective rings. The shortfall in the expected mass of the rocky satellites lies in difference in the rate of segregation of the condensate material onto the central circular orbit  $R_p$  of each

TABLE III  
Uranus System

$R_p$ , the radius of the contracting subsolar nebula in units of 1000 km	Annular ring ( $R_{p-1}, R_p$ )	Known object in annular ring ( $R_{p-1}, R_p$ )	Observed mean distance of the known object in the annular ring ( $R_{p-1}, R_p$ ) in units of 1000 km
$R_0 = 27.00$	( $R_1, R_0$ )	Ring	-
$R_1 = 34.02$	( $R_2, R_1$ )	Ring	-
$R_2 = 42.87$	( $R_3, R_2$ )	Ring	-
$R_3 = 53.95$	( $R_4, R_3$ )	Ring	-
$R_4 = 67.98$	( $R_5, R_4$ )	-	-
$R_5 = 85.70$	( $R_6, R_5$ )	-	-
$R_6 = 107.90$	( $R_7, R_6$ )	Miranda	131
$R_7 = 133.60$	( $R_8, R_7$ )	-	-
$R_8 = 171.40$	( $R_9, R_8$ )	Ariel	192
$R_9 = 216.00$	( $R_{10}, R_9$ )	Umbriel	267
$R_{10} = 272.10$	( $R_{11}, R_{10}$ )	-	-
$R_{11} = 343.00$	( $R_{12}, R_{11}$ )	-	-
$R_{12} = 432.10$	( $R_{13}, R_{12}$ )	Titania	438
$R_{13} = 544.50$	( $R_{14}, R_{13}$ )	Oberon	586
$R_{14} = 686.10$			

gaseous ring compared with the life time of the gaseous rings. The gaseous rings play a vital role both in focussing the condensing grains onto the central orbit  $R_p$  as well as in damping out the excess kinetic energy of the larger aggregating satellitesimals which form on these orbits (Hourigan, 1977). If the gaseous ring evaporates or disperses or even vigorously stirred by the passage of thermal energy from the central protoplanet, then the distribution of angular velocity and the density is destroyed and all segregation and aggregation cease.

It is clear that if the segregation time,  $t_{\text{seg}}$ , exceeds the life-time,  $t_f$ , of a gaseous ring then only a fraction of the available condensate material is able safely to migrate onto the central orbit  $R_p$  for accumulation into a satellite mass. The remaining fraction remains suspended in the gas and is swept away from the region of that orbit when the gas ring disperses. Thus, for a satellite to contain the full share of available condensate, we shall require  $t_{\text{seg}} < t_f$ . In addition, for the material which does successfully settle onto the orbit  $R_p$  to aggregate together in time, we also require  $t_{\text{agg}} < t_f$ , where  $t_{\text{agg}}$  is the aggregation time, if this condition is not met the circular stream of satellitesimals will be left strewn around the mean orbit  $R_p$  as Prentice (1978a) has suggested was the case for the asteroids.

Tables I-V shows that the satellite systems are like central body surrounded with disc of material having closely packed separate rings. Some rings even contain two or more objects. They are not separate satellites but could be looked upon as parts of the same ring which perhaps because of the vicinity of the giant

TABLE IV  
Neptune System

$R_p$ , the radius of the contracting subsolar nebula in units of 1000 km	Annular ring ( $R_{p-1}, R_p$ )	Known object in the annular ring ( $R_{p-1}, R_p$ )	Observed mean distance of the known object in the annular ring ( $R_{p-1}, R_p$ ) in units of 1000 km
$R_0 = 24.50$	$(R_1, R_0)$	Ring	—
$R_1 = 30.87$	$(R_2, R_1)$	Ring	—
$R_2 = 38.90$	$(R_3, R_2)$	Ring	—
$R_3 = 49.02$	$(R_4, R_3)$	Ring	—
$R_4 = 61.77$	$(R_5, R_4)$	—	—
$R_5 = 77.84$	$(R_6, R_5)$	—	—
$R_6 = 98.09$	$(R_7, R_6)$	—	—
$R_7 = 123.60$	$(R_8, R_7)$	—	—
$R_8 = 155.80$	$(R_9, R_8)$	—	—
$R_9 = 196.20$	$(R_{10}, R_9)$	—	—
$R_{10} = 247.30$	$(R_{11}, R_{10})$	—	—
$R_{11} = 311.60$	$(R_{12}, R_{11})$	Triton	353.4
$R_{12} = 392.60$	$(R_{13}, R_{12})$	—	—
$R_{13} = 494.70$	$(R_{14}, R_{13})$	—	—
$R_{14} = 623.40$	$(R_{15}, R_{14})$	—	—
$R_{15} = 785.60$	$(R_{16}, R_{15})$	—	—
$R_{16} = 990.00$	$(R_{17}, R_{16})$	—	—
$R_{17} = 1247.00$	$(R_{18}, R_{17})$	—	—
$R_{18} = 1572.00$	$(R_{19}, R_{18})$	—	—
$R_{19} = 1953.00$	$(R_{20}, R_{19})$	—	—
$R_{20} = 2461.00$	$(R_{21}, R_{20})$	—	—
$R_{21} = 3101.00$	$(R_{22}, R_{21})$	—	—
$R_{22} = 3908.00$	$(R_{23}, R_{22})$	—	—
$R_{23} = 4924.00$	$(R_{24}, R_{23})$	Nereid	5560
$R_{24} = 6205.00$			

TABLE V  
Mars System

$R_p$ , the radius of the contracting subsolar nebula in units of 1000 km	Annular ring ( $R_{p-1}, R_p$ )	Known object in the annular ring ( $R_{p-1}, R_p$ )	Observed mean distance of the known object in the annular ring ( $R_{p-1}, R_p$ ) in units of 1000 km
$R_0 = 3.400$	$(R_1, R_0)$	—	—
$R_1 = 4.285$	$(R_2, R_1)$	—	—
$R_2 = 5.399$	$(R_3, R_2)$	—	—
$R_3 = 6.803$	$(R_4, R_3)$	—	—
$R_4 = 8.572$	$(R_5, R_4)$	Phobos	9.38
$R_5 = 10.800$	$(R_6, R_5)$	—	—
$R_6 = 13.610$	$(R_7, R_6)$	—	—
$R_7 = 17.150$	$(R_8, R_7)$	—	—
$R_8 = 21.620$	$(R_9, R_8)$	Deimos	23.5
$R_9 = 27.240$	$(R_{10}, R_9)$	—	—
$R_{10} = 34.320$			



planets could not have formed a full big satellite. In other words for some rings, due to the vicinity of giant planets. The conditions:  $t_{\text{seg}} < t_f$  and  $t_{\text{agg}} < t_f$  have not met and hence the circular stream of satellitesimals have left strewn around their mean orbit.

Prentice (1978a, b), in his modern Laplacian theory, gets the ratio of the orbital radii  $R_p$  of the successively disposed gaseous rings to be a constant, given by

$$R_p/R_{p-1} = \left[ 1 + \frac{m}{Mf} \right]^2 = \text{constant}, \quad (3.7)$$

where  $m$  is the mass of a disposed gaseous ring;  $M$ , is the remaining mass of the protoplanetary nebula and  $f$ , the moment of inertia coefficient. Prentice (1978a, b) and Rawal (1984) have discussed the validity of this equation in the case of the Solar System. We want, to discuss now the validity of this equation in the cases of satellite systems of Jupiter, Saturn, Uranus, Neptune, and Mars.

As discussed in the beginning of this section, we have for each satellite system,  $M \approx 660 M_{\oplus}$ . Also by analogy of the Solar System,  $0.05 M = 33 M_{\oplus}$  material has gone to form a satellite system. By analogy, we also assume that there are twenty five rings in a system. Distributing, therefore, the material  $33 M_{\oplus}$  among twenty five rings, we get  $m = 1.3 M_{\oplus}$ . Hence taking the appropriate value of  $f$  to be 0.0165 we have

$$R_p/R_{p-1} = \left[ 1 + \frac{m}{Mf} \right]^2 = \text{constant} = 1.26. \quad (3.8)$$

This shows the agreement between the modified Laplacian theory for the formation of the Solar System and the modified Laplacian theory for the formation of the satellite systems.

The discussion here clearly shows that there may be several hitherto unknown objects in each satellite system. Scientific and technological advancement and future space probes may reveal their existence.

In order that the Equation (3.6) reconciles with the Kepler's third law, we have

$$T_p = T_o(a^{3/2})^p \quad (3.9)$$

where  $T_o$  is the rotational period of the planet at the time when it attained the present radius (also see, Dermott, 1968a, b, 1973; Rawal, 1986).

It has been found that the Kepler's third law with the radial distance equal to the present radius of the planet with the whole mass supposed concentrated at the centre well approximates  $T_o$ . For Jupiter and Uranus, it turns out to be about 3 h, for Saturn about 5.1 h, for Neptune 2.6 h and for Mars it turns out to be 1.6 h. We can calculate various  $T_p$  at the corresponding  $R_p$  shown in Table I-V and also discuss the resonance relation with the help of these  $T_p$ .

### Acknowledgements

Thanks are due to Professor J. V. Narlikar and Professor S. M. Chitre of the Tata Institute of Fundamental Research, Bombay and Professor S. Ramadurai of the Indian Institute of Science, Bangalore for helpful discussions and useful suggestions.

### References

- Alfvén, H. and Arrhenius, G.: 1976, *Evolution of the Solar System*, NASA SP 345, U.S. Government Printing Office, Washington D.C. p. 4.
- Cameron, A. G. W.: 1978, in S. F. Dermott (ed.), *The Origin of the Solar System*, John Wiley, London, p. 49.
- Dermott, S. F.: 1968a, *Monthly Notices Roy. Astron. Soc.* **141**, 349.
- Dermott, S. F.: 1968b, *Monthly Notices Roy. Astron. Soc.* **141**, 363.
- Dermott, S. F.: 1973, *Nature* **244**, 18.
- Hourigan, K.: 1977, *Proc. Astr. Soc. Australia* **3**, 169.
- Hoyle, F.: 1960, *Quart. J. Roy. Astron. Soc.* **1**, 28.
- Innanen, K. A.: 1979, *Astr. J.* **84**, 96.
- Kuiper, G. P.: 1951, in J. A. Hynek (ed.), *Astrophysics*, McGraw-Hill, New York, Chap. 8.
- Prentice, A. J. R.: 1978a, in S. F. Dermott (ed.), *The Origin of the Solar System*. John Wiley, London, p. 111.
- Prentice, A. J. R.: 1978b, *The Moon and Planets* **19**, 341.
- Rawal, J. J.: 1984, *Earth, Moon and Planets* **31**, 175.
- Rawal, J. J.: 1986, *Earth, Moon and Planets* **34**, 93.
- Urey, H. C.: 1951, *Geochim. Cosmochim. Acta* **1**, 209.
- Whipple, F. L.: 1971, *Earth, Moon and Planets*, Harvard Univ. Press, Cambridge, Mass.