BACK-SCATTERING OF SUNLIGHT BY ICE GRAINS IN THE MESOSPHERE

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Abstract. Small dielectric ice particles of radius $\sim 0.25\ \mu$m, which are known to be present in the mesophere, scatter a fraction of incident sunlight in backward directions that do not reach the Earth. This back-scattered fraction is rigorously calculated using Mie theory for a uniform distribution of particles distributed over a sunlit hemisphere. Such calculations provide necessary information for estimating equilibrium surface temperatures of the Earth under different conditions.

1. Introduction

The presence of ice particles in the mesosphere, above an altitude of ~ 75 km is indicated both by observations of twilight and by data concerning the noctilucent clouds (Rosenburg, 1966; Bronshten and Grishin, 1975). Both sets of data point to dielectric grains of typical radius 2×10^{-5} cm and with mass loadings at the present day of 10^{12} to 10^{13} g for the whole Earth. Such particles scatter sunlight mainly in the forward direction, but a fraction that is scattered backwards will be lost and unavailable for heating the Earth. An accurate computation of this fraction is important for several reasons, not least of all for calculating the energy budget of the planet. In this paper we calculate the effects of back-scattering using Mie theory for spherical particles and performing appropriate integrations over a sunlit planetary hemisphere.

2. Phase Function and Forward Directivity of Scattering

The Mie formulae for spherical homogeneous particles lead to two complex amplitude functions

$$S_1(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \{a_n \pi_n(\cos\theta) + b_n \tau_n(\cos\theta)\}\,, \tag{1}$$

$$S_2(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \{b_n \pi_n(\cos\theta) + a_n \tau_n(\cos\theta)\}\,,$$

where π_n, τ_n are given in terms of the Legendre polynomial P_n by

$$\pi_n(\cos\theta) = P_n'(\cos\theta)\,, \tag{2}$$

TABLE I

The phase parameter for ice spheres with $m = 1.31$

x	$\langle\cos\theta\rangle$
0.2	0.0073
0.4	0.0289
0.6	0.0649
0.8	0.1157
1.0	0.1831
1.2	0.2703
1.4	0.3793
1.6	0.5020
1.8	0.6100
2.0	0.6707
2.2	0.6882
2.4	0.6966
2.6	0.7195
2.8	0.7548
3.0	0.7850
3.2	0.8004
3.4	0.8073
3.6	0.8142
3.8	0.8235
4.0	0.8326
4.2	0.8402
4.4	0.8476
4.6	0.8534
4.8	0.8555
5.0	0.8550

$$\tau_n(\cos\theta) = \cos\theta\,\pi_n(\cos\theta) - \sin^2\theta\,\frac{d}{d\cos\theta}\,\pi_n(\cos\theta)\,;$$

and a_n, b_n are the Mie coefficients which involve the Riccati–Bessel functions (Wickramasinghe, 1973). The total scattering amplitude function $S(\theta)$ at wavelength λ, where θ is the angle between a scattered ray and the direction of propagation of the incident beam, is given by

$$S(\theta) = \frac{1}{2}\left(\frac{\lambda}{2\pi}\right)^2 \{|S_1(\theta)|^2 + |S_2(\theta)|^2\}\,. \tag{3}$$

From Equation (3) the phase parameter $\langle\cos\theta\rangle$ can be calculated according to

$$\langle\cos\theta\rangle = \frac{\int_0^\pi S(\theta)\cos\theta\sin\theta\,\mathrm{d}\theta}{\int_0^\pi S(\theta)\sin\theta\,\mathrm{d}\theta}\,; \tag{4}$$

and for the case of ice particles of radius a with refractive index $m = 1.31$ this quantity is set out in Table I as a function of $x = 2\pi a/\lambda$. To determine the extent of forward-scattering around the direction of the propagation vector a more useful parameter to compute is $\theta_{1/2}$ given by

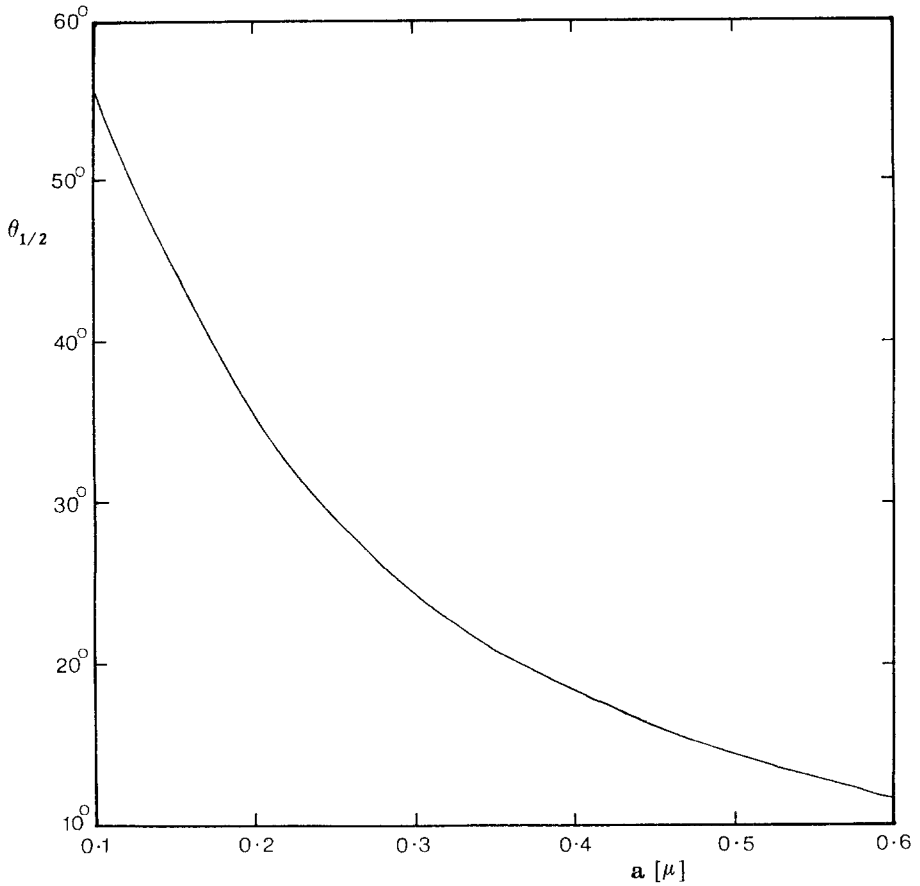

Fig. 1. The scattering angle which leads to an intensity equal to half that appropriate for the forward direction. The calculations are for $m = 1.31$ and $\lambda = 5000$ Å.

$$S(\theta_{1/2})/S(0) = 0.5\,. \tag{5}$$

Figure 1 shows the plot of $\theta_{1/2}$ as a function of particle radius for monochromatic light of wavelength 5000 Å. Particles of radii $a = 0.25\ \mu$m which have $\theta_{1/2} \simeq 30°$ are in general agreement with the requirements imposed by measurements on noctilucent clouds and on the light distribution of twilight.

3. Fraction of Scattered Sunlight Lost to the Earth

Consider a scattering particle at a point O on the Earth receiving sunlight from a direction that makes an angle ψ with the radius vector to the centre of the Earth (Fig. 2). If $\mathbf{k}$ denotes a unit vector along the direction EO, sunlight that is scattered into directions $\mathbf{a}$ such that $\mathbf{a} \cdot \mathbf{k} > 0$ will be lost to the Earth provided single scattering prevails. Let $b(\psi)$ be the fraction that is thus scattered. The average

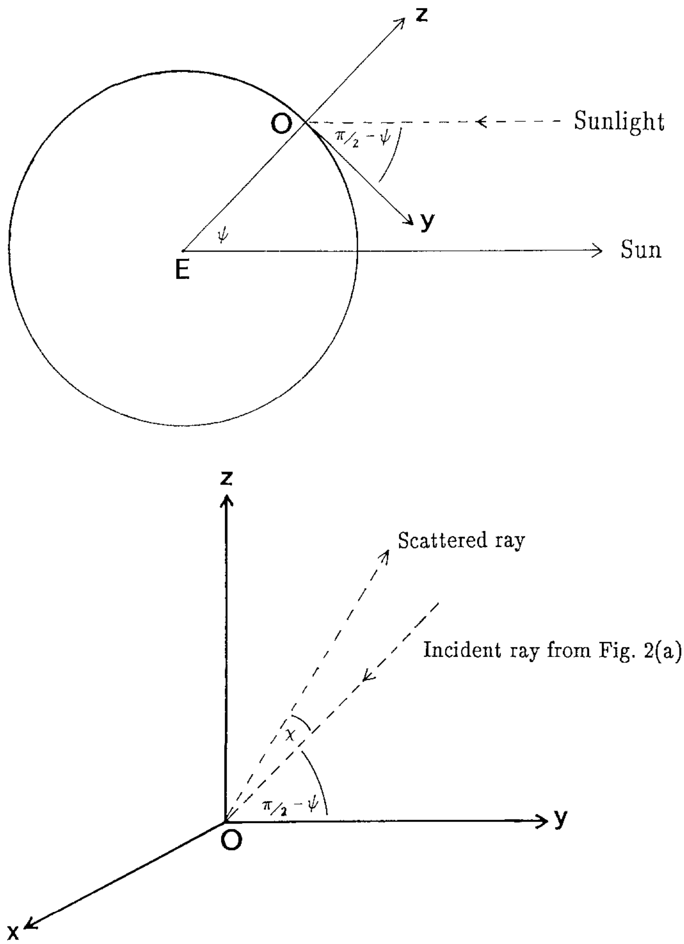

Fig. 2. (a, b) Geometry of Earth–Sun–scattering particle.

fraction lost from particles distributed uniformly over the sunlit hemisphere is clearly

$$f = \frac{2\pi \int_0^{\pi/2} \sin\psi \cos\psi b(\psi)\, \mathrm{d}\psi}{\pi}. \tag{6}$$

The problem of obtaining f reduces, therefore, to one of calculating the function $b(\psi)$ for a prescribed set of grain properties.

Figure 2(a) represents a plane through the centre of the Earth E, a scattering particle O and the direction of the Sun. Oy, Oz are axes in this plane. The right-handed set of axes is completed to define the axis Ox at O as is shown in Figure 2(b). The incident ray from Figure 2(a) is shown in Figure 2(b) as also is a scattered ray making angle χ with the incident ray. All such scattered rays are lost from the Earth for $\chi \leq \pi/2 - \psi$. The solid angle of this cone is

$$2\pi \int_{\pi/2+\psi}^{\pi} \sin\theta \, d\theta \,, \tag{7}$$

where θ is the scattering angle in the forward direction, i.e. $\theta = \pi - \chi$. Inserting $S(\theta)$, we find the fraction of the incident light scattered back in this cone to be

$$\int_{\pi/2+\psi}^{\pi} \sin\theta S(\theta) \, d\theta \Big/ \int_{0}^{\pi} \sin\theta S(\theta) \, d\theta \,. \tag{8}$$

For $\pi/2 + \psi \geqslant x \geqslant \pi/2 - \psi$ imagine the scattered ray to lie in the plane yOz. Then a point at unit distance along the scattered ray is at a height $\cos(\psi - \chi)$ above the xOy plane. Turning the scattered ray by angle λ about the incident ray lowers this point by $\sin\psi \sin\chi(1 - \cos\lambda)$. Thus for

$$\sin\psi \sin\chi(1 - \cos\lambda) = \cos(\psi - \chi) \,;$$

i.e., (9)

$$\lambda = \cos^{-1}(-\cot\psi \cot\chi) \,;$$

the point at unit distance along the scattered ray is lowered into the xOy plane; whence (9) gives the largest angle of turning at which the scattered ray is lost from the Earth. The solid angle for which scattered rays are lost from the Earth is therefore

$$2 \int_{\pi/2-\psi}^{\pi/2+\psi} \sin\chi \cos^{-1}(-\cot\chi \cot\psi) \, d\chi =$$

$$= 2 \int_{\pi/2-\psi}^{\pi/2+\psi} \sin\theta \cos^{-1}(\cot\theta \cot\psi) \, d\theta \,,$$

in which we have set $\theta = \pi - \chi$. Inserting $S(\theta)$ we obtain

$$\frac{1}{\pi} \int_{\pi/2-\psi}^{\pi/2+\psi} S(\theta) \sin\theta \cos^{-1}(\cot\theta \cot\psi) \, d\theta \Big/ \int_{0}^{\pi} S(\theta) \sin\theta \, d\theta \tag{10}$$

for the fraction lost.

For $\chi \geqslant \pi/2 + \psi$ there is again a cone in which the scattered fraction is

$$\int_{0}^{\pi/2-\psi} S(\theta) \sin\theta \, d\theta \Big/ \int_{0}^{\pi} \sin\theta S(\theta) \, d\theta \,. \tag{11}$$

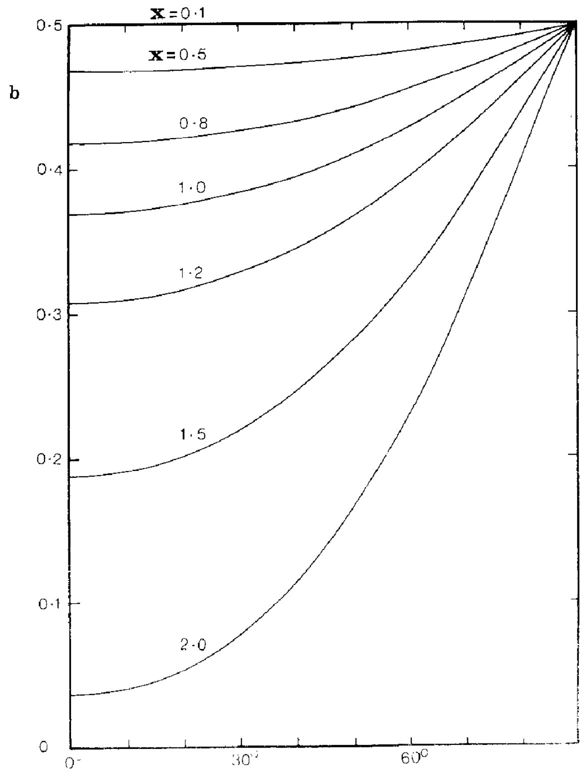

Fig. 3. Plot of the back-scattered fraction $b(\psi)$ as functions of ψ for various values of $x = 2\pi a/\lambda$.

But this is all below the plane xOy and comes through to the Earth. Hence the total scattered fraction is given by

$$b(\psi) = \left\{ \int_{\pi/2+\psi}^{\pi} S(\theta) \sin\theta \, d\theta + \right.$$

$$\left. + \frac{1}{\pi} \int_{\pi/2-\psi}^{\pi/2+\psi} \sin\theta \, S(\theta) \cos^{-1}(\cot\theta \cot\psi) \, d\theta \right\} \bigg/ \int_0^{\pi} S(\theta) \sin\theta \, d\theta. \tag{12}$$

Equations (1), (2), (3), (6–12) can now be used to calculate both $b(\psi)$ and f as functions of $x = 2\pi a/\lambda$ and the complex refractive index m. For the case of ice spheres with $m = 1.31$. Figure 3 shows the plots of $b(\psi)$ for values of x ranging from $x = 0.1$ to $x = 2$. Predominantly Rayleigh scattering particles are seen to be 50% back-scatters for all values of ψ, whereas for Mie scatterers with large x a 50% back-scatter is achieved only for particles illuminated near the Earth's limb at sunset or sunrise.

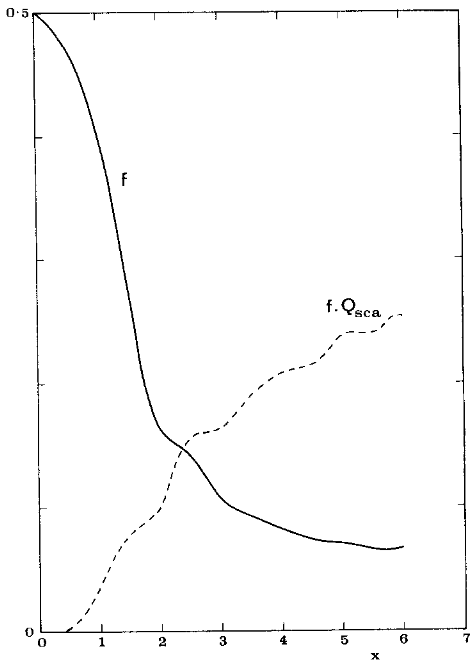

Fig. 4. The average back-scatter fraction f over the sunlight hemisphere as well as the function $f \cdot Q_{sca}$ for ice spheres.

Figure 4 shows the function $f(x)$ which gives the average back-scatter fraction over the entire sunlit hemisphere, and also the function $f \cdot Q_{sca}$. A back-scatter fraction of $f > 0.25$ is now seen to hold for x up to ~1.6. Table II sets out values of f for the cases $m = 1.16$, 1.31 and 1.6.

4. Integrated Solar Flux Excluded from the Earth

To evaluate the fraction of the solar flux lost to the Earth at all wavelengths it is necessary to compute the average

$$\langle fQ_{sca}\rangle = \int_0^\infty \frac{fQ_{sca}B_\lambda(5800°\text{K})\,d\lambda}{B_\lambda(5800°\text{K})\,d\lambda}, \tag{13}$$

and thence calculate a mass exclusion coefficient

$$\bar{\kappa} = \frac{\langle fQ_{sca}\rangle \pi a^2}{\frac{4}{3}\pi a^3 s}; \tag{14}$$

where s is the density of water–ice taken as 0.917 g cm^{-3}. The quantity $\bar{\kappa}$ is plotted in Figure 5 as a function of particle radius a. We note that $\bar{\kappa}$ has a broad maximum centred on the radius $a = 0.25\mu$, with a maximum value of $\bar{\kappa}$ close to 4225 cm^2 g^{-1}.

TABLE II

The function $f(x)$

x	$m = 1.16$	$m = 1.31$	$m = 1.6$
0.2	0.4964	0.4923	0.4918
0.4	0.4862	0.4815	0.4795
0.6	0.4692	0.4636	0.4594
0.8	0.4448	0.4384	0.4309
1.0	0.4129	0.4049	0.3917
1.2	0.3727	0.3615	0.3371
1.4	0.3243	0.3073	0.2656
1.6	0.2701	0.2463	0.2047
1.8	0.2165	0.1926	0.1956
2.0	0.1734	0.1657	0.2057
2.2	0.1469	0.1537	0.1950
2.4	0.1344	0.1494	0.1655
2.6	0.1274	0.1380	0.1462
2.8	0.1188	0.1203	0.1489
3.0	0.1070	0.1053	0.1598
3.2	0.0942	0.0975	0.1586
3.4	0.0837	0.0941	0.1441
3.6	0.0768	0.0906	0.1426
3.8	0.0722	0.0860	0.1602
4.0	0.0685	0.0816	0.1750
4.2	0.0648	0.0777	0.1649
4.4	0.0609	0.0740	0.1708
4.6	0.0571	0.0711	0.2014
4.8	0.0532	0.0700	0.2131
5.0	0.0497	0.0703	0.2112

In earlier papers we have discussed the possibility of an atmospheric mass loading amounting to $\sim 10^{12}$ g arising from either extraterrestrial dust accumulation or ice particles of terrestrial origin (Hoyle and Wickramasinghe, 1990; Wickramasinghe *et al.*, 1989). A total mass of $\xi\ 10^{12}$ g distributed over the Earth's area gives an average surface density

$$2 \times 10^{-7} \xi \text{ g cm}^{-2}.$$

For particles with $\bar{\kappa} = 4225 \text{ cm}^2 \text{ g}^{-1}$ the fraction of sunlight excluded from the Earth is then

$$\frac{\Delta F}{F} \cong -(1 - e^{-(4225 \times 2 \times 10^{-7} \xi)}) \simeq -8.4 \times 10^{-4} \xi . \tag{15}$$

If all other factors controlling the Earth's surface temperature remain unaltered, the balance of absorption of sunlight and re-emission gives the approximate relation

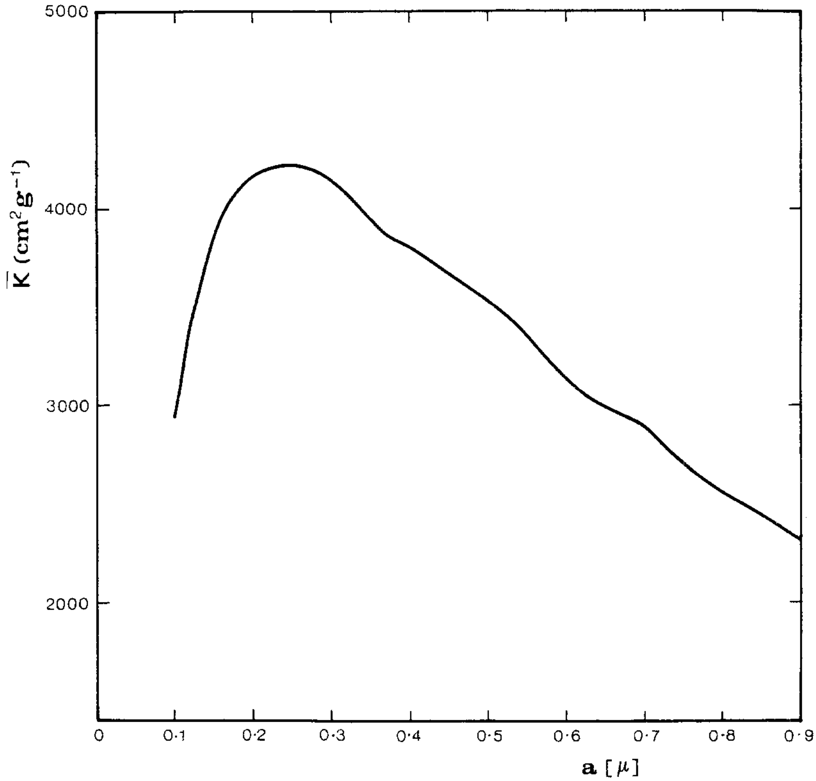

Fig. 5. The back-scatter mass coefficient averaged over the solar spectrum and the sunlit hemisphere, as a function of particle radius a.

$$\frac{\Delta F}{F} \cong 4T^3\frac{\Delta T}{T^4} = 4\frac{\Delta T}{T}. \tag{16}$$

Thus we have

$$\frac{\Delta T}{T} \cong \frac{1}{4}\frac{\Delta F}{F} \tag{17}$$

due to the scattering effect considered. Using (15) we obtain

$$\frac{\Delta T}{T} = -2.1 \times 10^{-4}\xi \tag{18}$$

with $\xi \simeq 10$, $T \cong 300$ °K we obtain a decrease of temperature of ~0.6°C, which is comparable to the upward temperature fluctuations due to the greenhouse effect being discussed at the present time. This is even without the inclusion of a forcing factor ~3 due to water vapour feedback effects that are used in greenhouse model calculations. With such a forcing factor and for ξ at the upper limit of particle loading in the high atmosphere the cooling is by ~2°C, which would be highly significant in discussions of worldwide climatology. The extra opacity which small particles provide in the infrared, working to increase the greenhouse, is a relatively small effect it may be noted. There is the possibility that ξ attained values of 10–10^2 at earlier epochs, as for instance during the so-called Little Ice Age.

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