LOCAL HARMONIC ANALYSIS OF PLANETARY DOPPLER GRAVITY DATA

A. W. GERHARD KUNZE

Department of Geology, University of Akron, Akron, Ohio U.S.A.

(Received 20 July, 1984)

Abstract. Planetary gravity fields represented in terms of spherical harmonics or surface mass distributions do not have the necessary resolution to permit gravity analysis of local features. Doppler gravity maps representing residual line-of-sight (LOS) accelerations have much greater resolution but cannot be used for conventional geophysical analysis due to the geometric distortions inherent in LOS gravity patterns and lack of normalization of LOS data. However, LOS gravity data may be converted to vertical gravity anomalies by expressing the anomalous local gravitational potential over small rectangular areas in terms of a modified double Fourier series constrained by local Doppler gravity data. The vertical derivative of the resulting potential yields the vertical gravity components at desired altitudes. The resolution of the resulting normalized free air anomaly maps is limited only by that of the original Doppler gravity data. Extended gravity maps may be constructed this way using a moving window approach. It is anticipated that much of the lunar frontside can be mapped at resolutions ranging from 1 to 4 deg of arc.

1. Introduction

Most of our knowledge of the gravity fields of the Moon, Mars and Venus is derived from Doppler ratio tracking data of orbiting spacecraft. Various methods of converting spacecraft tracking data to gravity field parameters are in use and are discussed by Muller and Sjogren (1968), Lorell (1970), Phillips (1974), Sjogren *et al.* (1976), Ferrari and Ananda (1977), and Phillips *et al.* (1978), among others.

Three basic types of gravity field representations have emerged:

(1) Global gravity field models, usually in terms of spherical harmonics, based on analysis of long period variations of the spacecraft mean orbital elements. This approach was used to construct the 16th order/degree lunar gravity field of Ferrari (1977) and Bills and Ferrari (1980), the 18th order/degree gravity field of Mars by Balmino *et al.* (1982), and the 6th and 7th degree/order gravity fields of Venus by Ananda *et al.* (1980) and Williams *et al.* (Reported by Sjogren, 1983). Unfortunately, the resolution of these global gravity models is too low (on the order of 20 degrees of arc in the case of the Moon and Mars) for detailed geophysical analysis. Theoretically the resolution of global spherical harmonic representations can be improved by including correspondingly higher order/degree terms; however, the coefficient matrices required to achieve local feature resolution become prohibitively large. The number of coefficients in a spherical harmonic expansion of degree N is equal to $(N + 3)N + 1$. Thus, to achieve a resolution of 1 degree of arc ($N = 360$), well over 100 000 coefficients need to be determined.

(2) Regional gravity models, generally in terms of surface mass distributions and/or corresponding vertical gravity components, usually based on dynamic reduction of

Earth, Moon, and Planets **32** (1985) 173–181. 0167–9295/85/0322–0173$01.35.

Doppler residuals through iterative adjustments of free gravity field model parameters utilizing orbit determination computer programs. Gravity models of this type include the lunar frontside gravity representations of Gottlieb (1970), Wong *et al.* (1971), and Sjogren (1974), and the Venus gravity models of Reasenberg *et al.* (1981, 1982) and Esposito *et al.* (1982). Again, the resolution of even the most detailed these models is too low (on the order of 200 km) for local geophysical analysis.

(3) Direct mapping of the anomalous line-of-sight (LOS) acceleration component, utilizing spline fitting and differentiation of Doppler residuals. The first of these so-called Doppler gravity representations was the now famous lunar mascon anomaly map constructed by Muller and Sjogren (1968). The resulting Doppler gravity representations have horizontal feature resolutsions of approximately twice the spacecraft altitude (Sjogren *et al.*, 1980). Thus, for the lowest Apollo orbits analysed, the optimum resolutions are on the order of 30 km (e.g. Gottlieb *et al.*, 1970; Sjogren *et al.*, 1972a, b; Sjogren *et al.*, 1974a, b; Muller *et al.*, 1974). In addition to these, there are numerous other Doppler gravity representations of lower resolution published for the moon, Mars and Venus by the same and other investigators.

In view of the relatively low resolution of planetary gravity field representations utilizing spherical harmonics or surface mass distributions, most local geological and geophysical interpretations have been based on the more detailed raw LOS gravity data available for many regions of the Moon, Mars and Venus. Unfortunately, two basic properties of LOS gravity representations preclude conventional, precise geophysical analysis: LOS gravity data on a given map are not normalized to a common reference level, and LOS gravity anomalies do not represent conventional, vertical gravity patterns except near the planetary sub-earth point (SEP). Elsewhere they suffer from the various geometric distortions discussed by Kane (1969). For example, the LOS gravity anomaly of a surface point mass m is given by the expression

$$\Delta g_{\mathrm{LOS}} = \frac{Gm}{r^3}\left[h\cos\delta - \sin\delta(\Delta x\sin\alpha + \Delta y\cos\alpha)\right], \tag{1}$$

where h is the spacecraft altitude, Δx and Δy are the spacecraft coordinates with respect to the point mass position, r is the straight line spacecraft – point mass distance ($\sqrt{h^2 + \Delta x^2 + \Delta y^2}$), δ is the LOS deflection from the normal, and α is the LOS azimuth. Figure 1 shows the resulting LOS anomaly (normalized to its maximum possible value) at

(a) the sub-earth point (taken as 0° latitude and longitude);
(b) 15° N, 15° E, corresponding to a LOS deflection of 21° and LOS azimuth of 046°;
(c) 30 E, corresponding to a LOS deflection of 41° and LOS azimuth of 049°;
(d) 45° N, 45° E, corresponding to a LOS deflection of 60° and LOS azimuth of 055°;
(e) 60° N, 60° E, corresponding to a LOS deflection of 76° and LOS azimuth of 063°.

Figure 1 illustrates the major LOS anomaly modifications:

(1) A decrease in peak amplitude.

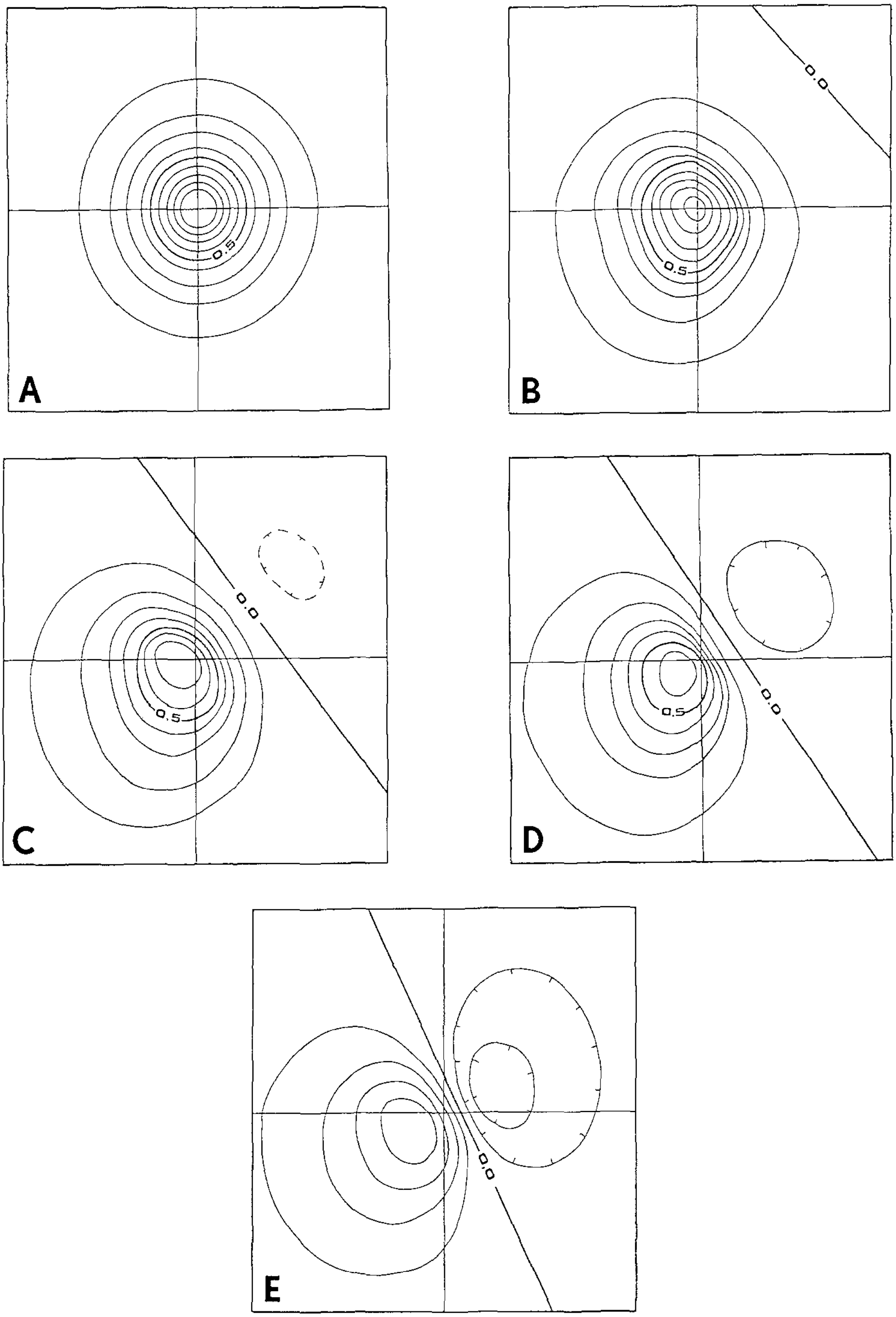

Fig. 1. Point mass LOS gravity anomalies at: (A) SEP (vertical anomaly with peak value of 1.00); (B) 15° N, 15° E (peak value 0.93); (C) 30° N, 30° E (peak value 0.75); (D) 45° N, 45° E (peak value 0.61); (E) 60° N, 60° E (peak value 0.49).

(2) A shift in peak position away from the anomaly source.
(3) The appearance of spurious anomalies of opposite sign.

These modifications clearly rule out conventional geophysical modelling techniques. To apply such techniques, raw LOS gravity data must first be converted to vertical gravity components and normalized to a common altitude. This may be accomplished using the approach discussed below.

2. Fourier Gravity Representation

The planetary gravity potential on a global scale is best expressed in terms of spherically harmonic functions. However, the gravitational potential over a small rectangular or square planetary surface area of negligible curvature may be expressed with much greater resolution in terms of a modified double Fourier series whose coefficients are determined through the constraints provided by the LOS acceleration data above that area. The vertical derivative of the resulting gravitational potential expression provides the vertical gravity pattern at desired altitudes.

The theoretical basis of this approach is Laplace's equation which must be satisfied by the gravitational potential V in free space. A proper solution to Laplace's equation in Cartesian (x, y, z) coordinates is of the form

$$V = -\sum_m \sum_n (A_{mn} \sin mx \sin ny + B_{mn} \sin mx \cos ny + \\ + C_{mn} \cos mx \sin ny + D_{mn} \cos mx \cos ny) \exp(-\sqrt{m^2 + n^2}\, z). \tag{2}$$

The directional derivative of V in the LOS direction yields the Doppler gravity component and may be expressed as

$$\frac{\mathrm{d}V}{\mathrm{d}l} = \mathbf{a} \cdot \nabla V, \tag{3}$$

where $\mathrm{d}l$ is the virtual displacement in the LOS direction and $\mathbf{a}$ the unit vector in the LOS direction. $\mathbf{a}$ may be expressed in terms of the Cartesian unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ and the LOS direction cosines q, r, s (Figure 2a) as

$$\mathbf{a} = q\mathbf{i} + r\mathbf{j} + s\mathbf{k}. \tag{4}$$

Normally, the LOS direction is most easily expressed as a deflection from the normal δ and a local azimuth α (Figure 2b). In terms of these quantities

$$\mathbf{a} = \sin\delta \sin\alpha\, \mathbf{i} + \sin\delta \cos\alpha\, \mathbf{j} - \cos\delta\, \mathbf{k}. \tag{5}$$

Writing ∇V as $(\partial V/\partial x)\mathbf{i} + (\partial V/\partial y)\mathbf{j} + (\partial V/\partial z)\mathbf{k}$, we now evaluate Equation (3) as

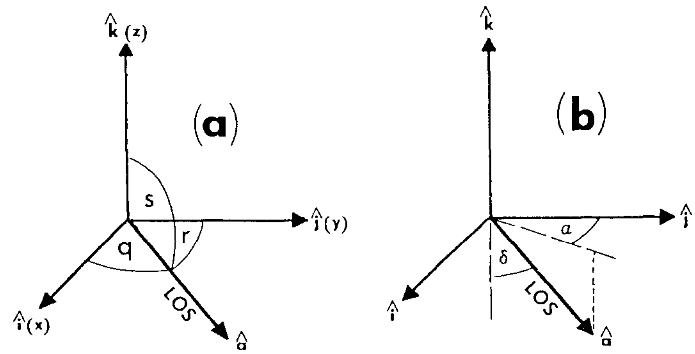

Fig. 2. LOS geometry in terms of (a) direction cosines q, r, s; (b) LOS deflection δ and azimuth α.

$$\frac{\mathrm{d}V}{\mathrm{d}l} = \frac{\partial V}{\partial x}\sin\delta\sin\alpha + \frac{\partial V}{\partial y}\sin\delta\cos\alpha - \frac{\partial V}{\partial x}\cos\delta. \tag{6}$$

Taking partial derivatives of V (Equation (2)) with respect to x, y, and z, and substituting the resulting expressions into Equation (6) we obtain the working equation for the proposed local gravity analysis in the form

$$\begin{aligned}
\frac{\mathrm{d}V}{\mathrm{d}l} = & -\sum_{mn}\sum\{A_{mn}[m\cos mx\sin ny\sin\delta\sin\alpha + n\sin mx\cos ny \times \\
& \times \sin\delta\cos\alpha + \sqrt{m^2+n^2}\sin mx\sin ny\cos\delta] + \\
& + B_{mn}[m\cos mx\cos ny\sin\delta\sin\alpha - n\sin mx\sin ny \times \\
& \times \sin\delta\cos\alpha + \sqrt{m^2+n^2}\sin mx\cos ny\cos\delta] - \\
& - C_{mn}[m\sin mx\sin ny\sin\delta\sin\alpha - n\cos mx\cos ny \times \\
& \times \sin\delta\cos\alpha - \sqrt{m^2+n^2}\cos mx\sin ny\cos\delta] - \\
& - D_{mn}[m\sin mx\cos ny\sin\delta\sin\alpha + n\cos mx\sin ny \times \\
& \times \sin\delta\cos\alpha - \sqrt{m^2+n^2}\cos mx\cos ny\cos\delta]\}\exp \\
& (-\sqrt{m^2+n^2}z).
\end{aligned} \tag{7}$$

This equation is used to determine the unknown coefficients A_{mn}, B_{mn}, C_{mn}, D_{mn} defining the local anomalous gravitational potential within a square surface area from the Doppler gravity values $\mathrm{d}V/\mathrm{d}l$ in that area. The vertical gravity components Δg_z in the area are then calculated at desired coordinates (x, y, z) from the expression

$$\begin{aligned}
\Delta g_z = -\frac{\partial V}{\partial z} = & -\sum_{mn}\sum\sqrt{m^2+n^2}(A_{mn}\sin mx\sin ny + \\
& + B_{mn}\sin mx\cos ny + C_{mn}\cos mx\sin ny + \\
& + D_{mn}\cos mx\cos ny)\exp(-\sqrt{m^2+n^2}z).
\end{aligned} \tag{8}$$

In the actual application of Equations (7) and (8), it is necessary to replace x, y, and z by $2\pi x/L$, $2\pi y/L$, and $2\pi z/L$, respectively, where L is the fundamental Fourier wavelength of a square local area.

3. Application and Discussion

Equation (7) represents the known LOS gravity field over a small area of the planet. Hence, the only unkown quantities in Equation (7) are the Fourier coefficients A_{mn}, B_{mn}, C_{mn}, and D_{mn}. Each known Doppler gravity value $\mathrm{d}V/\mathrm{d}l$ is associated with known planetary coordinates (x, y, z) and a known time of observation. The LOS deflection from the normal (δ) and local azimuth (α) at each point of observation are functions of the local coordinates (x, y) and of the sub-earth point coordinates (x_0, y_0) and are determined through the spherical laws of cosines and sines as

$$\delta = \arccos [\sin y_0 \sin y + \cos y_0 \cos y \cos (x - x_0)], \tag{9a}$$

$$\alpha = \arcsin [\cos y_0 \sin (x - x_0)/\sin \delta] \text{ (Observer } N \text{ of SEP)}, \tag{9b}$$

$$\alpha = \pi - \arcsin [\cos y_0 \sin (x - x_0)/\sin \delta] \text{ (Observer } S \text{ of SEP)}; \tag{9c}$$

where x and y are longitude and latitude, respectively. Planetary sub-earth point coordinates are published for each time of observation in the Astronomical Almanac/American Ephemeris. Hence, a knowledge of N Doppler gravity values at known coordinates and times in the local region under investigation permits, in principle, the construction of a system of N equations from Equation (7) and the calculation of up to N harmonic coefficients. The resolution of the corresponding vertical gravity field is similar to that of the original Doppler gravity data. Thus for the lunar frontside, an optimum resolution of 30 km (1 deg of arc) may be realized. The number of Fourier coefficients required to achieve this resolution depends on the dimensions of the area being analysed. For a double Fourier series of order N of the type presented in Equation (8), the number of non-zero coefficients is $4N (N + 1)$. If the local area in question has dimensions $10^\circ \times 10^\circ$ and a resolution of 1° is sought, then a 10th order double Fourier series with 440 coefficients is required. The average number of available Doppler gravity data per $10^\circ \times 10^\circ$ region of the lunar frontside is estimated to be on the order of 1000 – well in excess of the minimum number required.

Equations (7) and (8) do not permit representation of a DC level in the LOS or vertical gravity pattern because the inversion matrix elements of the corresponding harmonic coefficients $(A_{00}, B_{00}, C_{00}, D_{00})$ as well as certain others $(A_{0n}, B_{0n}, A_{m0}, C_{m0})$ vanish. To accommodate a possible DC level in the data, it is necessary to choose the fundamental Fourier wavelength L to be greater than the dimensions of the study area.

This proposed LOS gravity conversion method was tested with the synthetic point mass LOS anomalies presented earlier (Figure 1). The LOS anomalies were sampled at 48 points and a 3rd order Fourier Series solution with 48 non-zero harmonic coefficients was obtained for each case. The resulting vertical gravity anomalies are shown in Figure 3.

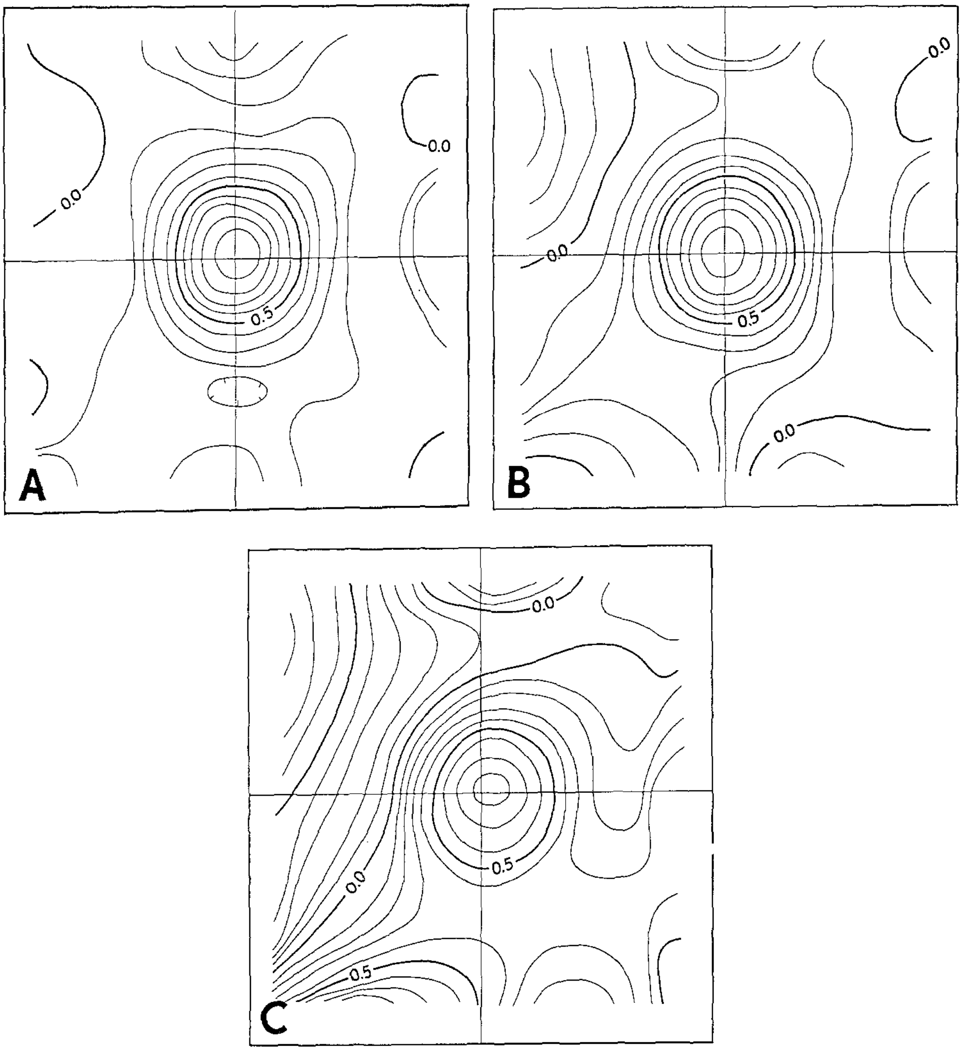

Fig. 3. Results of LOS gravity conversion. Point mass vertical gravity anomaly at: (A) 15° N, 15° E (peak value 0.99); (B) 30° N, 30° E (peak value 0.97); (C) 45° N, 45° E (peak value 0.81).

In all cases shown the initial vertical point mass anomaly with its characteristic half width and maximum value is largely restored in its proper, centered position although with increasing distance from the sub-earth point the results are progressively disturbed by short wavelength noise and an increasingly ragged edge effect. Beyond the 60° of arc distance from the sub-earth point corresponding to Figure 3c the results are severely degraded and are not shown. Apparently the recovery of vertical gravity from largely horizontal LOS components is problematic if not impossible.

The short wavelength noise is primarily the result of the oscillatory nature of the Fourier series. It precludes meaningful downward continuation. Hence, as expected, the resolution of the resulting vertical anomaly map cannot exceed that of the constraining Doppler data.

4. Conclusions and Summary

A modified double Fourier series representation of the anomalous planetary gravitational potential may be used to generate local vertical gravity anomaly maps from local LOS gravity constraints. This method produces acceptable results out to an arc distance of approximately 60° from the planetary sub-earth point.

Thus, free air gravity anomaly maps of greatly improved resolution may be constructed in many regions of Mars, Venus and the Moon with adequate LOS gravity coverage. The resolution of such vertical gravity anomaly maps depends on the resolution of the constraining Doppler gravity data. Optimum expected resolution is on the order of 1, 3, and 10 deg of arc for the lunar frontside, Venus and Mars respectively. Much of the lunar frontside may be mappable at resolutions ranging from 1 to 4 deg of arc.

In addition, extended free air anomaly maps may be constructed from a composite of overlapping local maps. Such a moving window approach has the effect of an inefficient high pass filter by eliminating wavelengths significantly greater than the window dimensions. This, however, will not impair the utility of the resulting regional maps for the analysis of local features.

Acknowledgements

This investigation was supported by the University of Akron Summer Research Fellowship Grant 778/1983.

References

Ananda, M. P., Sjogren, W. L., Phillips, R. J., Wimberly, R. N., and Bills, B. G.: 1980, *J. Geophys. Res.* **85**, 8303.

Balmino, G., Moynot, B., and Vales, N.: 1982, *J. Geophys. Res.* **87**, 9735.

Bills, B. G. and Ferrari, A. J.: 1980, *J. Geophys. Res.* **85**, 1013.

Esposito, P. B., Sjogren, W. L., Mottinger, N. A., Bill,s B. G., and Abbott, E.: 1982, *Icarus* **51**, 443.

Ferrari, A. J.: 1977, *J. Geophys. Res.* **82**, 3065.

Ferrari, A. J. and Ananda, M. P.: 1977, *J. Geophys. Res.* **82**, 3085.

Gottlieb, P.: 1970, *Radio Sci.* **5**, 301.

Gottlieb, P., Muller, P. M., Sjogren, W. L. and Wollenhaupt, W. R.: 1970, *Science* **168**, 477.

Kane, M. F.: 1969, *J. Geophys. Res.* **74**, 6579.

Lorell, J.: 1970, *The Moon* **1**, 190.

Muller, P. M. and Sjogren, W. L.: 1968, *Science* **161**, 680.

Muller, P. M., Sjogren, W. L., and Wollenhaupt, W. R.: 1974, *The Moon* **10**, 195–205.

Phillips, R. J., Sjogren, W. L., Abbott, E. A., and Zisk, S. H.: 1978, *J. Geophys. Res.* **83**, 5455.

Reasenberg, R. D., Goldberg, Z. M., MacNeil, P. E., and Shapiro, I. I.: 1981, *J. Geophys. Res.* **86**, 7173.

Reasenberg, R. D., Goldberg, Z. M. and Shapiro, I. I.: 1982, *Geophys. Res. Let.* **9**, 637.
Sjogren, W. L.: 1974, *Geochimica et Cosmochimica Acta,* Supplement 5, Frontispiece.
Sjogren, W. L.: 1983, *Rev. Geophys. Space Phys.* **21**, 528.
Sjogren, W. L., Gottlieb, P., Muller, P. M., and Wollenhaupt, W. R.: 1972a, *Science* **175**, 165.
Sjogren, W. L., Muller, P. M., and Wollenhaupt, W. R.: 1972b, *The Moon* **4**, 411.
Sjogren, W. L., Wimberly, R. N., and Wollenhaupt, W. R.: 1974, *The Moon,* **9**, 115.
Sjogren, W. L., Wimberly, R. N. and Wollenhaupt, W. R.: 1974, *The Moon,* **11**, 41.
Sjogren, W. L., Anderson, J. D., Phillips, R. J., and Trask, D. W.: 1976, *IEEE Trans, Geosci Electron,* **GE-14**, 172.
Sjogren, W. L., Phillips, R. J., Birkeland, P. W., and Wimberly, R. N.: 1980, *J. Geophys. Res.* **85**, 8295.
Wong, L., Buechler, G., Downs, W., Sjogren, W., Muller, P., and Gottlieb, P.: 1971, *J. Geophys. Res.* **76**, 6220.